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GROUP INVESTING

**QWAFEFW BOSTON
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REFERENCE: *GROUP INVESTING*, WITH STEVE SATCHELL, JPM 2024



A SURPRISING CONTRADICTION

- Investment portfolios with multiple stakeholders are all around us, yet ...
- Guidance on optimal group investing is hard to find.



WHAT WOULD YOU DO?

- Richard and Linda, with no children and minimal unrealized gains and losses, have come to you for advice on stock versus bond allocation:
 - Joint investment account \$900,000
 - Richard's IRA \$50,000
 - Linda's IRA \$50,000
- What additional information do you need?
- How would you use it?



HAZARDS FOR RICHARD AND LINDA

- Differences in group member situations and goals
 - Appropriate risk aversion
 - Non-investment risks to future saving and funds needed
- Differences in returns to be considered:
 - Nominal versus real after inflation
 - Tax rates to be applied
 - Time between reallocations
- Network interdependencies – weights given to each group member and to each of their portfolio stakes.



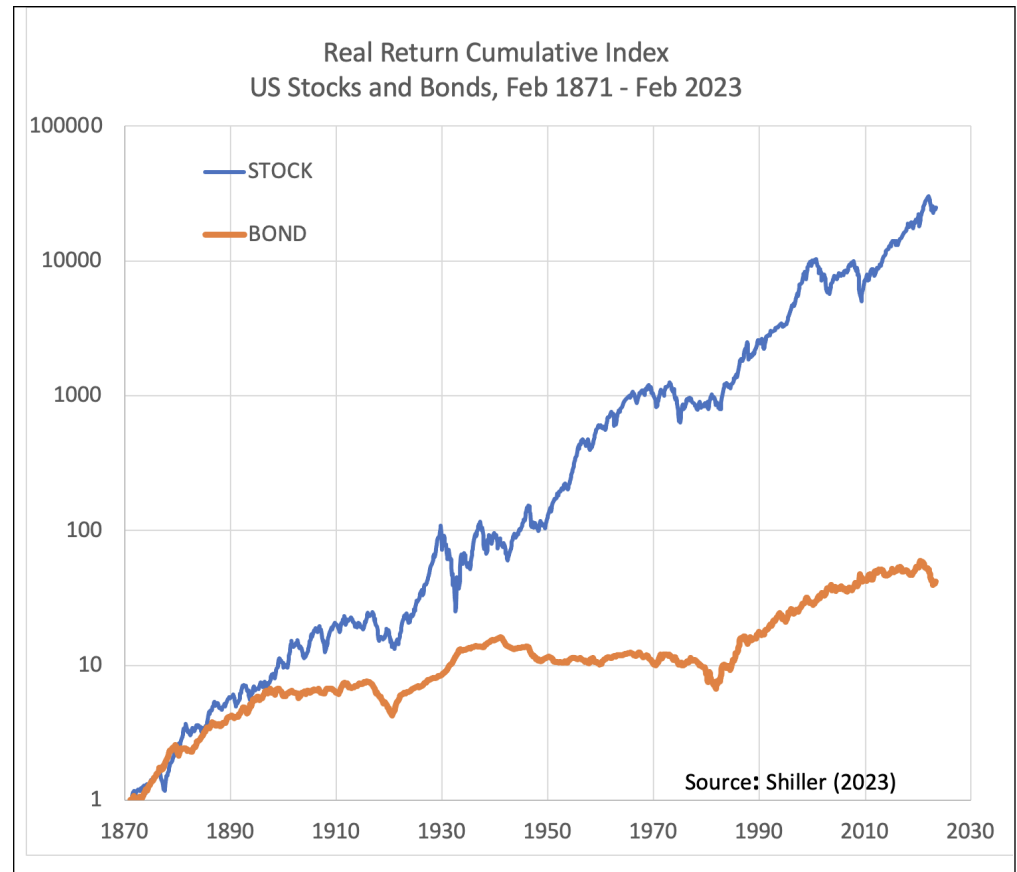
GOAL-BASED INVESTING CHANGES THE RISK PARADIGM

- Appropriate risk aversion becomes partially dependent on the probability of failing to have sufficient funds.
- It connects risk aversion with financial planning surplus:
 - more objective than attitude assessment
 - explains contingent risk aversion
 - reveals investment role of non-investment funding risks.



INFLATION

- Most goals require returns to be net of inflation
 - To prevent mistakes
 - To promote hedges against inflation risk
- Exceptions:
 - Paying off debt
 - Committing to buy at a fixed future price



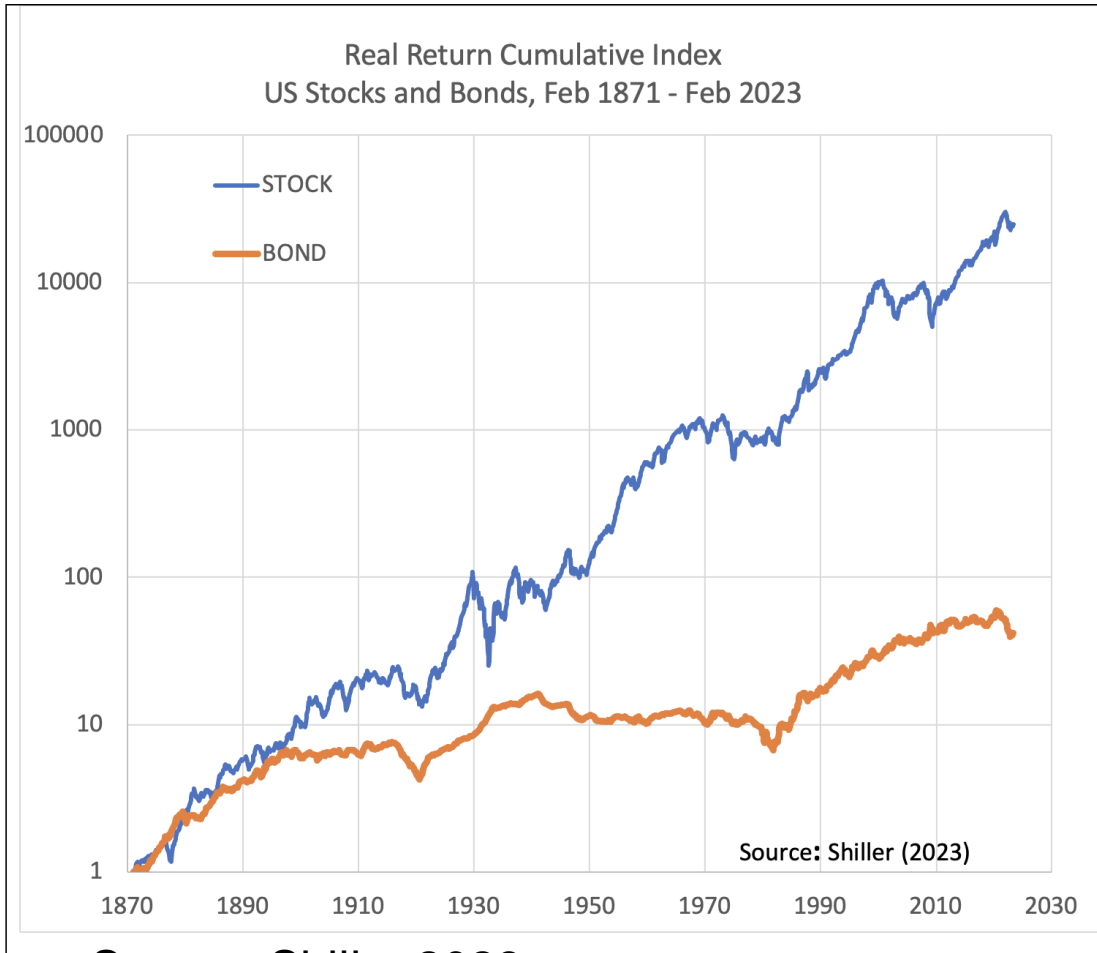
Source: Shiller 2023.

TAX IMPACT

1. Direct effects → 1. $R(1-\tau)$
 2. Higher tax rate → 2. $(r + i)^*(1-\tau) - i$
investors suffer more
from inflation
 3. Higher tax rates reduce
variance more than
means → 3. Mean = $E[R] * (1-\tau)$
Variance = $V(R) * (1-\tau)^2$
 4. Long-term deferral
reduces effective tax
rate τ^* for high return
assets → 4. $(1+R(1-\tau^*))^n = (1+R)^n * (1-\tau) + \tau$
- Where:
R: pretax return
r: real return
 τ : nominal tax rate
 τ^* : effective tax rate
i: inflation rate

REALLOCATION FREQUENCY

- Irrelevant, if decisions were based on CRRA utility.
- Instead, goal-based investing tilts long-term lock-ins toward stocks.
- Implicit dynamic hedging can also tilt high-frequency decisions toward stocks.



Source: Shiller 2023.

WHY THE GAP BETWEEN PRACTICAL IMPORTANCE AND RESEARCH?

- The group investing problem is too complex to optimize?
- Social fairness issues distract economists?
- Practitioners with quantitative skills:
 - start from the mean-variance paradigm?
 - are distracted by benchmark relative incentives
 - groups are assumed to have homogeneous members?

WHAT DO YOU THINK?



A SIMPLIFIED APPROACH

1. Discrete probability distributions of returns ...
 - transparent, granular detail
 - easily modified views to suit each group member.
2. Expected utilities appropriate for goal-based investing
3. Network of pooled and personal portfolios with exogeneous weights
4. Strict convex optimization problem definition



GOAL-ORIENTED UTILITY

LIFE BALANCE SHEET

ASSETS		LIABILITIES & DISCRETIONARY	
Investments	\$ 250,000	Investment Debt	\$ 0
Other Assets	\$ 100,000	Other Debt	\$ 0
Committed Saving	\$ 660,393	Committed Withdrawals	\$ 994,616
		Discretionary Wealth	\$ 15,777
TOTAL	\$ 1,010,393	TOTAL	\$ 1,010,393
		Investment Leverage on Discretionary Wealth	15.8

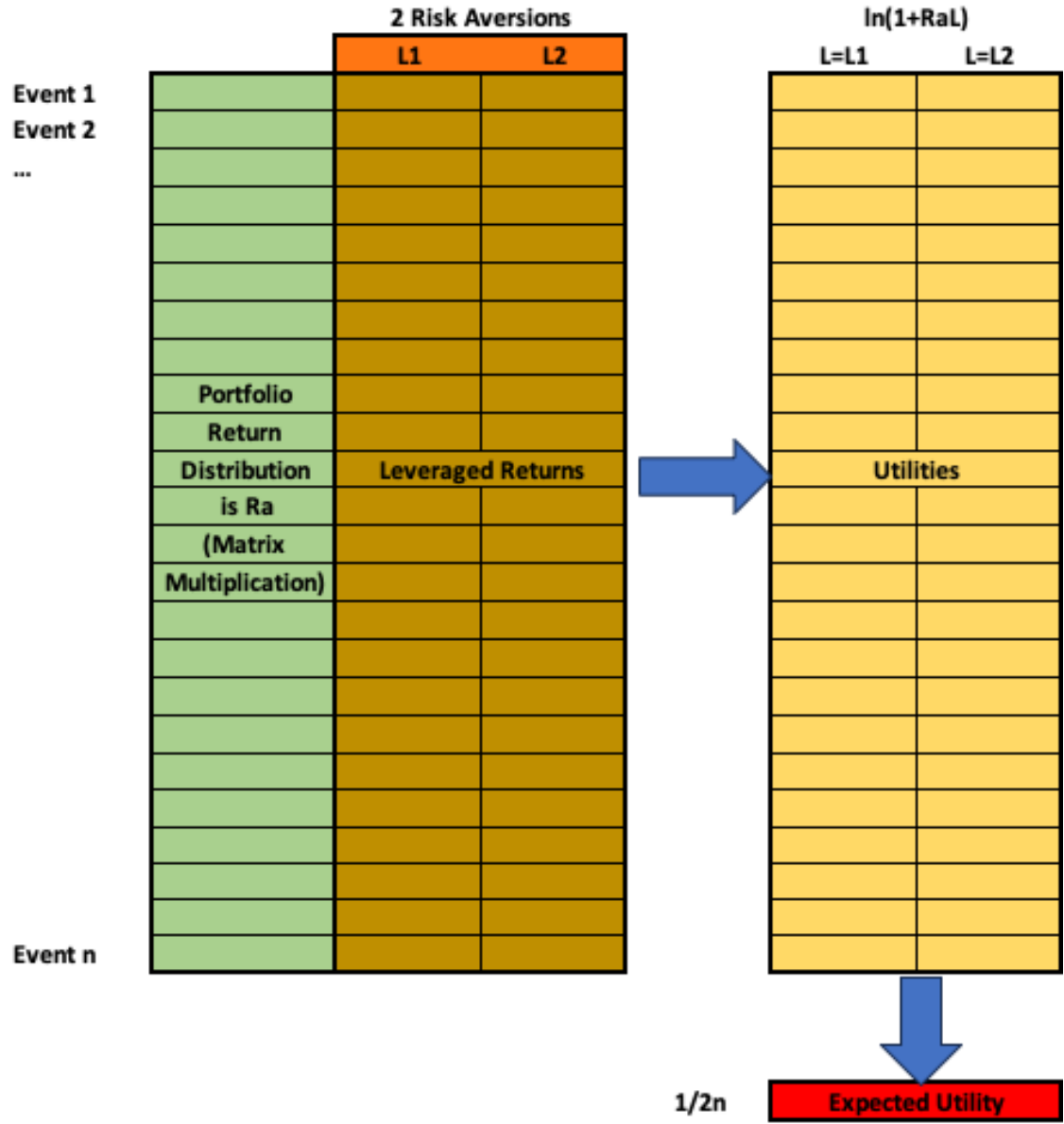
$L = 250,000 / 15,777 = 15.8$, Wilcox (2003)

Rubinstein Utility = $\ln(1+Lr)$, Rubinstein (1976)

An event $r = -1/L$ or -6.3% results in utility of minus infinity.

INDIVIDUAL EXPECTED UTILITY

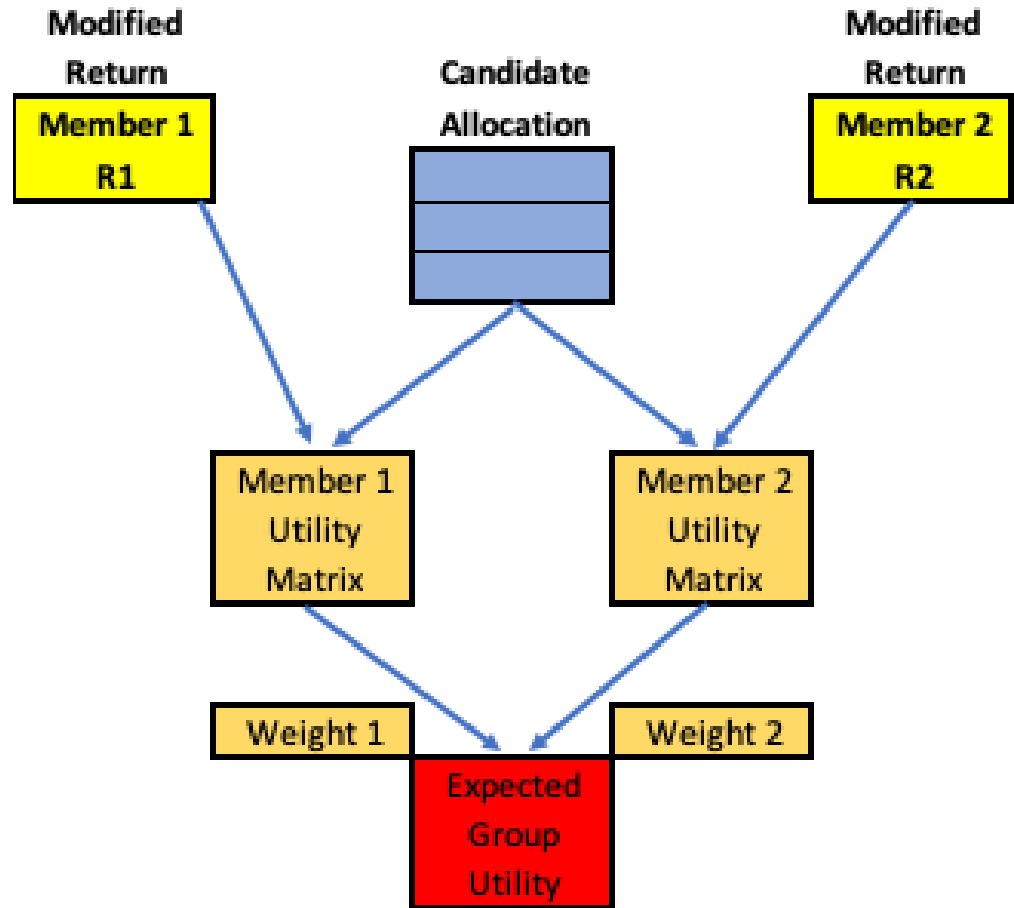
For each group member, we generate utilities from a customized view of portfolio return distribution and appropriate risk aversion distribution.



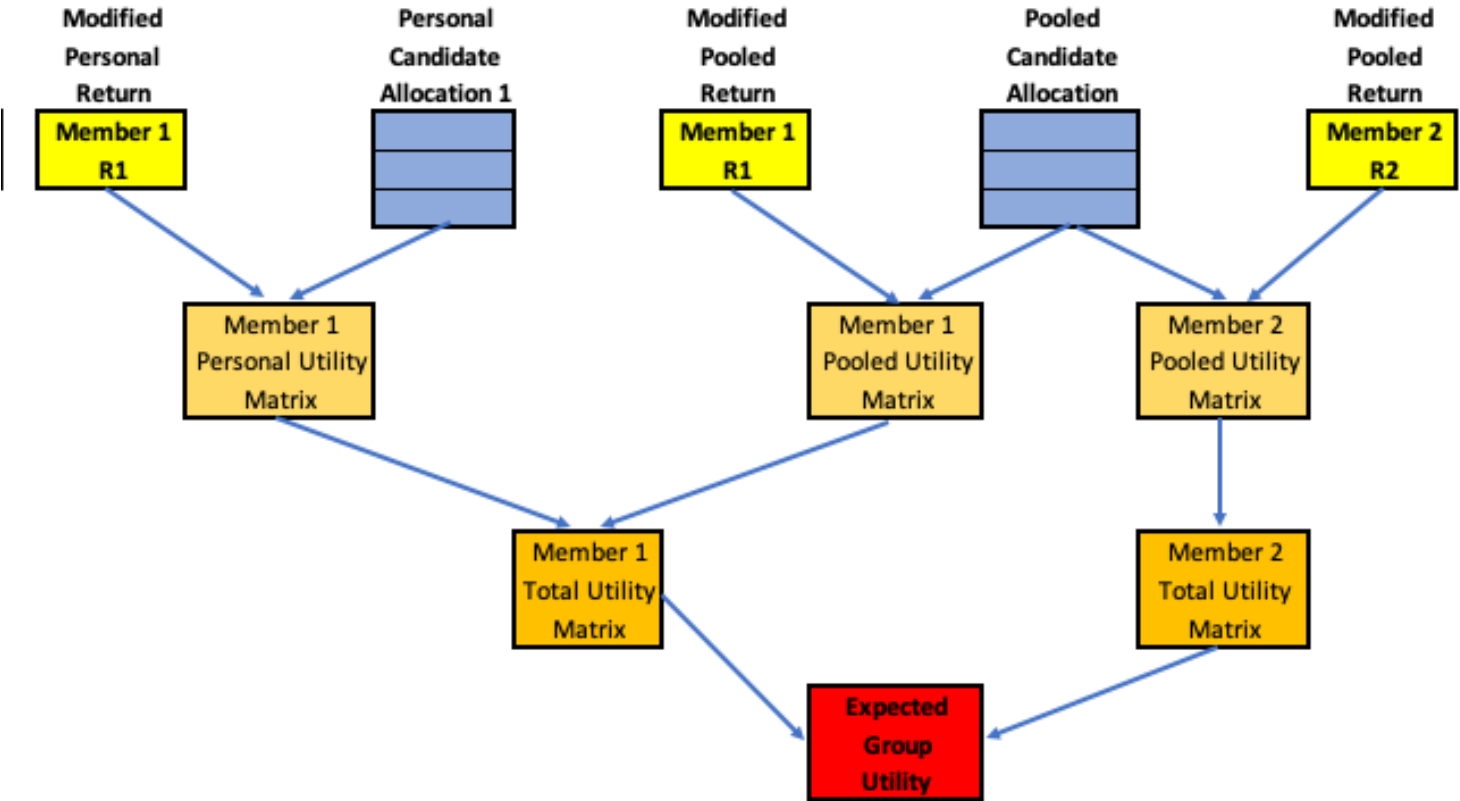
POOLED EXPECTED UTILITY

The same candidate allocation is evaluated differently for each group member.

The resulting expected utilities are weighted with weights summing to 1 to obtain the expected group utility.



ADDING A PERSONAL PORTFOLIO



CONVEX OPTIMIZATION PROBLEM

- *Choice Variables:*

$a_i \in \text{Real}^M$: individual asset allocations for person i , $i \in \{1, \dots, N\}$

$a_g \in \text{Real}^M$: pooled asset allocation

- *Objective:*

Maximize $E[U] = E_w[d_i \odot U_i]$ by searching among choices for a_g and a_i where

$$U_i = E_p[\log 1p (v_{i,pooled} \odot R_i a_g L_i + v_{i,own} \odot R_i a_i L_i)]$$

- *Subject to:*

a_i, a_g are each vectors containing only non-negative real numbers

$$\text{sum}(a_i) = \text{sum}(a_g) = 1$$



DIVERGENT RISK AVERSION

- Appropriate risk aversion is the most obvious divergence factor among group members.
- These percentage allocations are based on 150 years of monthly US history.

Real Returns after Taxes
Monthly Decisions
Deflated After-Tax S=25%, B=40%

		Richard	Linda
	<u>L=2</u>	<u>L=4</u>	<u>L=8</u>
STOCK	100	77.5	33.2
BOND	0	22.5	66.8

POOLED PORTFOLIO COMPROMISE

- Here, equal weighting on utility functions does not imply equal movement toward compromise allocations.

Compromise Favors the Steepest Utility Slope

Allocation Interval = 1 month

	POOLED	SEPARATE	
		Richard L=4	Linda L=8
STOCK	38.3	77.5	33.2
BOND	61.7	22.5	66.8

- The pooled fund allocation is equally uncomfortable for both.

EMERGENT COMPENSATION BEHAVIOR

- Networks often provide surprising emergent behavior.
- In this case, the the IRA allocations compensate to reduce stress.

Allocation Interval = 1 month

	TAXABLE	IRA PLANS	
	Pooled	Richard L=4	Linda L=8
STOCK	41.4	100	0
BOND	58.6	0	100

CONCLUSION

- Group investing optimization appears surprisingly practical.
- Diverse group member pooling causes sacrifice, but it can be mitigated by including compensatory personal portfolios.
- Best use case at this stage: strategic asset allocation targets.



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