Risk Parity with Uncertain Risk Contributions

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See paper for details and math

Shah, A. (2019). Uncertain Risk Parity
http://ssrn.com/abstract=3406321

Link to Python code (coming)

Criticism and questions:
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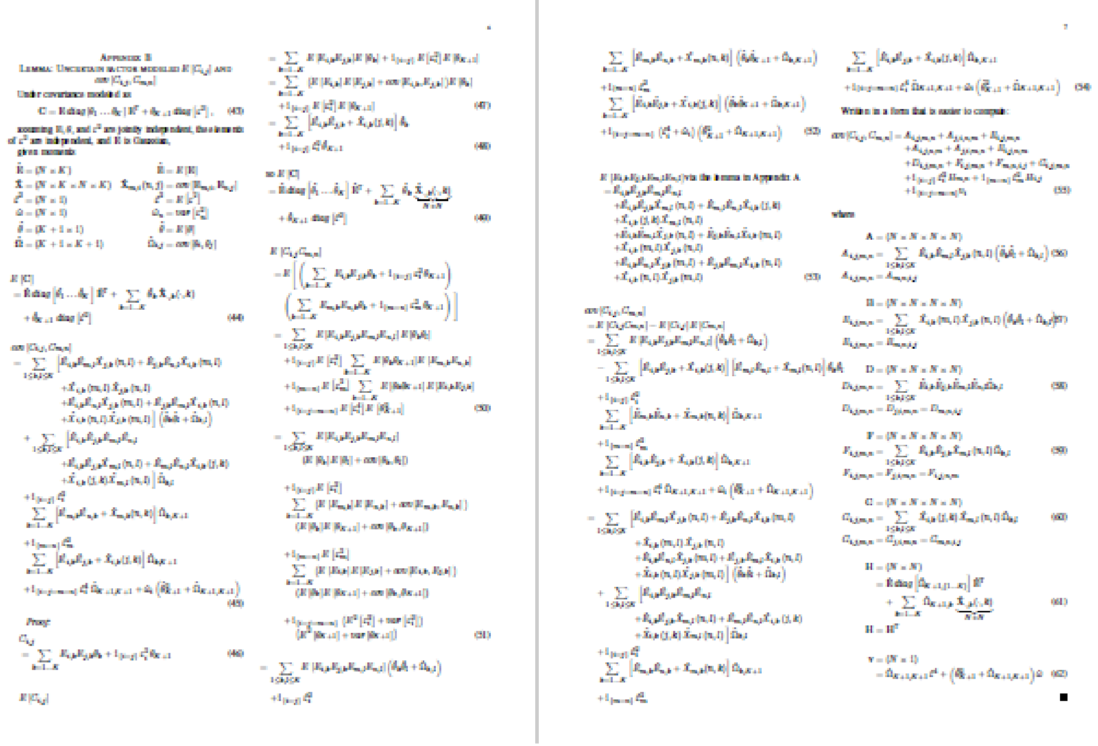


Risk parity

Qian, E. (2005). Risk parity portfolios: Efficient portfolios through true diversification

- Almost always at asset class level – stocks, levered bonds, commodities, currencies, …

Finds weights where **each investment contributes its targeted fraction of portfolio risk**

- To normalize, fix risk of portfolio or force weights to sum to 1

Motivations

- Get diversified exposure to risk premium
- Prudent approach to an uncertain future. If covariance is diagonal, weights by inverse of std dev
- Robust way of doing minimum variance (4 slides ahead)

Risk contributions

Risk can be reported in both variance and standard deviation

- For contributions, it makes no difference – an investment's fraction of total risk is the same either way
- Let **w** = (n × 1) investment weights, **C** = (n × n) covariance matrix

In variance

- $\sigma^2(\mathbf{w})$ = variance of portfolio **w** = $\sum_i w_i \sum_j C_{i,j} w_j = \sum_i v_i$ where v_i = investment i's var contribution

In standard deviation

- $\sigma(\mathbf{w})$ = std dev of portfolio **w** = $[\ \sigma^2(\mathbf{w})\ ]^{½}$
- $\sigma(t\mathbf{w}) = t\ \sigma(\mathbf{w}) \rightarrow \sigma$ is homogenous of degree 1 and can apply Euler's theorem
- $\sigma(\mathbf{w}) = \sum_i w_i\ \partial\sigma / \partial w_i = \sum_i w_i \sum_j C_{i,j} w_j\ / \sigma = \sum_i s_i$ where s_i = investment i's std dev contribution

$v_i / \sigma^2 = s_i / \sigma$, so fractional contributions are the same

Std dev contributions, geometrically

Menchero and Davis(2011). Risk contribution is exposure times volatility times correlation

Risks = directions (like north-south, east-west)

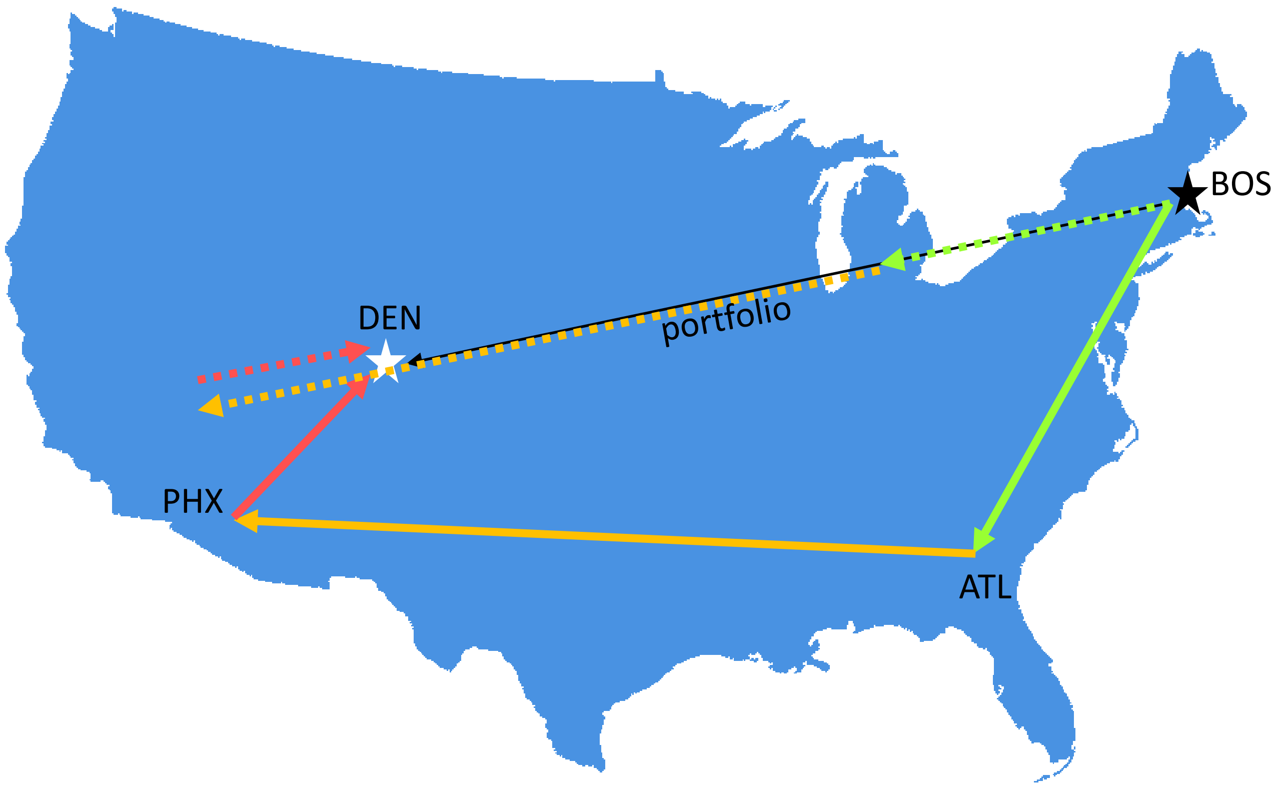
An investment (travel leg here) is a weighted combination of risks

Standard deviation = length

A portfolio is a sum of investments

Std dev contribution = length in the direction of the portfolio

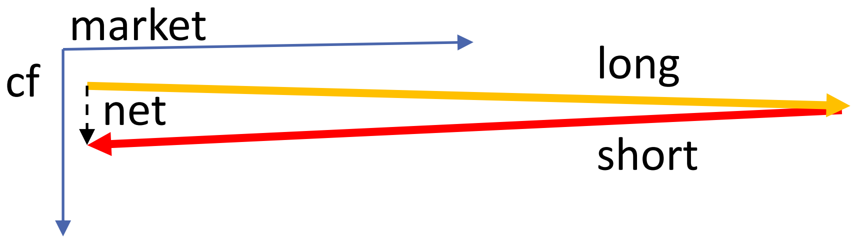
All the movement in other directions nets to zero



Getting real risks wrong

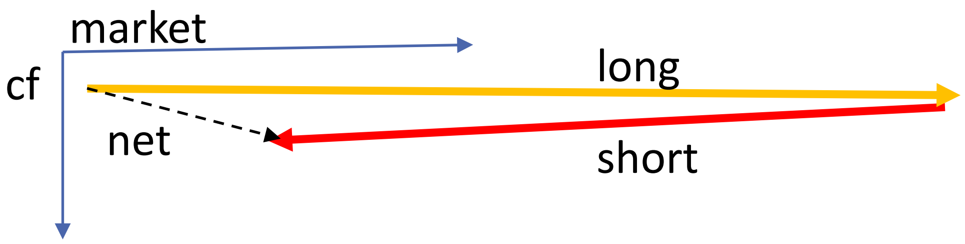
Imagine two directions of risk – market and chicken feed – and investing long-short

Beta hedged → net risk only in chicken feed → no market in risk contributions



What if perception is off and beta isn't hedged?

- The large market component in each side, mismatched, drives the risk of the net



Perceived-as-hedged directions vanish regardless of sensitivity to underlying offsetting risks

- Imagine looking at the equity of a levered company without separately seeing the debt
- Hidden unstable risks at both the individual investment and portfolio levels

Optimization finds perceived hedges

Risk parity is optimization

Risk parity is the solution to minimum variance with a penalty on small weights

- Maillard, S., Roncalli, T., and Teïletche, J. (2010). The properties of equally weighted risk contribution portfolios

Minimize$_{\mathbf{w}}$ $\frac{1}{2}\,\mathbf{w}^T\mathbf{C}\mathbf{w} + \sigma^2 \sum_i -\log(w_i)\, f_i$

- σ^2 is the target portfolio variance, and f_i is the risk fraction of investment i
- function is convex, and $d/d\mathbf{w} = 0 \leftrightarrow$ the portfolio satisfies risk parity

Large literature on problems of estimation error in optimization

A different take: risk parity is a way to minimum variance under uncertainty

- de Jong, M. (2018). Portfolio optimisation in an uncertain world

Recap of motivation

Why consider uncertainty?

- **Risks perceived as hedged are unseen** at both the portfolio and individual contribution levels
- Optimization finds perceived hedges
- Risk parity, like portfolio optimization, is a form of optimization
- Knowing that perception is imperfect, **use your coconut and consider how things could be wrong**

Next: add uncertainty

Uncertain contributions

Recall

- σ^2 = portfolio variance = $\sum_i w_i \sum_j C_{i,j} w_j$
- v_i = variance contribution of investment i = $w_i \sum_j C_{i,j} w_j$

Because these are linear combinations of the entries $C_{i,j}$

- Means – $E[\sigma^2]$ and $E[v_i]$ – are linear combinations of the $E[C_{i,j}]$'s
- Variances – $var[\sigma^2]$, $cov[v_i,v_j]$, and $cov[v_i,\sigma^2]$ – are linear combinations of the $cov[C_{i,j}, C_{k,l}]$'s
- **Easily computable given $E[C_{i,j}]$'s and $cov[C_{i,j}, C_{k,l}]$'s**

Note: std dev volatility, σ, and contributions, v_i / σ, are nonlinear in the entries $C_{i,j}$

- Thus, machinery ahead is in variance
- Earlier, we saw quantities in std dev were their variance counterparts / σ
- Can get 'feel' of std dev by dividing variance quantities by $E[\sigma^2]^{½}$
- Can approximate errors by linearizing, e.g. $s_i = v_i / \sigma \rightarrow var(s_i) \approx [\partial s/v_i \; \partial s/\sigma^2] \; cov(v_i \; \sigma^2) \; [\partial s/v_i \; \partial s/\sigma^2]$

Baby example of uncertain contributions

Long-short portfolio

- Each side has beta 1 ± 0.05 Beta uncertainty is uncorrelated and Gaussian. Market vol is 15%. No other risks or uncertainty
- Portfolio beta is then 0 ± 0.07

Conventional calculation

- Net beta of 0 → portfolio risk is 0 → each side's contribution is 0
- No market risk in portfolio

Considering uncertainty

- Portfolio variance has mean 1.13 and std deviation 1.6
- Each side's variance contribution is 0.56 ± 15. (The correlation between the contributions is -0.995)
- Market, previously latent, causes all the risk

An uncertain risk decomposition report sees the risks from sensitivity in hedges

Uncertain risk parity

Recall to get risk parity

- Peg the portfolio volatility and the fractions contributed by each investment
- (Equivalent) peg the contributions (not fractions), which sum to the portfolio volatility

Given uncertainty, these slightly differ. (We'll proceed using the second)

And numbers aren't fixed, so pegging is impossible. So

- **Employ a loss function to measure deviation from targets**
- **Find investment weights to minimize expected loss**

Uncertain risk parity (II)

Squared error is the loss function

Minimize mean squared error between variance contributions and targets

- Minimize$_{\mathbf{w}}$ $\sum_i$ E[(investment i's variance contribution – target)^2]

Minimize $\sum_i$ (E[var contribution$_i$] – its target)^2 + var(var contribution$_i$)

- Easily computed given the E[$C_{i,j}$]'s and cov[$C_{i,j}$, $C_{k,l}$]'s of entries in the investment covariance matrix (3 slides ago)
- The function minimized is a 4th degree polynomial in **w**

Uncertain risk parity (III): regimes

Suppose one believes regimes, obeying different covariance, are possible with some probability

- e.g., risk-on covariance with probability p, risk off with probability 1-p
- Variance contribution targets can also be regime dependent

Expected loss is easily computed

- E[loss] = E[E[loss | regime = m]] = probability weighted average of each regime's expected loss

One final piece: the $E[C_{i,j}]$'s and $cov[C_{i,j}, C_{k,l}]$'s of the covariance matrix entries

Modeling uncertain covariance

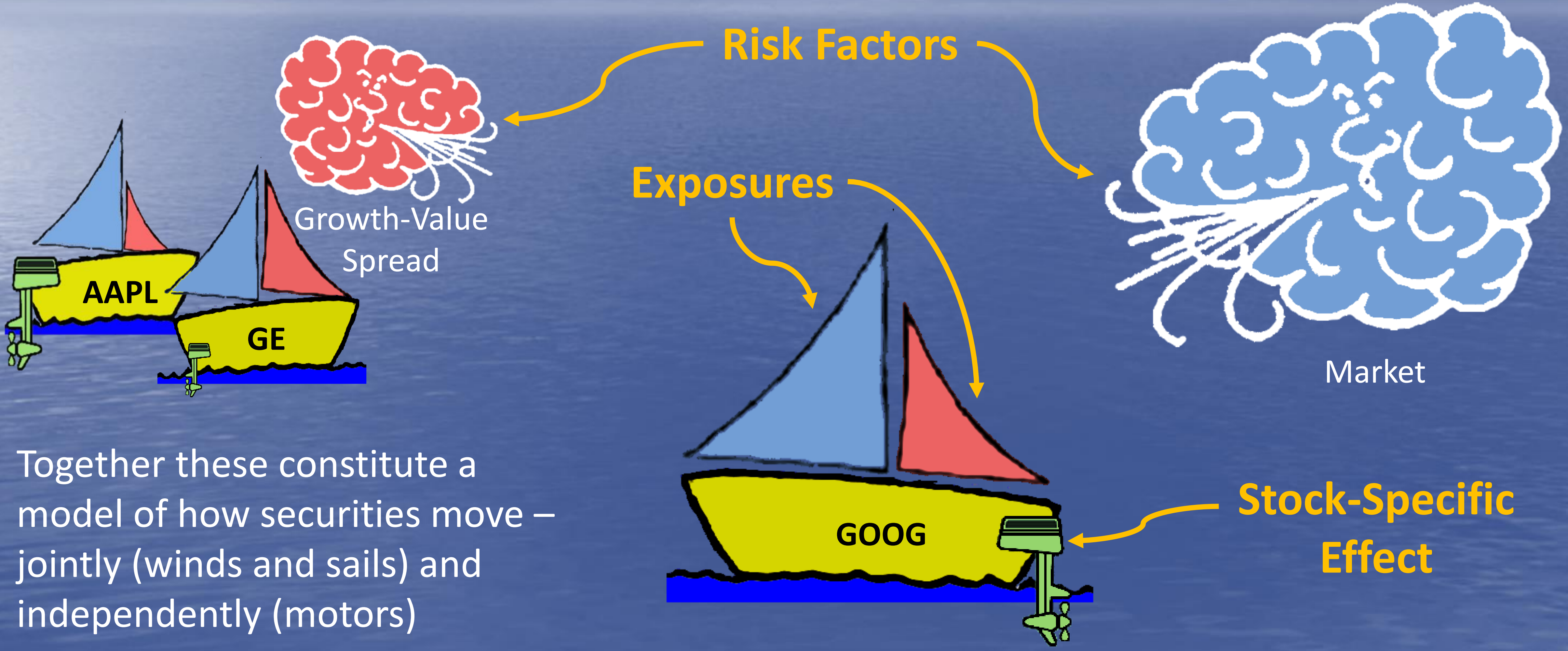
Shah, A. (2015). Uncertain Covariance Models

- http://ssrn.com/abstract=2616109

Presentations

- http://www.slideshare.net/AnishShah23/uncertain-covariance-62814740
- https://www.slideshare.net/AnishShah23/uncertaintypenalized-portfolio-optimization

A Factor Covariance Model in Pictures



Together these constitute a model of how securities move – jointly (winds and sails) and independently (motors)

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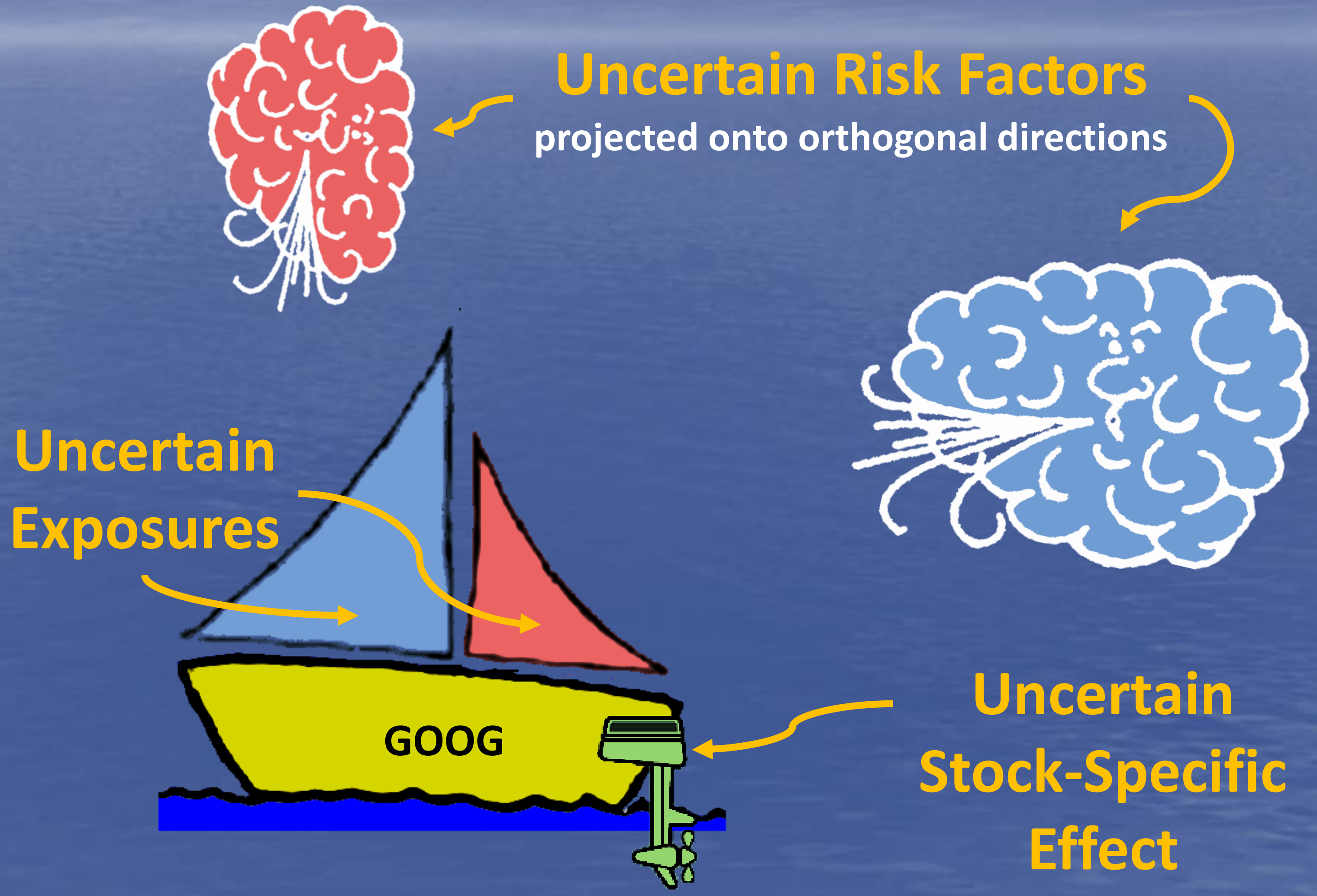
An Uncertain Factor Covariance Model

Welcome to reality!
Nothing is known with certainty.
But some forecasts are believed more accurate than others

A portfolio's risk has – according to beliefs – an expected value and a variance

Good decisions come from considering uncertainty explicitly.
Ignoring doesn't make it go away



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Summary

Standard calculations for risk contributions – whether for reports or for use in risk parity portfolio construction

- Assume hedges are real
- Don’t see sensitivity, **miss the consequences of being wrong**

Optimization finds (perceived) hedges

- On the reporting side, risks on these portfolios can be invisible
- Risk parity is a form of optimization

A saner course is modeling with uncertainty

- Reveals what is hiding, by explicitly modeling and integrating across the distribution of what‘s possible
- In risk parity and other optimization, creates robust portfolios that don’t count on perfect alignments in a random world