MARKET IMPLIED GDP

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OUTLINE

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BACKGROUND

- IN 2012, INVESTORS JOINED GREECE ON A SWAP AGREEMENT (PSI)

- NEW BONDS, SHORT-TERM NOTES AND A WARRANT FOR THE EXCHANGE OF THE PREVIOUS DEBT

- THE WARRANTS ARE A PUBLIC LISTED SECURITY WITH SOME SPECIAL FEATURES:
    1. MULTIPLE ANNUAL EXERCISE DATES (TREAT IT LIKE A SERIES OF CAPLETS)
    2. CASH-OR-NOTHING UP-AND-IN DUAL STRIKE LOOK BACK BARRIER
    3. BARRIERS RESET ON A ANNUAL BASIS UNTIL 2020, STABLE THEREAFTER
    4. PAYMENT IS CAPPED TO UP 1% OF THE NOTIONAL AMOUNT
    5. LIFETIME: 2012-2042

BARRIERS FOR THE PAYMENT

Barrier levels for warrant to be "in the money"		
Reference Year	Reference Nominal GDP in billion euros	Reference Real GDP Growth Rate
2014	210.10	2.35%
2015	217.90	2.90%
2016	226.35	2.85%
2017	235.72	2.80%
2018	245.47	2.60%
2019	255.88	2.50%
2020-2041	266.47	2.00%

WHEN BOTH GDP AND GDPR ARE ABOVE THE THESE THRESHOLDS A PAYMENT TAKES PLACE IN THE OF THE Q3 OF THE NEXT YEAR. GDP LEVEL AND REAL GDP GROWTH RATE ARE PUBLISHED BY EUROSTAT.

RESEARCH QUESTION

SINCE WARRANTS ARE LISTED, DO THEIR PRICES HAVE INFORMATION ABOUT GDP?

Two approaches:

- Regression pricing modeling
- Option Pricing

RESEARCH APPROACH

- REGRESSION BASED

$$GDP_t \equiv a + b_1 * WP_t$$

$$GDPR_t \equiv a + b_1 * WP_t$$

OPTION PRICING MODELS

BLACK SCHOLES

$$c = FN(d_1) - KN(d_2)e^{-rt}$$

$$d_1 = \frac{\ln(F/K) + t\sigma^2/2}{\sigma\sqrt{t}}$$

BACHELIER

$$c = e^{-rt}\left[(F-K)\Phi(d) + \sigma\sqrt{t}\varphi(d)\right]$$

$$d = \frac{F-K}{\sigma\sqrt{t}}$$

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$$c = e^{-rt}M(d^l, d^r; \rho)$$

$$d = \frac{(F-K) - t\sigma^2/2}{\sigma\sqrt{t}}$$

OPTION PRICING RESEARCH APPROACH

- WARRANT PRICING

$$c_{i,j} = e^{-rt_{i,j}} K_j M(d^l_{i,j}, d^r_{i,j}; \rho)$$

$$C_j = \sum_{i=j}^{n} c_{i,j}$$

$$d^l_{i,j} = \frac{(S^l_{i,j} - X^l_{i,j}) - t_{i,j}(\sigma^l_j)^2 / 2}{\sigma^l_j \sqrt{t_{i,j}}}$$

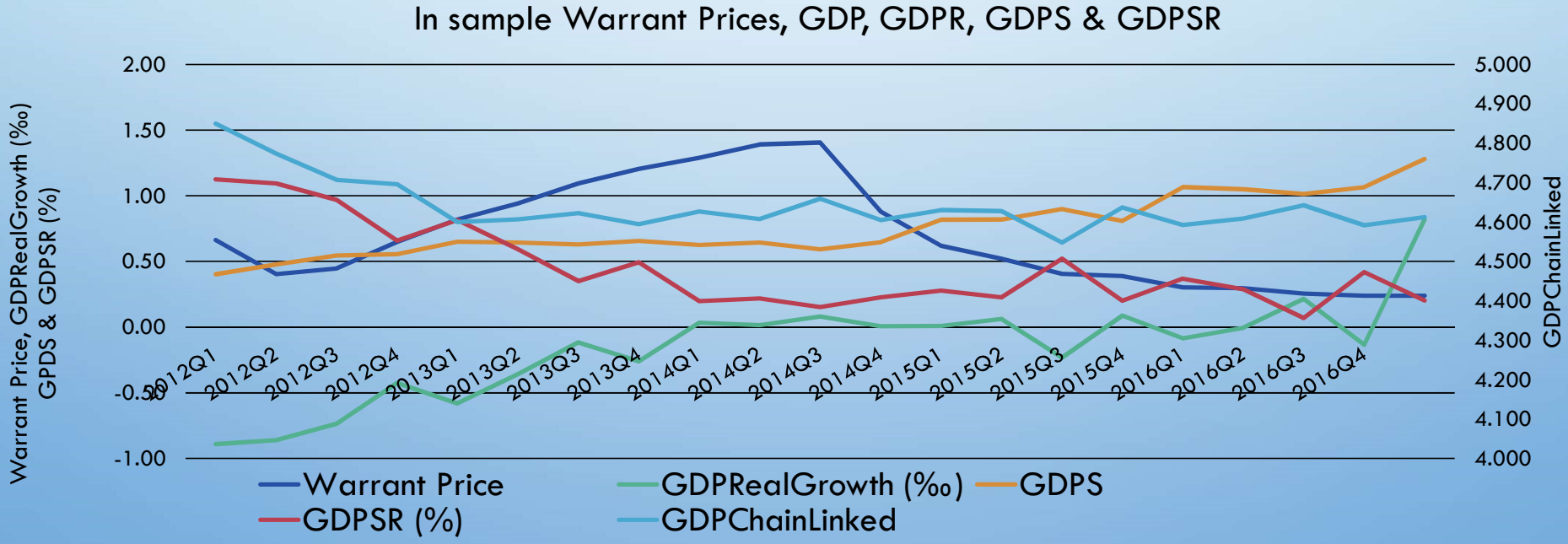
$$S^l_{i+1,j} = S^l_i * \sigma^l_j$$

$$d^r_{i,j} = \frac{(S^r_{i,j} - X^r_{i,j}) - t_{i,j}(\sigma^r_j)^2 / 2}{\sigma^r_j \sqrt{t_{i,j}}}$$

$$S^r_{i+1,j} = S^r_i + \sigma^r_j$$

DATA

- GDP NOMINAL AND REAL GDP GROWTH RATE
- WARRANT QUARTERLY AVERAGE OF DAILY PRICES
- QUARTERLY FROM 2012 TO PRESENT
- BASED ON THE BARRIERS WE ARE CALCULATING TWO MORE VARIABLES (SPREAD LEVEL AND RATE)



ESTIMATION

- CONVENTIONAL

$$GDP_t = a^w + b_1^w * WP_t + b_2^w * \exp(WP_t) + \varepsilon_t \quad \text{(ESTIMATE)}$$

$$WIGDP_t \equiv a^w + b_1^w * WP_t + b_2^w * \exp(WP_t) \quad \text{(FORECAST)}$$

- OPTION IMPLIED

FOR EVERY QUARTER *t* WE CALCULATE THE VALUES OF S^L AND S^R BY SOLVING SIMULTANEOUSLY FOR THE σ_j^l AND σ_j^r THAT EQUATES THE CAP C_j TO WARRANT PRICE. THE OPTION IMPLIED GDP (OIGDP) AND GDPR (OIGDPR) EQUAL THE VALUES OF $S_{1,j}^l$ AND $S_{1,j}^r$.

IMPROVING FORECASTS

- $GDP_t = a + b_1 * GDP_{t-1} \ldots + \varepsilon_t$ (BASIC AUTOREGRESSION EQUATION)
- $GDP_t = a + b_1 * GDP_{t-1} + b_2 * WIGDP_{t-1} + \varepsilon_t$ (CONVENTIONAL FORECAST)
- $GDP_t = a + b_1 * GDP_{t-1} + b_2 * OIGDP_{t-1} + \varepsilon_t$ (OPTION IMPLIED FORECAST)

RESULTS

ANOVA F Test		
Variables	Warrant Implied	Option Implied
GDP	0.03 (0.86)	7.56 (0.01)
GDPR	0.90 (0.35)	6.91 (0.02)
GDPS	1.09 (0.31)	1.28 (0.27)
GDPSPR	1.31 (0.27)	6.18 (0.02)

CONCLUSIONS

Conventional forecasts do not improve forecasts

Option Implied forecasts DO improve forecasts of GDP Rate

QUESTIONS?