Optimal Volatility

Mechanics of Dynamic Risk Control

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QWAFAFEW

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Achieving Good Reward for Risk

- Smart Beta strategies continue to gain favor
- Low Volatility strategies are successful and gaining AUM
- Optimization strategies provide good reward for risk
- New dynamic risk optimization strategies are emerging
Smart Beta Classifications and Example Strategies

- **Fundamentals**
  - Book Value
  - Sales
  - Cash Flow
  - Dividends

- **Risk Premia**
  - Value
  - Size
  - Momentum
  - Quality

- **Optimized Risk**
  - Minimum Volatility
  - Risk Weighting
  - Risk Parity (Equal Risk Weighting)
  - Maximum Diversification
  - Dynamic Managed Volatility
Return vs. Risk for Smart Beta Strategies

Efficient Combinations

Optimized

Fundamental Weighted

Risk Premia

SSIA
Building Simple Portfolios Based on Trailing Risk

- Calculate trailing 24-month volatility for largest stocks in the USA (99.5%)
- Rank from lowest to highest volatility
- Form 10 decile and 5 quintile portfolio groups based on volatility ranks
- Create equal-weighted and capitalization-weighted portfolios for each group
- Calculate return for each portfolio over the next month
- Repeat procedure using a new 24-month window including latest month

See paper for details:
- “Low Risk Stocks Outperform within All Observable Markets of the World”, SSRN, 2012
Risk and Return of Deciles: 1990-2017

United States

Excess Return vs. Risk of Decile Portfolios

Excess Return

Risk (std. dev.)

SSIA
Risk and Return of Deciles: 1990-2017

**United States**

**Sharpe Ratio vs. Risk of Decile Portfolios**

Return / Std Dev

<table>
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<tr>
<th>Return / Std Dev</th>
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<tbody>
<tr>
<td>120%</td>
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**United States**

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## Return and Risk Results for Portfolios Grouped by Volatility

### Annualized Results for Volatility Sorted Groups: 1980 - 2017

<table>
<thead>
<tr>
<th></th>
<th>USA</th>
<th>All Stocks</th>
<th>Low Half</th>
<th>High Half</th>
<th>Q1</th>
<th>Q5</th>
<th>D1</th>
<th>D5</th>
<th>Low - High</th>
<th>Q1 - Q5</th>
<th>D1 - D10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Return</strong></td>
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<td></td>
<td></td>
<td>12.2%</td>
<td>15.6%</td>
<td>9.0%</td>
<td>15.7%</td>
<td>4.4%</td>
<td>15.5%</td>
<td>-0.5%</td>
<td>6.5%</td>
<td>11.3%</td>
<td>16.0%</td>
</tr>
<tr>
<td><strong>Pct &gt; 0</strong></td>
<td></td>
<td>72.6%</td>
<td>84.2%</td>
<td>65.6%</td>
<td>87.7%</td>
<td>55.5%</td>
<td>88.4%</td>
<td>45.8%</td>
<td>72.6%</td>
<td>74.6%</td>
<td>78.3%</td>
</tr>
<tr>
<td><strong>Risk (std dev)</strong></td>
<td></td>
<td>16.1%</td>
<td>11.9%</td>
<td>21.1%</td>
<td>9.8%</td>
<td>24.7%</td>
<td>8.9%</td>
<td>26.5%</td>
<td>-9.2%</td>
<td>-14.9%</td>
<td>-17.6%</td>
</tr>
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<td><strong>Pct &gt; 0</strong></td>
<td></td>
<td>100.0%</td>
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<tr>
<td><strong>Sharpe</strong></td>
<td></td>
<td>1.03</td>
<td>1.68</td>
<td>0.65</td>
<td>2.06</td>
<td>0.33</td>
<td>2.22</td>
<td>0.09</td>
<td>1.04</td>
<td>1.73</td>
<td>2.12</td>
</tr>
<tr>
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<td>55.5%</td>
<td>88.4%</td>
<td>45.8%</td>
<td>88.8%</td>
<td>89.5%</td>
<td>91.7%</td>
</tr>
</tbody>
</table>
Return and Risk of USA Quintiles: 1990-2017

Return and Risk of Quintiles

Excess Return (net of average) Excess Risk (net of average)
Sharpe Ratio of Quintiles: 1990-2017

3-year Sharpe Ratio net of the Average Sharpe Ratio

United States

Sharpe Q1
Sharpe Q5

SSIA
Portfolio Risk Depends on Average Stock Volatilities and Correlations

\[
\text{Risk} = \text{Volatility} \times \text{Correlation}
\]

Portfolio Volatility = weighted average volatility of stocks \cdot weighted average correlation

\[
\sigma_p^2 = \sum_i x_i \cdot \sigma_i^2 \cdot \sum_i x_i \cdot \rho_i
\]

where:
- \( \sigma_p^2 \) = portfolio volatility
- \( \sum_i x_i \cdot \sigma_i^2 \) = weighted average volatility of stocks in the portfolio
- \( \sum_i x_i \cdot \rho_i \) = weighted average correlation among stocks in the portfolio
- \( \rho_i \) = weighted average correlation of stock i with all other stocks

Price information drives Returns, Volatilities and Correlations
Dynamic Approach - Optimal Volatility

Expected Reward and Risk

Efficient Frontier

Min Vol

Opt Vol

S&P 500
Frontiers: Opt Vol portfolio is selected to Maximize Portfolio Reward-to-Risk

Opt Vol Portfolio at tangent

S&P 500

S&P 500
Frontiers Depend on Recent Risk and Return Market Conditions
Opt Vol Risk is close to Min Vol Risk. It is only 7% of the way to the Max Vol Risk

Aggressiveness: \[ \frac{\text{Opt Vol risk} - \text{Min Vol risk}}{\text{Max Vol risk} - \text{Min Vol risk}} \]
Sample Efficient Frontier: When Market Risk is High and Reward is Low

Opt Vol Risk Equals the Max Vol Risk. It is 100% of the way to through the risk range.
Opt Vol moves to higher risk as the frontier shifts. Efficient frontiers can both move and tilt.

Efficient Frontiers Under Various Market Conditions

Opt Vol Positions on Different Frontiers
Percentage Along Frontier Risk Range as Efficient Frontiers Shift

Opt Vol Positions on Different Frontiers

Percent Along Frontier
Function for Determining the Optimal Volatility Index Holdings

A Reward-to-Risk characteristic can be calculated using four factors for each stock in the Selection Universe:

- Volatility (60 months)
- Correlation (60 months)
- Change in Volatility (last 24 months less last 60 months)
- Change in Correlation (last 24 months less last 60 months)

\[
R_{12} = \beta_0 + \beta_1 F_1 + \beta_2 F_2 + \beta_3 F_3 + \beta_4 F_4 + \epsilon
\]

Where:

- \( R_{12} \) = Vector of returns for stocks over the last 12 months
- \( F_{1-4} \) = Vectors of volatility or correlation factors for stocks

Optimization

Maximize:

\[
\frac{E(RR)}{E(PR)} = \frac{Expected \ Reward-to-Risk}{Expected \ Portfolio \ Risk}
\]
Optimal Volatility Increases Risk Dynamically Based on Market Conditions

- Estimated based on 4 factors

Nonlinear Volatility Relationship

\[
Max: \left( \frac{E.return}{E.risk} \right) = \frac{x \cdot Er}{\sqrt{(x \cdot COV \cdot x)}}
\]
Optimal Volatility Index Construction: GLCOV Index by S&P

Selection Universe

• Constituents of the Standard and Poor’s 500 index.
• Market capitalization of 5 USD billion or more
• Annual dollar value traded to float adjusted cap > 1.0
• Minimum of 250,000 shares traded in prior 6 months

Constraints

• Sector constraints:
  +/- 10 % relative to capitalization weighted sectors
• Stock upper limits:
  smaller of 3 % or 10x market weight
• Lower limit of 0% on all positions (no short selling)
• Weights in the portfolio must sum to 100%
Index Construction and Performance Results

- Index is rebalanced quarterly based on Optimal Volatility solution
- Stocks must be members of updated quarterly S&P 500
- S&P 500 calculates performance of GLCOV Index

Results:

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Return</td>
<td>5.4%</td>
<td>6.6%</td>
<td>10.2%</td>
</tr>
<tr>
<td>Risk (Std. Dev.)</td>
<td>14.5%</td>
<td>11.1%</td>
<td>13.0%</td>
</tr>
<tr>
<td>Risk / S&amp;P 500 Risk</td>
<td>100%</td>
<td>77%</td>
<td>90%</td>
</tr>
<tr>
<td>Return / Risk</td>
<td>37%</td>
<td>59%</td>
<td>78%</td>
</tr>
<tr>
<td>Beta</td>
<td>100%</td>
<td>70%</td>
<td>81%</td>
</tr>
</tbody>
</table>
Performance Comparison

Optimal Volatility Performance

- Minimum Volatility
- Optimal Volatility
- S&P 500

Performance in Up vs. Down Markets is Participation Ratio Difference - PRD

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<tr>
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<tbody>
<tr>
<td>Down Markets</td>
<td>-3.8%</td>
<td>-2.3%</td>
<td>-2.5%</td>
</tr>
<tr>
<td>Up Markets</td>
<td>3.2%</td>
<td>2.4%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Down Capture</td>
<td>100%</td>
<td>62%</td>
<td>66%</td>
</tr>
<tr>
<td>Up Capture</td>
<td>100%</td>
<td>76%</td>
<td>96%</td>
</tr>
<tr>
<td>PRD (Up – Down Capture)</td>
<td>0%</td>
<td>14%</td>
<td>30%</td>
</tr>
</tbody>
</table>
Up and Down Market Capture

<table>
<thead>
<tr>
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<td>3.1%</td>
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**UP AND DOWN MARKET PERFORMANCE**
Dynamic Volatility

- Optimal Volatility index often has lower exposure to market risk than S&P 500
- Risk is typically closer to that of the Minimum Volatility strategy
- Max Vol is constructed by maximizing portfolio beta using the same constraints

![Graph showing risk (std dev) over time for Opt Vol, Min Vol, and Max Vol strategies from 1990 to 2017.](image-url)

The graph illustrates the risk (standard deviation) for Optimal, Minimum, and Maximum Volatility strategies over a period from 1990 to 2017.
Heartbeat: Optimal Volatility Aggressiveness Ratio in High-Low Volatility Range

Aggressiveness:  \[ \frac{(\text{Opt Vol risk} - \text{Min Vol risk})}{(\text{Max Vol risk} - \text{Min Vol risk})} \]
Optimal Volatility Increases Risk Tactically Based on Market Conditions

- Optimal Volatility portfolio has risk lower than the market.
- Higher than market return suggest risk exposure is increase only when there is high reward.

### Return vs Volatility

- **Optimal Volatility**
- **Minimum Volatility**
- **S&P 500**
Optimal Volatility Summary and Investigation Directions

- Price Information: Returns, Correlations, and Volatilities
- Systematic optimization based on market reward
- Dynamically moves to higher risk when it is rewarded
- Defensive most of the time

Where does Optimal Volatility work?
- Sector Applications
- Asset Allocation
Appendix
Notes:

- The optimal volatility portfolio is created to maximize the ratio of reward-to-risk.
- Measure the market reward to four risk factors over the last 12 months.
- Single number expresses the expected reward-to-risk for each stock.
- Highest reward to risk portfolio combination is selected using an optimizer.
- Optimization is similar to finding a maximum Sharpe ratio portfolio.
- Expected reward-to-risk is used for the numerator (return axis).
- Portfolio volatility over the last 60 months is used for the denominator (risk axis).

- Efficient Frontier changes with market conditions.
- Cannot predict when the optimal volatility portfolio will take on high risk or low risk.
- When market volatility is low, optimal volatility portfolio generally will be low volatility.
- When does volatility of portfolio generally increase?
  - Market volatility has been high recently.
  - Market is stabilizing.
  - Higher risk stocks demonstrate greater reward-to-risk.

  Generally, Optimal volatility portfolio will then become more aggressive.

- There is no method to time or predict these shifts.
- Market information related to price determines shifts: returns, correlations, and risk.