The Divergence of High- and Low-Frequency Estimation: Causes and Consequences

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Outline

Evidence and causes of divergence

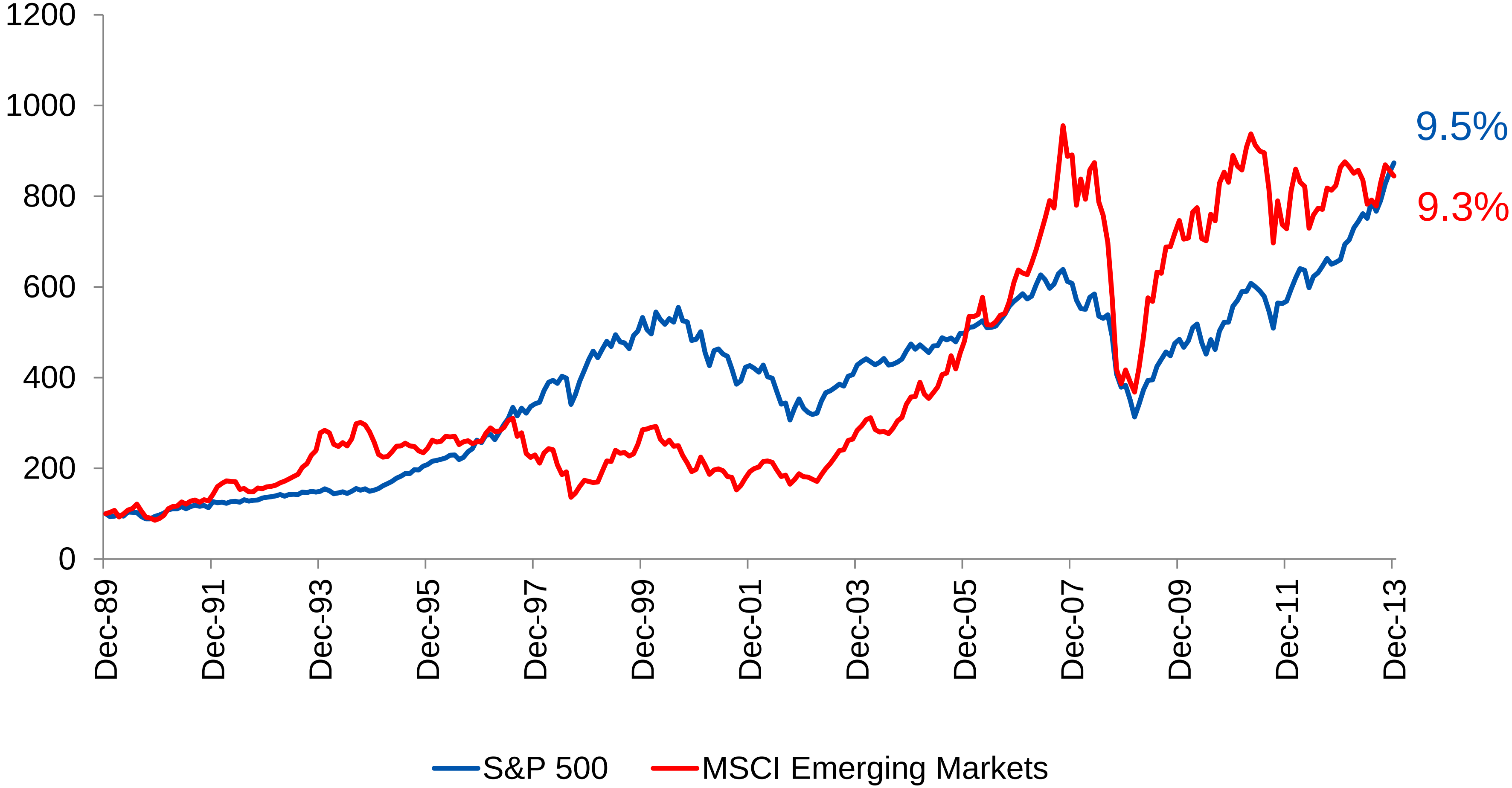
The divergence of high- and low-frequency estimation

- It is common practice to extrapolate monthly or higher-frequency volatility and correlation estimates to longer horizons.

- Most analysts assume implicitly that volatilities scale with the square root of time and correlations estimated from high-frequency returns are similar to correlations estimated from low-frequency returns.

- Evidence does not support this view. Asset values often evolve through time in ways that are highly inconsistent with their high-frequency volatilities and correlations.

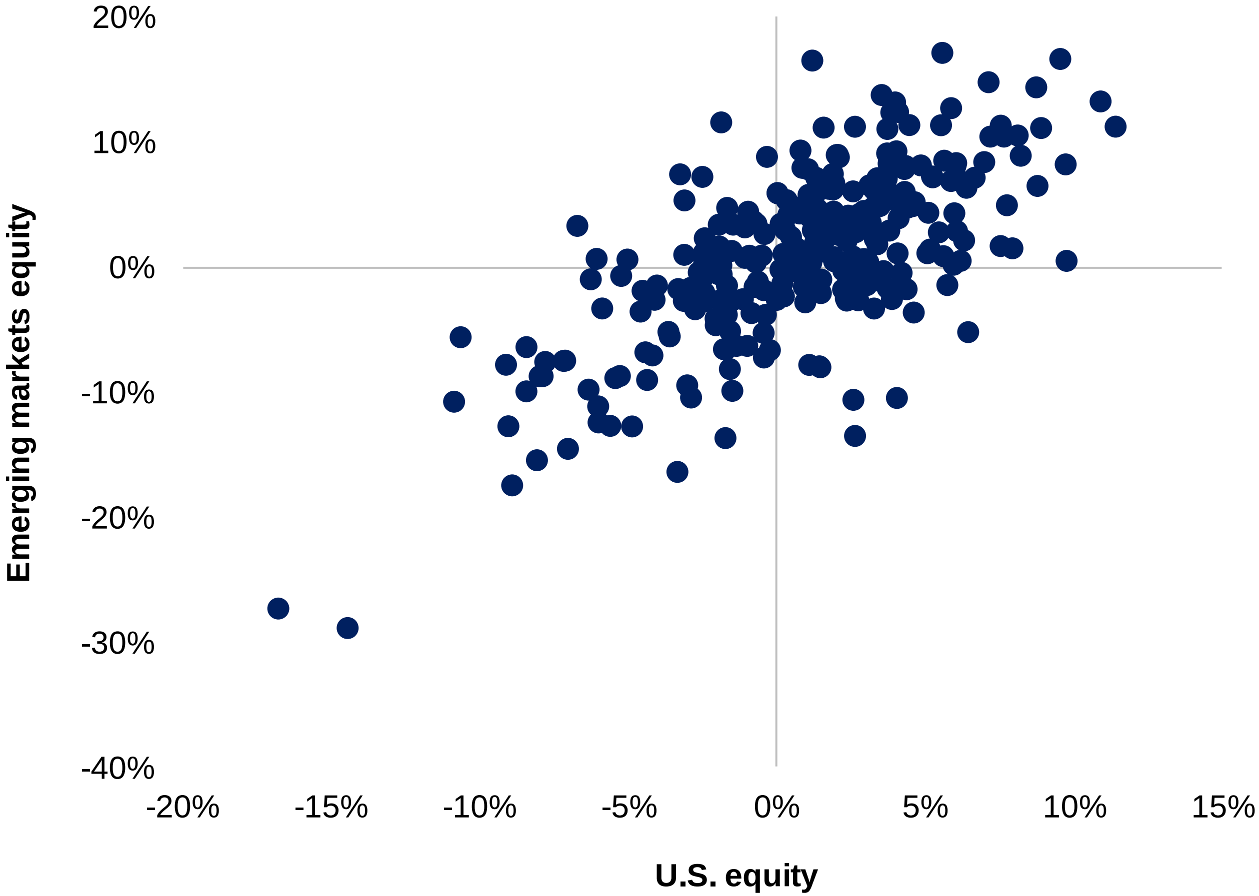
Emerging markets and U.S. equities: Cumulative returns (1990-2013)



Source: State Street Associates

Monthly returns

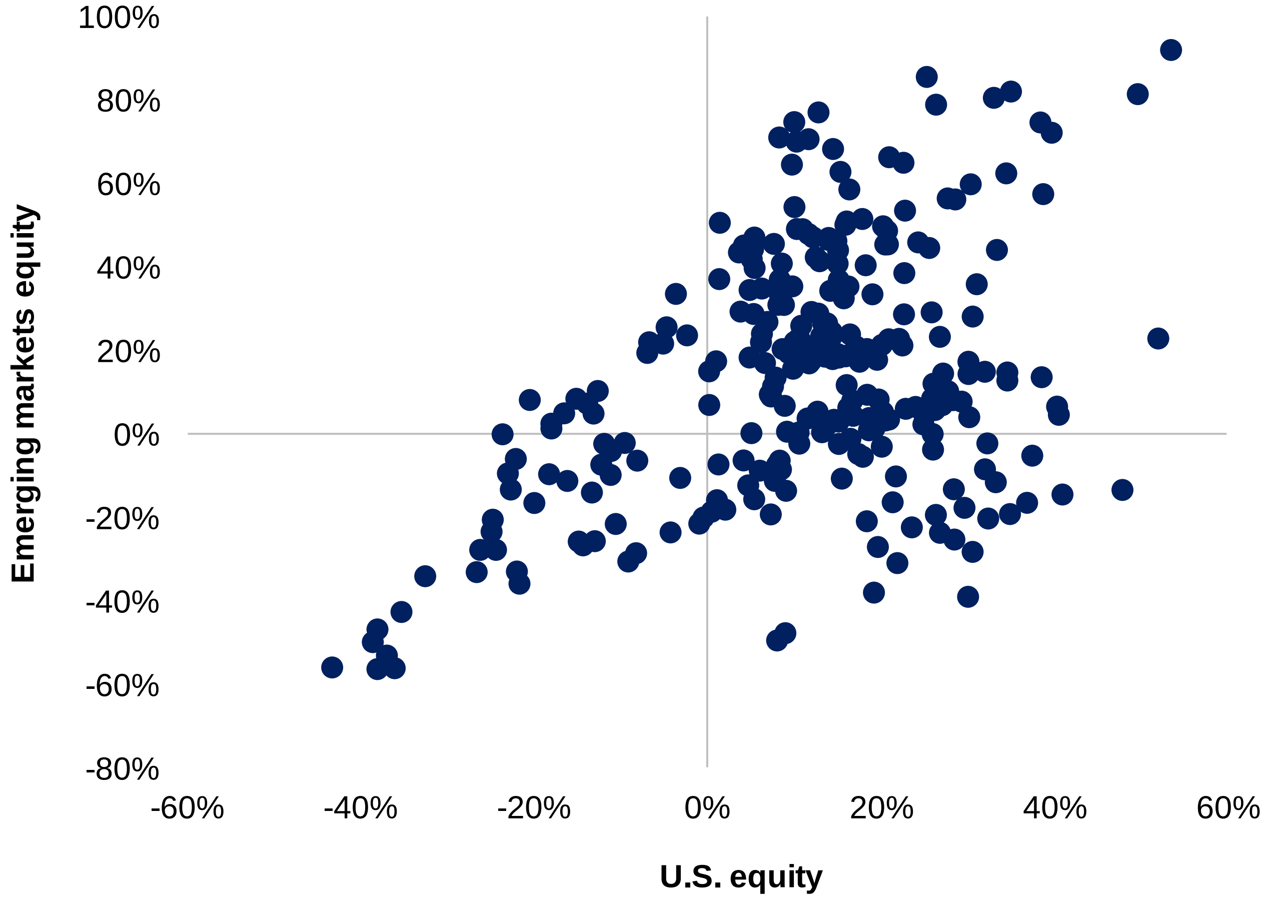
Correlation = 0.69



Source: State Street Associates

Annual returns

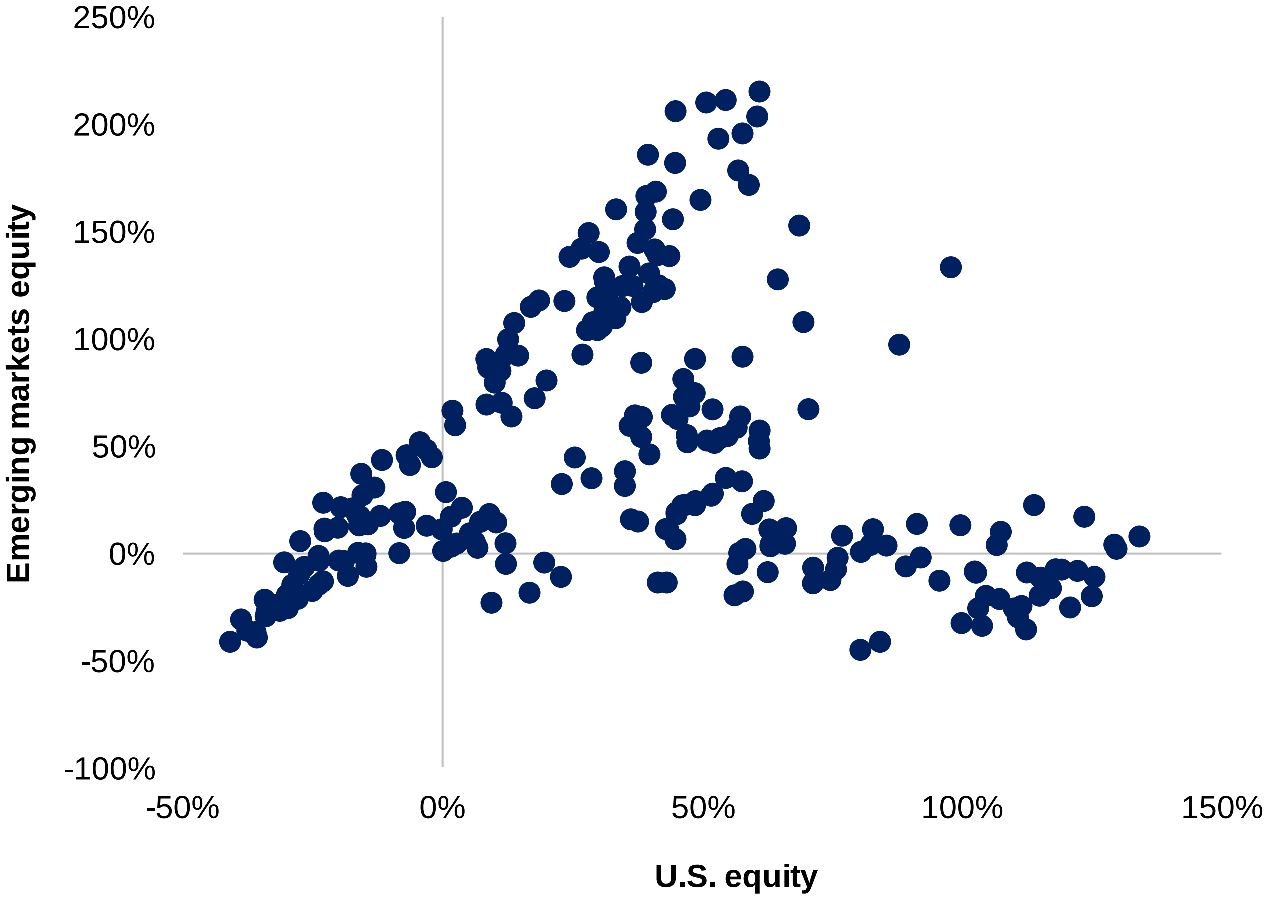
Correlation = 0.44



Source: State Street Associates

Triennial returns

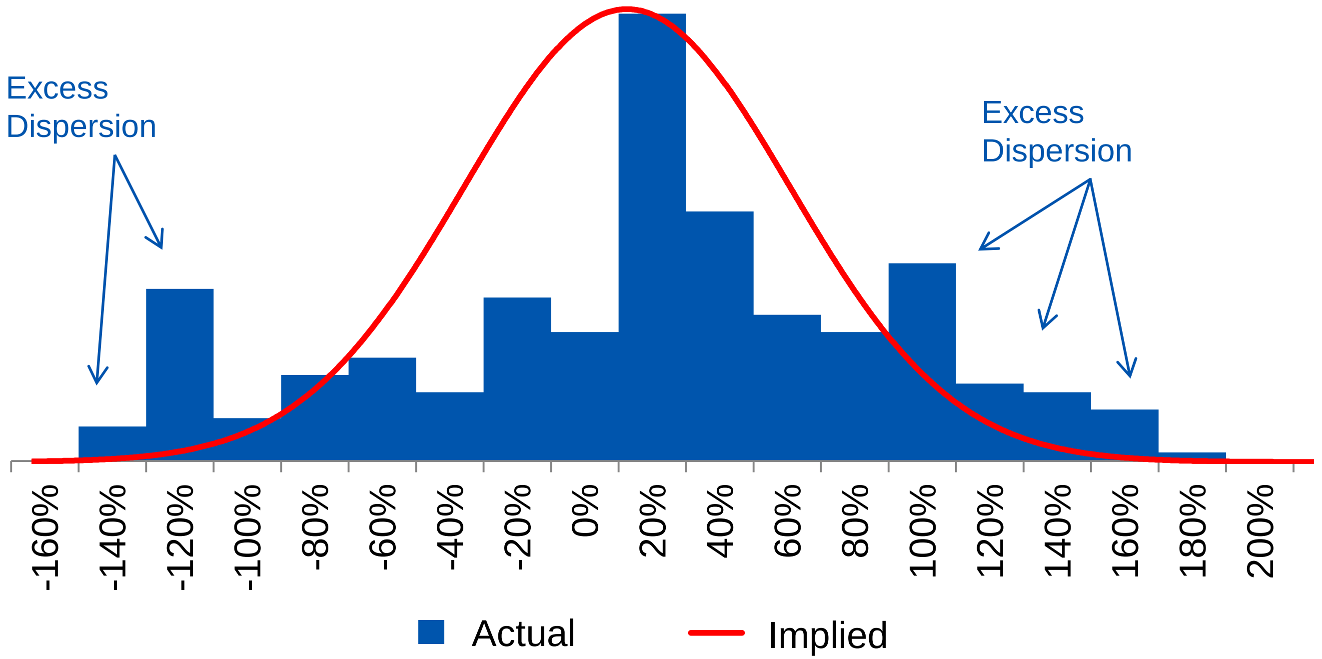
Correlation = 0.04



Source: State Street Associates

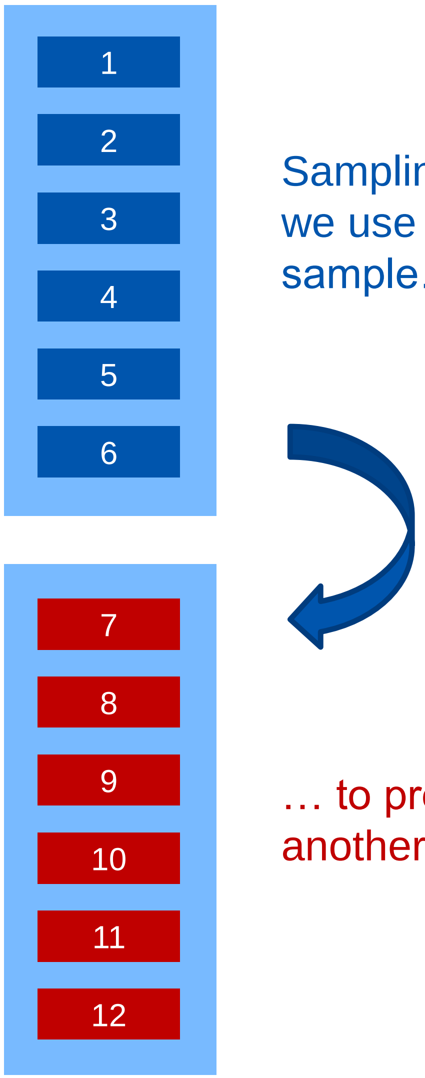
Emerging markets and U.S. equities: Relative return distribution

Excess dispersion refers to the fraction of a distribution that falls outside the one-standard deviation tails of the distribution implied by monthly returns.



Source: State Street Associates

Excess dispersion is different from sampling error

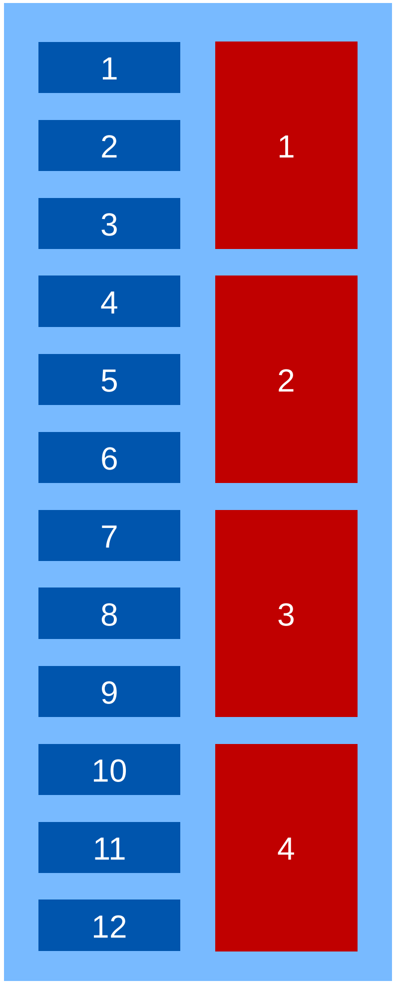


Sampling error arises when we use parameters from one sample…

… to predict parameters in another sample.

Excess dispersion is different from sampling error

Excess dispersion arises when we extrapolate parameters derived from high-frequency observations…



… to estimate low-frequency parameters within the same sample.

Attribution of excess dispersion of triennial relative returns

			Excess Dispersion				
		IID	Non-normality	Auto-correlation	Lagged Cross-correlation	Auto and Cross-correlation	Actual
U.S. Domestic Assets							
Hedge Funds	Private Equity	0.0%					
Real Estate	Government Bonds	0.0%					
Large Cap Equity	Government Bonds	0.0%					
Corporate Bonds	Government Bonds	0.0%					
Energy Stocks	Utilities Stocks	0.0%					
International Assets							
U.S. Equities	Emerging Equities	0.0%					
Canadian Equities	U.S. Equities	0.0%					
Global Portfolio in USD	USD/AUD	0.0%					
USD/GBP	USD/JPY	0.0%					
German Equities	U.K. Equities	0.0%					

All underlying data is monthly from Jan 1990 through Dec 2013, except for private equity and real estate which are quarterly from Q1 1996 through Q3 2013. Country equity data are MSCI total return indices in US dollars. Equity sector data are total returns for 10 sector portfolios from Ken French's website. Bond data are Barclays total return indices. Currency data are WM/Reuters spot rates.

Source: State Street Associates

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		IID	Non-normality	Auto-correlation	Lagged Cross-correlation	Auto and Cross-correlation	Actual
U.S. Domestic Assets							
Hedge Funds	Private Equity	0.0%	0.7%				
Real Estate	Government Bonds	0.0%	-0.8%				
Large Cap Equity	Government Bonds	0.0%	-0.4%				
Corporate Bonds	Government Bonds	0.0%	-1.9%				
Energy Stocks	Utilities Stocks	0.0%	-0.5%				
International Assets							
U.S. Equities	Emerging Equities	0.0%	-0.9%				
Canadian Equities	U.S. Equities	0.0%	0.3%				
Global Portfolio in USD	USD/AUD	0.0%	0.2%				
USD/GBP	USD/JPY	0.0%	-0.2%				
German Equities	U.K. Equities	0.0%	0.4%				

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		IID	Non-normality	Auto-correlation	Lagged Cross-correlation	Auto and Cross-correlation	Actual
U.S. Domestic Assets							
Hedge Funds	Private Equity	0.0%	0.7%	24.8%			
Real Estate	Government Bonds	0.0%	-0.8%	20.3%			
Large Cap Equity	Government Bonds	0.0%	-0.4%	9.7%			
Corporate Bonds	Government Bonds	0.0%	-1.9%	-13.4%			
Energy Stocks	Utilities Stocks	0.0%	-0.5%	-9.0%			
International Assets							
U.S. Equities	Emerging Equities	0.0%	-0.9%	-2.7%			
Canadian Equities	U.S. Equities	0.0%	0.3%	-1.2%			
Global Portfolio in USD	USD/AUD	0.0%	0.2%	-3.0%			
USD/GBP	USD/JPY	0.0%	-0.2%	-1.5%			
German Equities	U.K. Equities	0.0%	0.4%	-4.2%			

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Source: State Street Associates

Attribution of excess dispersion of triennial relative returns

				Excess Dispersion			
		IID	Non-normality	Auto-correlation	Lagged Cross-correlation	Auto and Cross-correlation	Actual
U.S. Domestic Assets							
Hedge Funds	Private Equity	0.0%	0.7%	24.8%	5.3%		
Real Estate	Government Bonds	0.0%	-0.8%	20.3%	4.3%		
Large Cap Equity	Government Bonds	0.0%	-0.4%	9.7%	1.4%		
Corporate Bonds	Government Bonds	0.0%	-1.9%	-13.4%	7.0%		
Energy Stocks	Utilities Stocks	0.0%	-0.5%	-9.0%	-18.7%		
International Assets							
U.S. Equities	Emerging Equities	0.0%	-0.9%	-2.7%	20.6%		
Canadian Equities	U.S. Equities	0.0%	0.3%	-1.2%	14.8%		
Global Portfolio in USD	USD/AUD	0.0%	0.2%	-3.0%	19.7%		
USD/GBP	USD/JPY	0.0%	-0.2%	-1.5%	13.2%		
German Equities	U.K. Equities	0.0%	0.4%	-4.2%	-11.8%		

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Hedge Funds	Private Equity	0.0%	0.7%	24.8%	5.3%	26.8%	
Real Estate	Government Bonds	0.0%	-0.8%	20.3%	4.3%	22.9%	
Large Cap Equity	Government Bonds	0.0%	-0.4%	9.7%	1.4%	10.8%	
Corporate Bonds	Government Bonds	0.0%	-1.9%	-13.4%	7.0%	-4.6%	
Energy Stocks	Utilities Stocks	0.0%	-0.5%	-9.0%	-18.7%	-25.7%	
International Assets							
U.S. Equities	Emerging Equities	0.0%	-0.9%	-2.7%	20.6%	21.7%	
Canadian Equities	U.S. Equities	0.0%	0.3%	-1.2%	14.8%	15.1%	
Global Portfolio in USD	USD/AUD	0.0%	0.2%	-3.0%	19.7%	17.3%	
USD/GBP	USD/JPY	0.0%	-0.2%	-1.5%	13.2%	11.1%	
German Equities	U.K. Equities	0.0%	0.4%	-4.2%	-11.8%	-17.9%	

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Real Estate	Government Bonds	0.0%	-0.8%	20.3%	4.3%	22.9%	22.7%
Large Cap Equity	Government Bonds	0.0%	-0.4%	9.7%	1.4%	10.8%	14.5%
Corporate Bonds	Government Bonds	0.0%	-1.9%	-13.4%	7.0%	-4.6%	-12.0%
Energy Stocks	Utilities Stocks	0.0%	-0.5%	-9.0%	-18.7%	-25.7%	-26.6%
International Assets							
U.S. Equities	Emerging Equities	0.0%	-0.9%	-2.7%	20.6%	21.7%	27.6%
Canadian Equities	U.S. Equities	0.0%	0.3%	-1.2%	14.8%	15.1%	24.4%
Global Portfolio in USD	USD/AUD	0.0%	0.2%	-3.0%	19.7%	17.3%	22.4%
USD/GBP	USD/JPY	0.0%	-0.2%	-1.5%	13.2%	11.1%	11.0%
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Source: State Street Associates

Excess dispersion of triennial returns: Major asset classes

	U.S. Large Cap	U.S. Small Cap	EAFE Equities	Emerging Equities	Global Sovereign Bonds	U.S. Govt. Bonds	U.S. Corporate Bonds	Commod-ities	Hedge Funds	Real Estate*	Private Equity*
U.S. Large Cap		-0.02	-0.06	-0.60	-0.29	-0.12	-0.11	-0.05	0.00	0.29	0.14
U.S. Small Cap	3.4%		0.09	-0.26	-0.12	-0.13	0.09	0.11	-0.04	0.32	-0.07
EAFE Equities	5.8%	-15.9%		-0.22	-0.49	-0.47	-0.39	-0.02	-0.20	0.44	0.07
Emerging Equities	27.6%	14.5%	1.1%		0.27	-0.02	-0.15	0.06	-0.32	0.21	-0.38
Global Sovereign Bonds	15.3%	-12.8%	3.4%	-0.1%		0.00	0.00	-0.03	0.23	-0.28	-0.32
U.S. Government Bonds	14.5%	-10.4%	-1.3%	3.8%	-4.9%		-0.20	0.02	0.44	-0.20	0.10
U.S. Corporate Bonds	14.1%	-15.1%	-0.1%	1.9%	-9.6%	-12.0%		0.06	-0.05	-0.13	-0.20
Commodities	3.1%	-15.1%	-4.5%	-4.1%	-1.7%	-3.7%	-5.2%		0.05	0.24	-0.29
Hedge Funds	14.5%	-15.5%	-2.1%	12.5%	5.4%	3.1%	4.2%	-3.7%		0.56	-0.08
Real Estate*	-7.2%	-17.7%	-12.4%	-5.4%	29.7%	22.7%	17.4%	-19.5%	-1.9%		0.23
Private Equity*	-5.4%	5.1%	1.6%	6.9%	31.4%	26.2%	27.9%	1.6%	26.2%	-1.9%	

Upper Triangle of Matrix: Triennial Correlation minus Monthly Correlation

Lower Triangle of Matrix: Actual Excess Dispersion of Triennial Returns

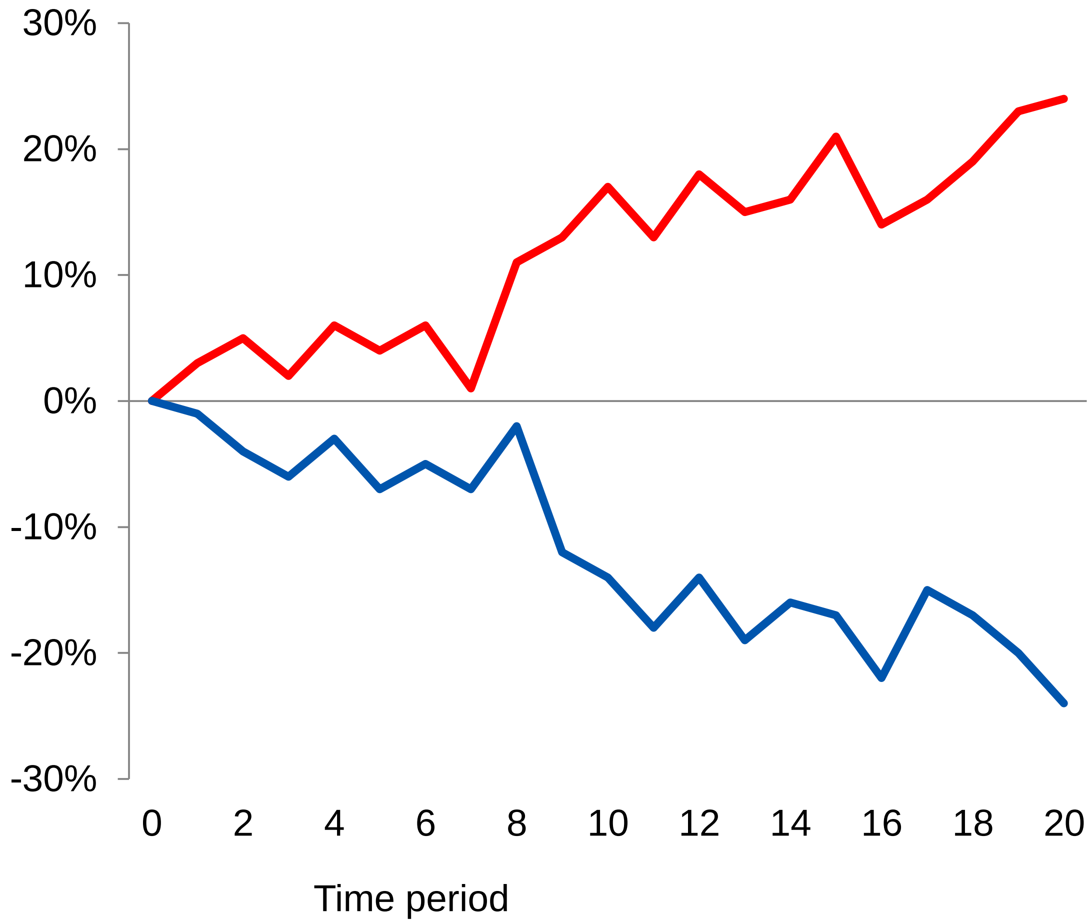
All data monthly from Jan 1990 - Dec 2013 except Real Estate and Private Equity, which are quarterly from Q1 1996 - Q3 2013.

Source: State Street Associates

Mathematics of divergence

Cumulative returns of two hypothetical assets

Monthly correlation	0.5
Correlation of 1st asset with lag of 2nd asset	-0.2
Correlation of 2nd asset with lag of 1st asset	-1.0
Autocorrelation of 1st asset	-0.5
Autocorrelation of 2nd asset	-0.5
Two month correlation	**-0.2**



Source: State Street Associates

The relation of high- and low- frequency volatility

The volatility of the cumulative continuous returns of x over q periods is given by:

$$\sigma(x_t + \cdots + x_{t+q-1}) = \sigma_x \sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t, x_{t+k}}}$$

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This term reflects annualization in the absence of lagged effects

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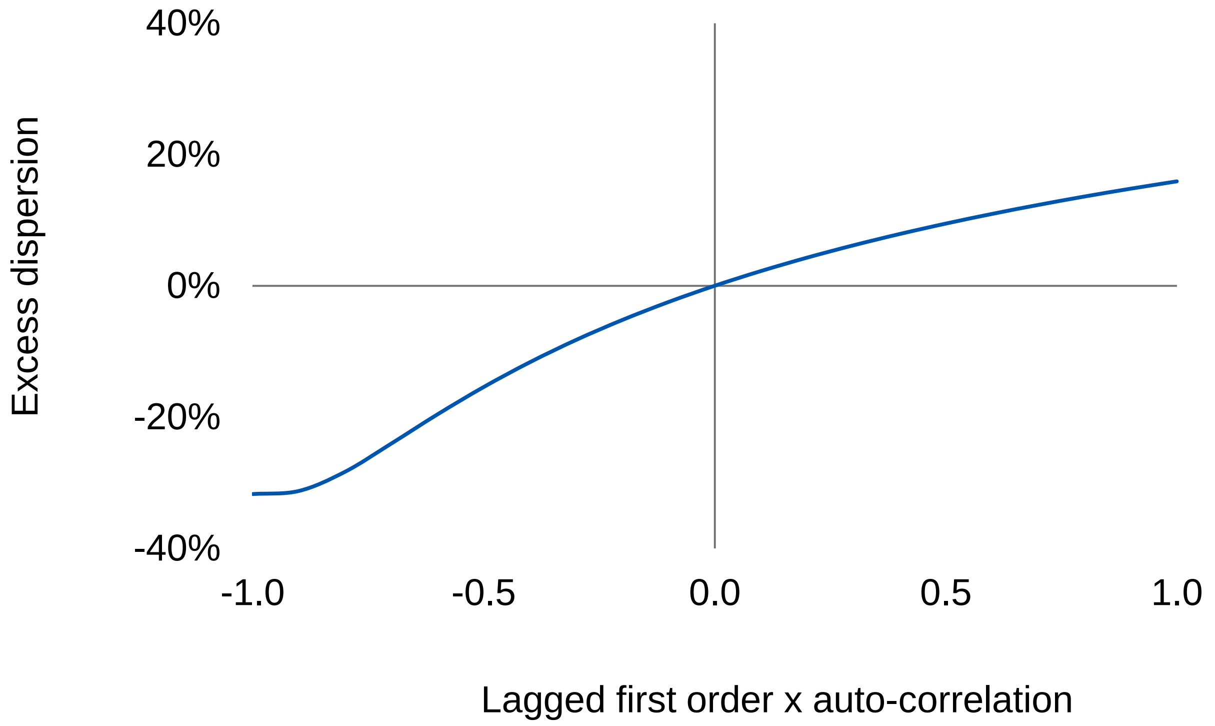
This term captures the impact of auto-correlation

The relation of high- and low- frequency volatility: Impact of first-order auto-correlation



Notes: This example assumes x and y have a contemporaneous correlation of zero.

Source: State Street Associates

The relation of high- and low- frequency correlation

The correlation between the cumulative returns of x and the cumulative returns of y over q periods is given by:

$$\rho(x_t + \cdots + x_{t+q-1}, y_t + \cdots + y_{t+q-1}) =$$

$$\frac{q\rho_{x_t, y_t} + \sum_{k=1}^{q-1}(q-k)(\rho_{x_{t+k}, y_t} + \rho_{x_t, y_{t+k}})}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t, x_{t+k}}}\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{y_t, y_{t+k}}}}$$

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This term captures the lagged cross-correlation between x and y

The relation of high- and low- frequency correlation

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$$\frac{q\rho_{x_t, y_t} + \sum_{k=1}^{q-1}(q-k)(\rho_{x_{t+k}, y_t} + \rho_{x_t, y_{t+k}})}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t, x_{t+k}}} \cdot \sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{y_t, y_{t+k}}}}$$

This term captures the auto-correlation of x

The relation of high- and low- frequency correlation

The correlation between the cumulative returns of x and the cumulative returns of y over q periods is given by:

$$\rho(x_t + \cdots + x_{t+q-1}, y_t + \cdots + y_{t+q-1}) =$$

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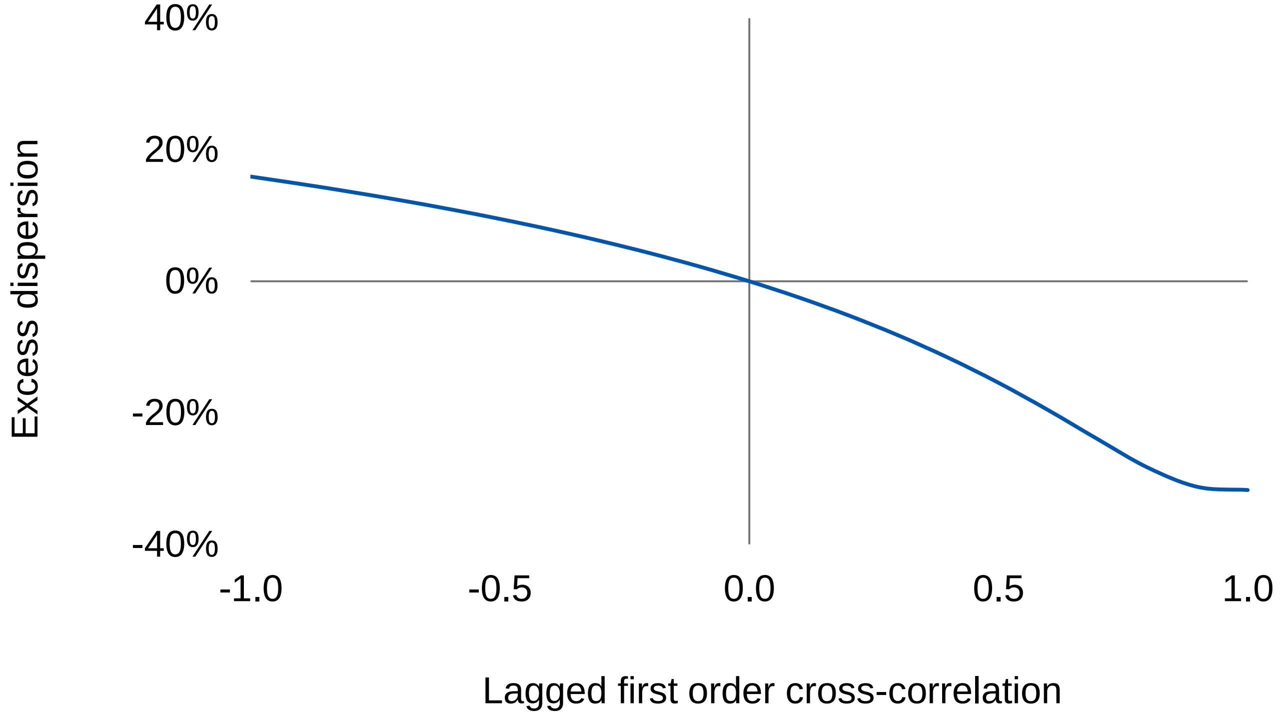
This term captures the auto-correlation of y

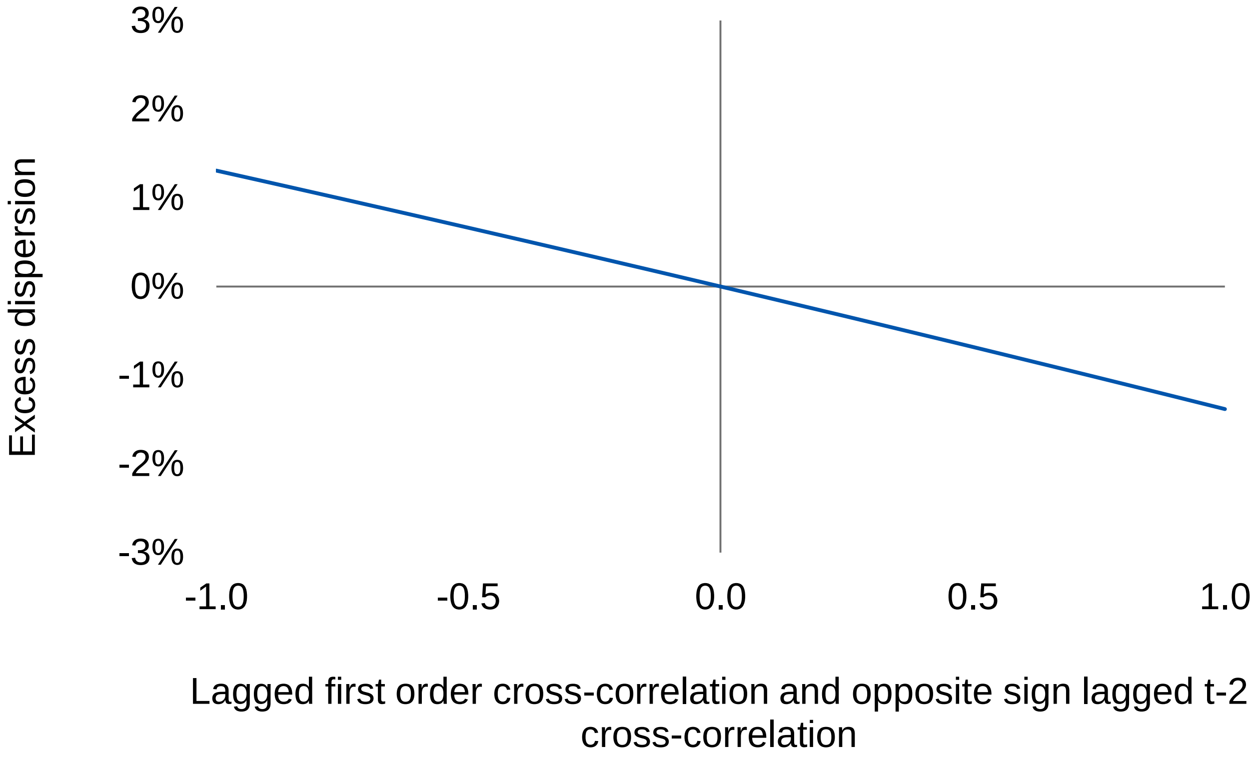
The relation of high- and low- frequency correlation: Impact of first-order cross-correlation



Notes: This example assumes x and y have a contemporaneous correlation of zero.

Source: State Street Associates

The relation of high- and low- frequency: Shorter versus longer lags



Notes: This example assumes x and y have a contemporaneous correlation of zero.

Source: State Street Associates

Low-frequency tracking error

We can now use the long-horizon standard deviations of x and y, together with their correlation, to compute the standard deviation of their relative performance after q periods, which we define as the tracking error between x and y:

$$TE(x,y) = \sqrt{\sigma_x^2 + \sigma_y^2 - 2\rho\sigma_x\sigma_y}$$

Implications for portfolio construction

Balancing high- and low-frequency optimality

- If investors care only about performance over short horizons or within long horizons they could construct portfolios that reflect aversion to risk based on covariances of high-frequency returns.

$$E(U) = \mu - \lambda_H \sigma_H^2$$

- Alternatively, if they are concerned only with performance at the conclusion of long horizons, they could estimate covariances of low-frequency returns.

$$E(U) = \mu - \lambda_L \sigma_L^2$$

- Or, as is the most likely case, they care about performance over both short and long horizons, they could include separate estimates of risk; one based on covariances of high-frequency returns and one based on covariances of low-frequency returns.

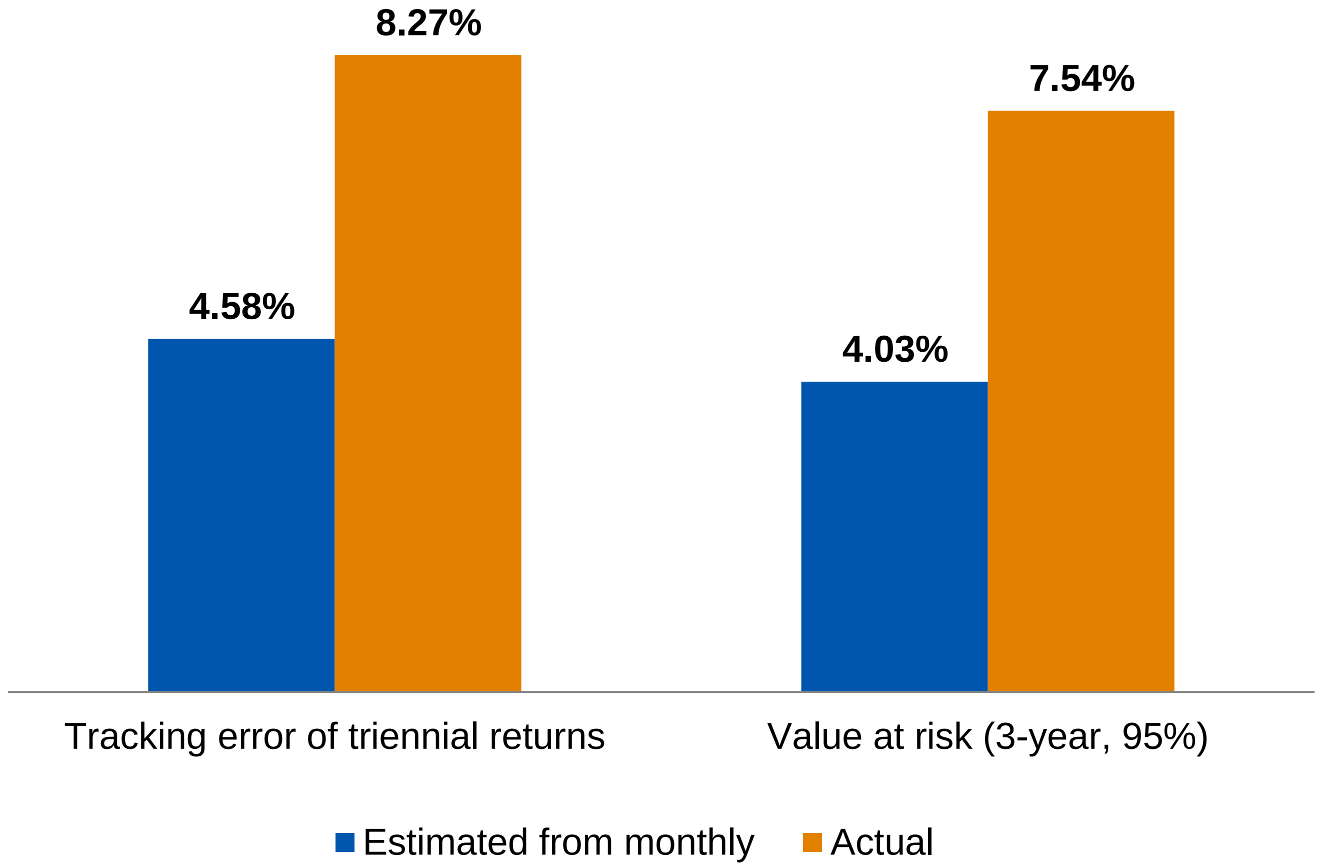
$$E(U) = \mu - \lambda_H \sigma_H^2 - \lambda_L \sigma_L^2$$

Benchmark and portfolio weights

Asset Class	Benchmark Weight	Portfolio Weight
U.S. Large Cap	24.33%	44.33%
U.S. Small Cap	2.70%	2.70%
EAFE Equities	19.64%	9.64%
Emerging Equities	5.80%	0.80%
Global Sovereigns	23.76%	18.76%
U.S. Government Bonds	11.88%	6.88%
U.S. Corporate Bonds	11.88%	16.88%

Source: State Street Associates

Low-frequency underperformance risk



Source: State Street Associates

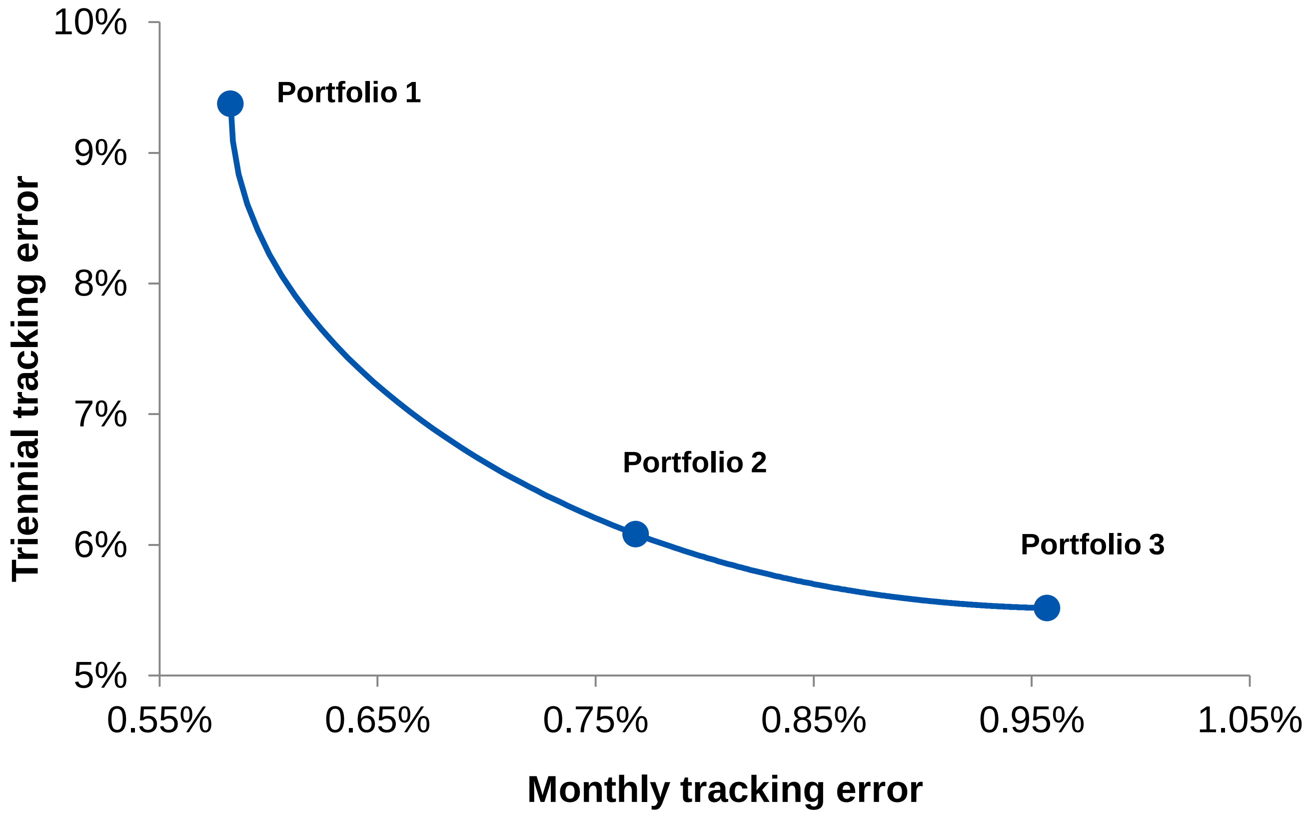
Standard deviations and correlations

MONTHLY	Standard Deviation	Correlation						
U.S. Large Cap	4.29%	1.00						
U.S. Small Cap	5.59%	0.81	1.00					
EAFE Equities	5.07%	0.75	0.66	1.00				
Emerging Equities	6.81%	0.69	0.70	0.73	1.00			
Global Sovereigns	1.93%	0.10	0.01	0.31	0.08	1.00		
U.S. Government Bonds	1.23%	-0.06	-0.16	-0.08	-0.16	0.61	1.00	
U.S. Corporate Bonds	2.51%	0.28	0.20	0.28	0.24	0.49	0.68	1.00

TRIENNIAL	Standard Deviation	Correlation						
U.S. Large Cap	42.98%	1.00						
U.S. Small Cap	33.49%	0.73	1.00					
EAFE Equities	35.34%	0.57	0.71	1.00				
Emerging Equities	62.30%	0.04	0.53	0.71	1.00			
Global Sovereigns	12.14%	-0.15	0.06	-0.06	0.26	1.00		
U.S. Government Bonds	8.10%	-0.04	-0.13	-0.51	-0.29	0.47	1.00	
U.S. Corporate Bonds	15.20%	0.23	0.42	-0.07	0.04	0.42	0.42	1.00

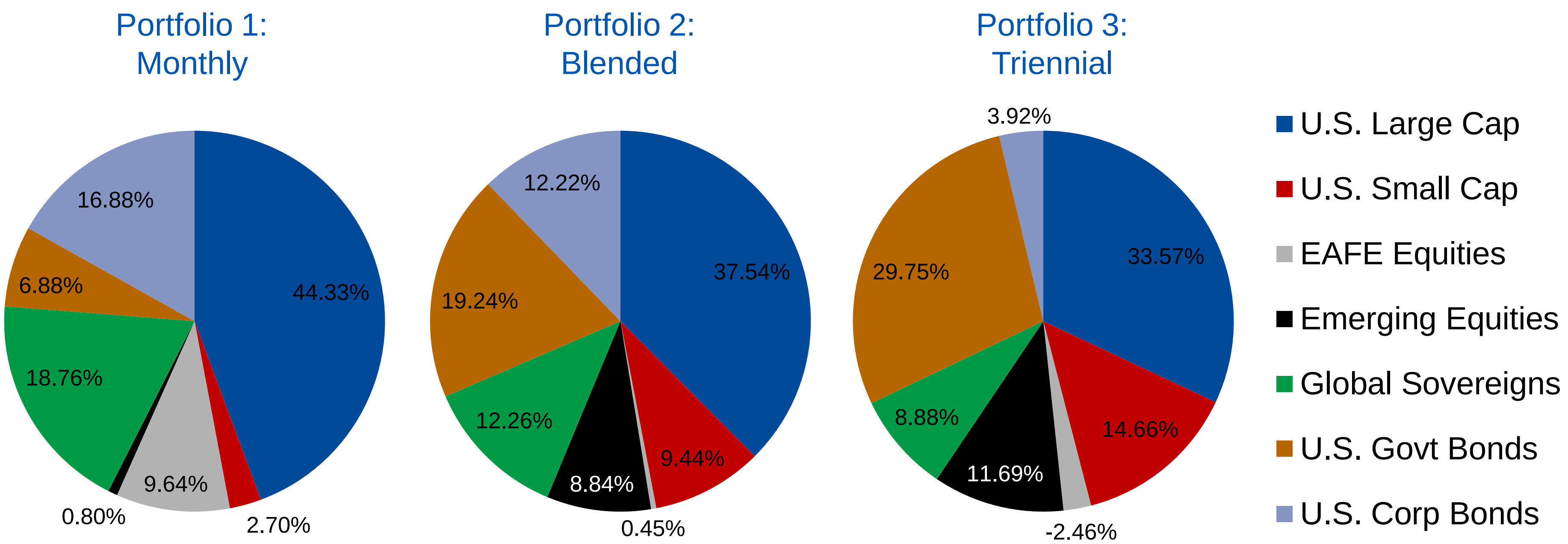
Source: State Street Associates

Iso-expected return curve balancing high- and low-frequency tracking error



Source: State Street Associates

Portfolio weights



Source: State Street Associates

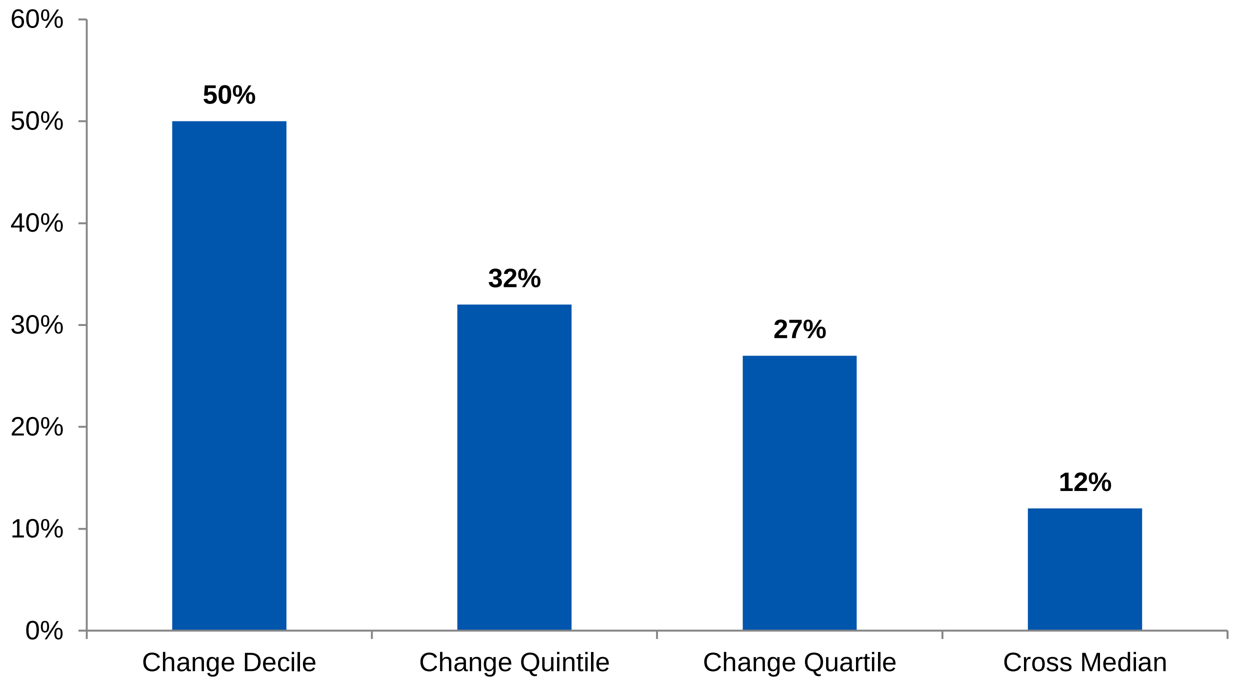
Implications for performance measurement

The distortion of hedge fund performance

- The conventional practice for evaluating the Sharpe ratios of hedge funds is to estimate their standard deviations from monthly returns and then to multiply these monthly standard deviations by the square root of 12.

- This approach implicitly assumes that hedge fund returns are serially independent at all lags, which is not the case.

Percentage of hedge funds that change quantile
Monthly versus triennial Sharpe ratio



Source: State Street Associates

Percentage of hedge funds within categories that change quantile

Monthly versus triennial Sharpe ratio

Hedge Fund Style	Number of Funds	Percentage of Funds that: Change Decile	Change Quintile	Change Quartile	Cross Median
U.S. Long/Short Equity	110	56%	29%	30%	13%
International Long/Short Equity	57	67%	40%	33%	21%
Global Macro	32	47%	38%	31%	6%
Debt	27	52%	44%	33%	22%
Long-Only Equity	16	56%	50%	44%	6%
Fund of Funds/Multi-strategy	209	43%	26%	22%	10%
Systematic Futures	73	52%	32%	25%	8%
Other	45	49%	36%	29%	18%
All Hedge Funds	**569**	50%	32%	27%	12%

Sample includes 569 hedge funds from the CISDM/Morningstar database that report monthly returns from January 2000 through December 2013. Sharpe ratios are computed from monthly and triennial return and standard deviation, as well as the monthly and triennial return of the JP Morgan U.S. Cash Index (0.20% and 7.39%, respectively) over the same period. To increase the sample size with each category, we consolidated the CISDM/Morningstar style categories as follows: U.S. Long/Short Equity includes Equity Market Neutral, U.S. Long/Short Equity, and U.S. Small Cap Long/Short Equity; International Long/Short Equity includes Asia/Pacific Long/Short Equity, Emerging Markets Long/Short Equity, Europe Long/Short Equity, and Global Long/Short Equity; Global Macro includes Global Macro and Currency; Debt includes Debt Arbitrage, Distressed Securityies, and Long/Short Debt; Long-Only Equity includes Long-Only Equity and Emerging Markets Long-Only Equity; Fund of Funds/Multistrategy includes Multistrategy funds as well as six Fund of Funds categories (Macro/Systematic, Debt, Equity, Event, Multistrategy, and Relative Value); Systematic Futures was not consolidated; Other includes Bear Market Equity, Convertible Arbitrage, Event Driven, Long-Only Other, and Merger Arbitrage.

Source: State Street Associates

The distortion of mutual fund performance

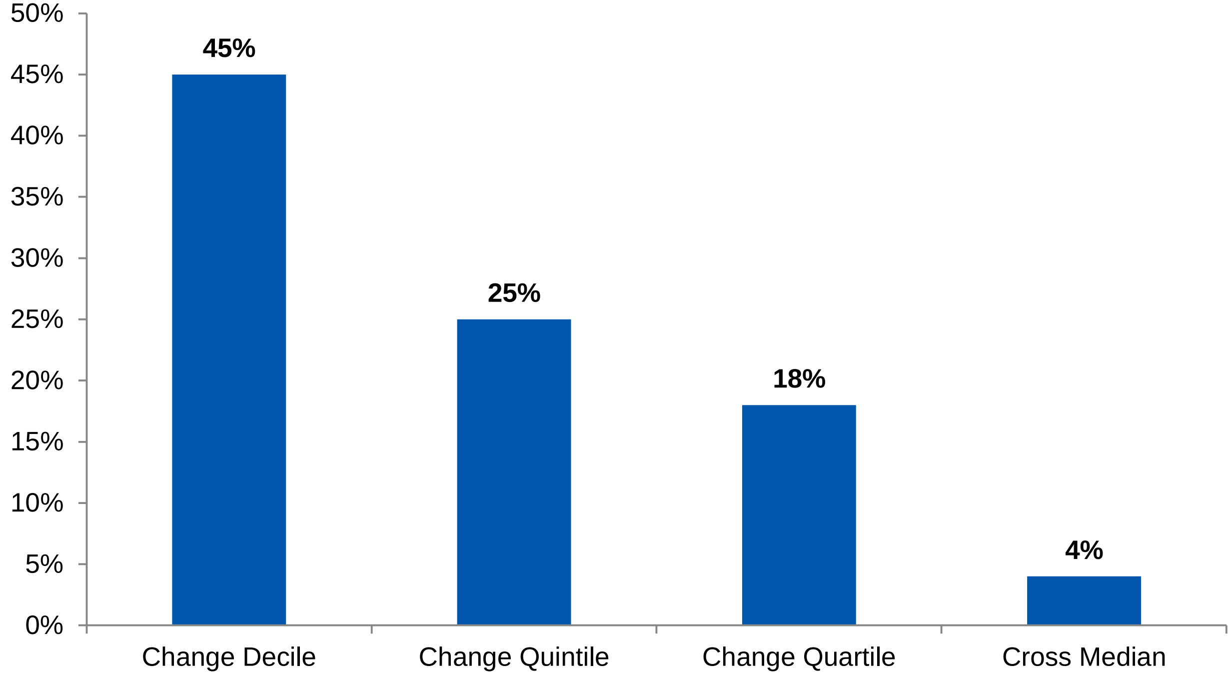
- Unlike hedge funds, mutual funds are usually evaluated by their performance relative to their given benchmarks.

- They are compared to each other based on their information ratios, which equals a fund's excess return relative to its benchmark divided by its excess risk relative to its benchmark.

- The conversion of monthly information ratios to longer-interval information ratios is therefore subject to three sources of distortion:
  - the auto-correlations of the fund's returns,
  - the auto-correlations of the benchmark's returns, and
  - the lagged cross-correlations between the fund's returns and the benchmark's returns.

Percentage of mutual funds that change quantile

Monthly versus triennial Sharpe ratio



Source: State Street Associates

Percentage of mutual funds within categories that change quantile

Monthly versus triennial Sharpe ratio

		Percentage of Funds that:			
Mutual Fund Style	**Number of Funds**	Change Decile	Change Quintile	Change Quartile	Cross Median
U.S. Large Cap Blend	786	49%	30%	22%	7%
U.S. Small Cap Blend	358	49%	28%	18%	2%
International Large Cap Blend	392	33%	15%	11%	2%
All Mutual Funds	**1536**	**45%**	**25%**	**18%**	**4%**

The U.S. Large Cap Blend sample includes 786 out of 1,428 mutual funds listed in the "Large Blend" category on Yahoo! Finance that meet the following two criteria: monthly returns are available from January 2008 through December 2013 and tracking error relative to the S&P 500 is greater than 1% per annum. We impose the latter filter to eliminate index funds. The U.S. Small Cap Blend sample includes 358 out of 640 funds listed in the "Small Blend" category that meet the same criteria. The Foreign Large Cap Blend sample includes 392 funds listed in the "Foreign Large Blend" category whose monthly returns are available for the same period and whose name does not include "Idx" or "Index" (unless it also includes "Enhanced"). Monthly total returns are from Bloomberg. Returns are net of fees and assume reinvestment of dividends. The benchmarks are S&P 500, Russell 2000, and MSCI All Country World ex USA, respectively.

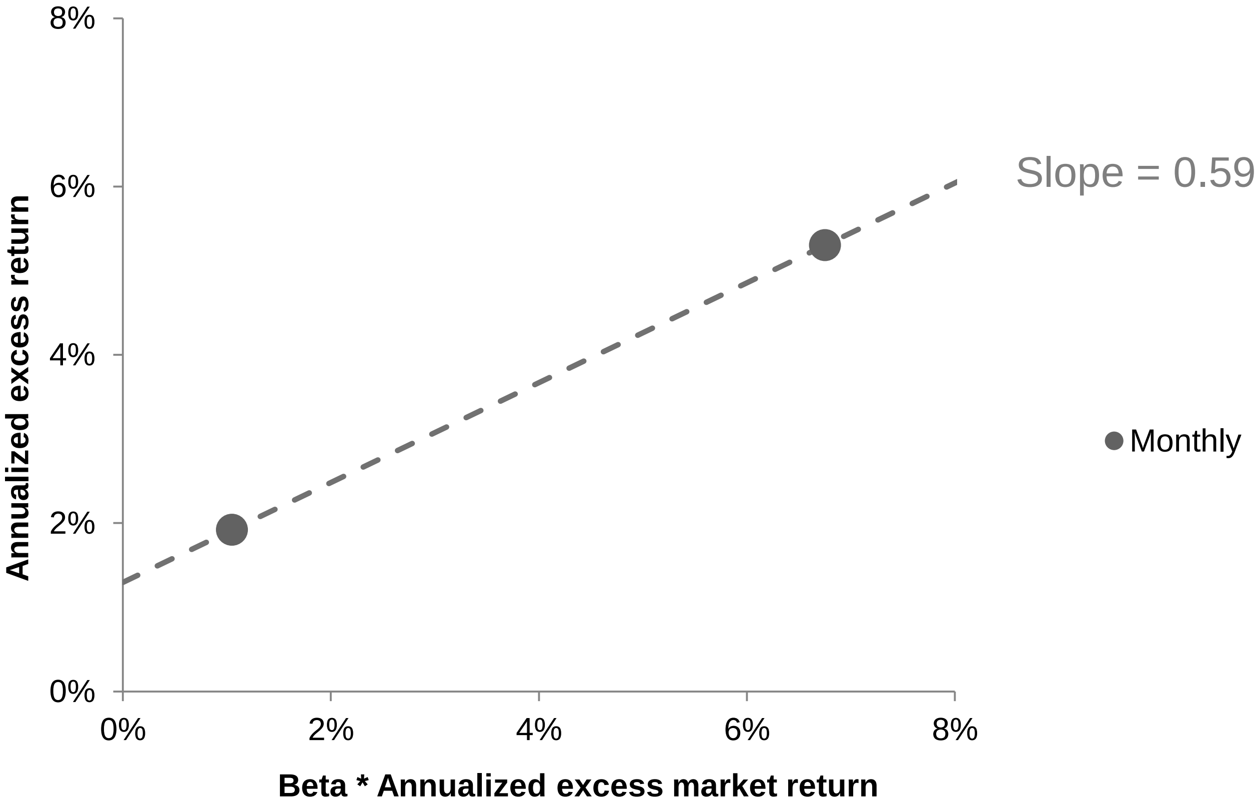
Source: State Street Associates

The fallacy of risk parity

The fallacy of risk parity

- Risk parity over weights low-risk assets relative to high-risk assets.
- It has been shown to outperform portfolios that are not biased toward low-risk assets.
- Conceptually, risk parity delivers the highest Sharpe ratio if all assets have identical Sharpe ratios and correlations.
- It is argued that its outperformance is enhanced because the security market line appears to have a slope that is significantly less than 1.0, implying that low-risk assets have higher risk-adjusted returns than high-risk assets.
- These observations do not hold when we account for lagged auto- and cross-correlations.

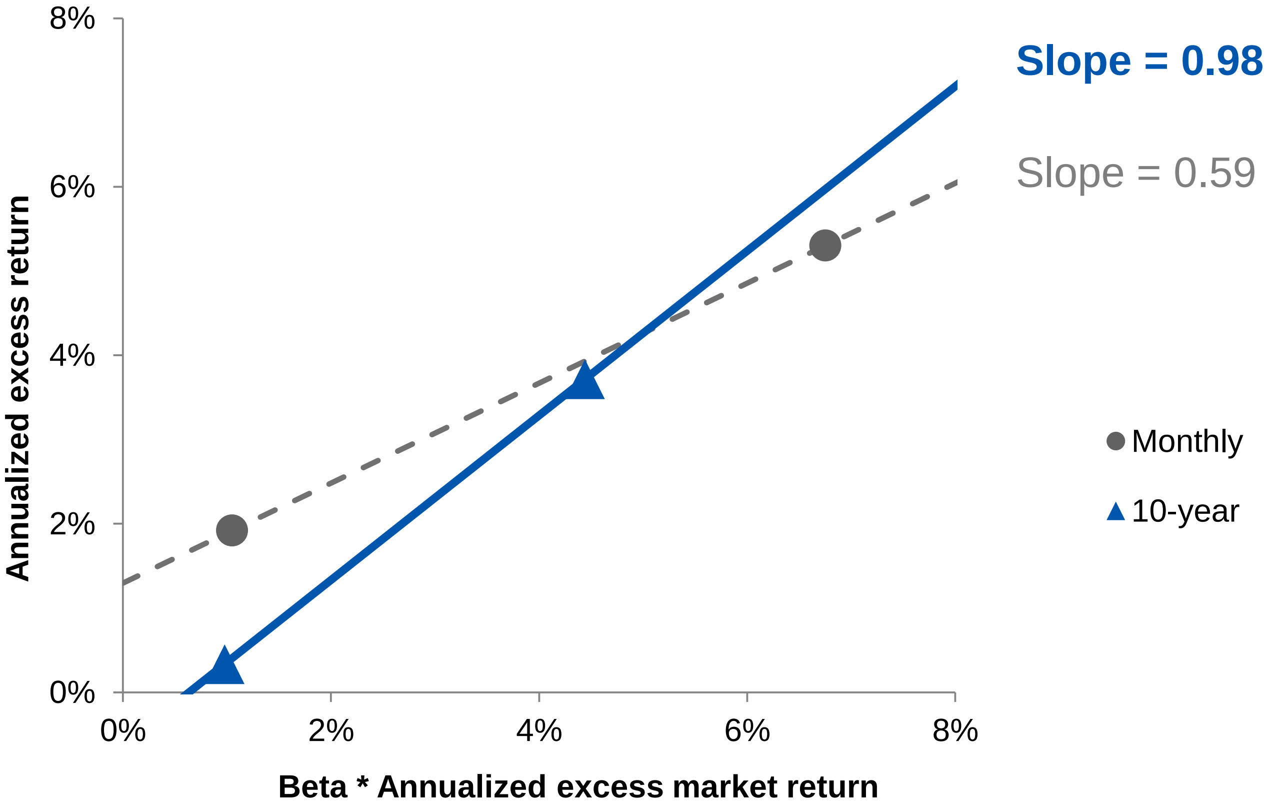
Security market line
U.S. stocks and government bonds (1929 – 2010)



Average annual excess returns are calculated as the annualized geometric mean of historical returns, minus the annualized geometric mean of historical 1-month treasury bill yields (or of 10-year treasury bond yields) over the same period. Standard deviations, betas and correlations for the 10-year holding period are calculated from historical rolling log returns in excess of the 10 year treasury yield, and splice the beginning of the data set onto the end to avoid unequal representation of returns in statistics. Returns for stocks and bonds are from Ibbotson. The market portfolio is a 60/40 stock/bond portfolio rebalanced monthly.

Source: State Street Associates

Security market line
U.S. stocks and government bonds (1929 – 2010)



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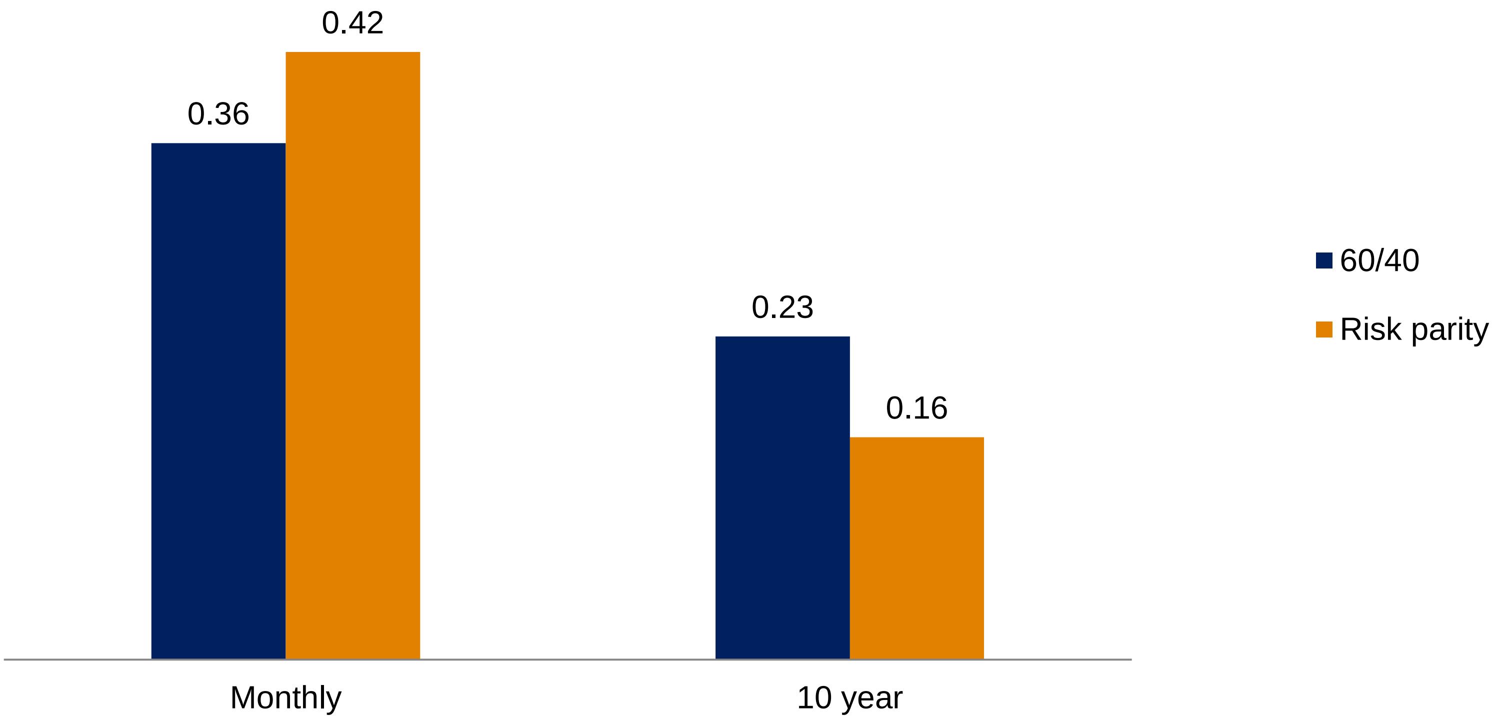
Source: State Street Associates

Annalized Sharpe ratios

U.S. stocks and government bonds (1929 – 2010)

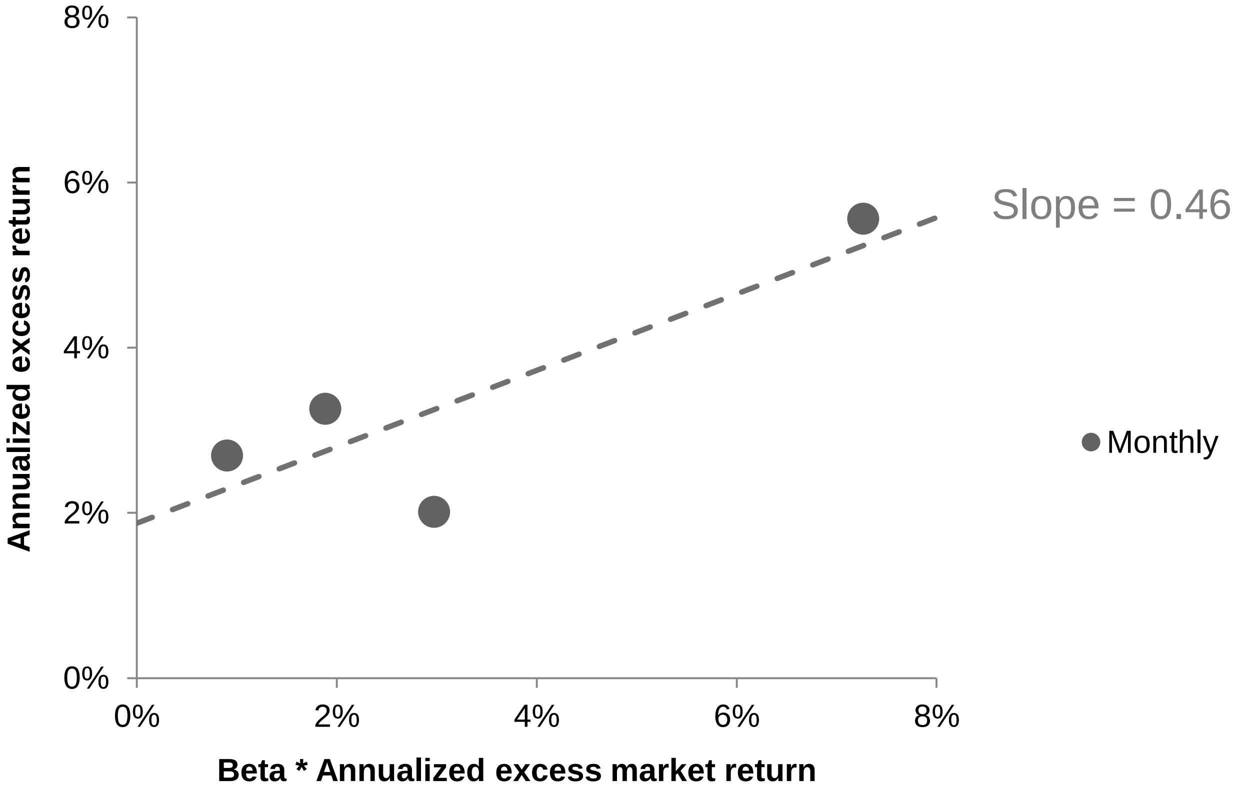


Risk parity weights each month are proportional to the inverse of the trailing 36 month asset volatilities, rescaled to sum to one so there is no leverage. Average annual excess returns are calculated as the annualized geometric mean of historical returns, minus the annualized geometric mean of historical 1-month treasury bill yields (or of 10-year treasury bond yields) over the same period. Standard deviations for the 10-year holding period are calculated from historical rolling log returns in excess of the 10 year treasury yield, and splice the beginning of the data set onto the end to avoid unequal representation of returns in statistics. Stock and bond data are from Ibbotson.

Source: State Street Associates

Security market line

U.S. stocks, treasuries, corporate bonds, and commodities (1976 – 2010)



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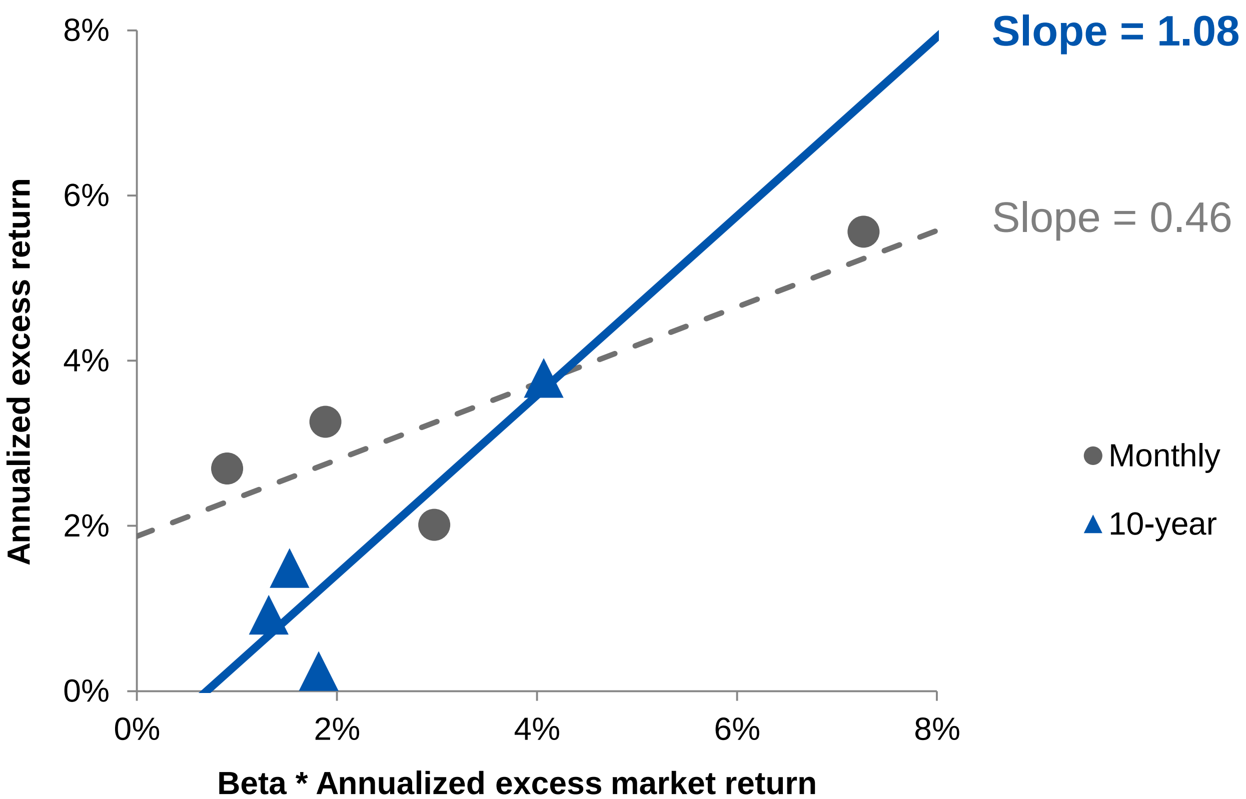
Source: State Street Associates

Security market line

U.S. stocks, treasuries, corporate bonds, and commodities (1976 – 2010)



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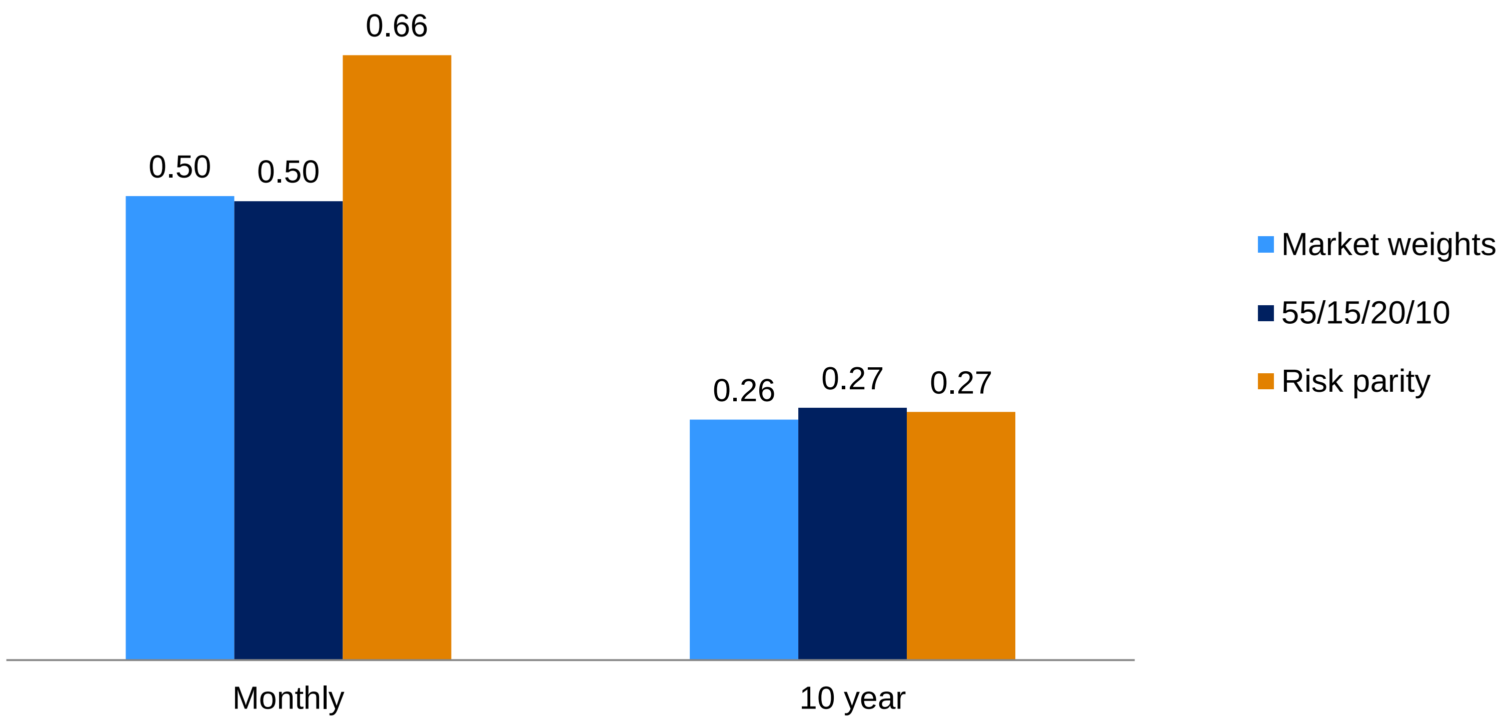

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Source: State Street Associates

Summary

- Financial analysts are often surprised by the extent to which assets that are thought to be strongly correlated diverge over time.
- This result can occur within the same sample from which the parameters are derived. It occurs because one or both assets may have non-normal returns, or because they are auto-correlated or cross-correlated at one or more lags.
- We find evidence of excess dispersion for a variety of asset pairs, and we attribute this excess dispersion to its sources. We find that most excess dispersion arises from non-zero auto-correlations and lagged cross-correlations.
- We mathematically relate high frequency estimates of mean, standard deviation, and correlation to their low frequency values, and investigate comparative statics.
- We introduce a portfolio construction framework that jointly accounts for aversion to high- and low-frequency risk.
- We provide evidence that the risk-adjusted performance of hedge funds, mutual funds, and risk parity strategies is highly sensitive to the return interval used to estimate it.

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