Multi-Asset Class Risk Models

Overcoming the Curse of Dimensionality

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Outline

- Motivation and Overview
- Candidate Models
- Evaluating the Accuracy of Correlation Forecasts
  - Factor-pair portfolios
- Evaluating the Quality of Optimized Portfolios
  - Out-of-sample volatility
  - Turnover
  - Leverage
- Ranking the Candidate Models
- MAC2 versus MAC1 Comparison
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Motivation and Overview

Multi-Asset Factor Covariance Matrices

- Portfolios may have exposure to multiple asset classes
- Each asset class is composed of multiple local markets
- Each local market is explained by many local factors

To obtain accurate risk forecasts for any portfolio requires a covariance matrix that combines all of the local factors

MAC Covariance Matrix

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{EQ} & \mathbf{F}_{FI}^{EQ} & \cdots & \mathbf{F}_{FX}^{EQ} \\ \mathbf{F}_{EQ}^{FI} & \mathbf{F}_{FI} & \cdots & \mathbf{F}_{FX}^{FI} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_{EQ}^{FX} & \mathbf{F}_{FI}^{FX} & \cdots & \mathbf{F}_{FX} \end{bmatrix}$$

Global Equity Block

$$\mathbf{F}_{EQ} = \begin{bmatrix} \mathbf{F}_{USA} & \mathbf{F}_{JAP}^{USA} & \cdots & \mathbf{F}_{EUR}^{USA} \\ \mathbf{F}_{USA}^{JAP} & \mathbf{F}_{JAP} & \cdots & \mathbf{F}_{EUR}^{JAP} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_{USA}^{EUR} & \mathbf{F}_{JAP}^{EUR} & \cdots & \mathbf{F}_{EUR} \end{bmatrix}$$

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The "Curse of Dimensionality"

- Forecasting accuracy requires a detailed factor structure spanning all markets and asset classes
  - Bloomberg MAC covariance matrix contains nearly 2000 factors
- Portfolio construction demands a *robust* covariance matrix
  - Risk model should not identify spurious hedges that fail out-of-sample
- With fewer observations than factors ($T<K$), sample covariance matrix contains one or more "zero eigenvalues"
  - Leads to spurious prediction of "riskless" portfolios
- This feature makes the sample covariance matrix unsuitable for portfolio construction

> *Special methods are required to simultaneously provide:*
> 1. *Accurate volatility forecasts (risk management)*
> 2. *Robust risk models (portfolio construction)*

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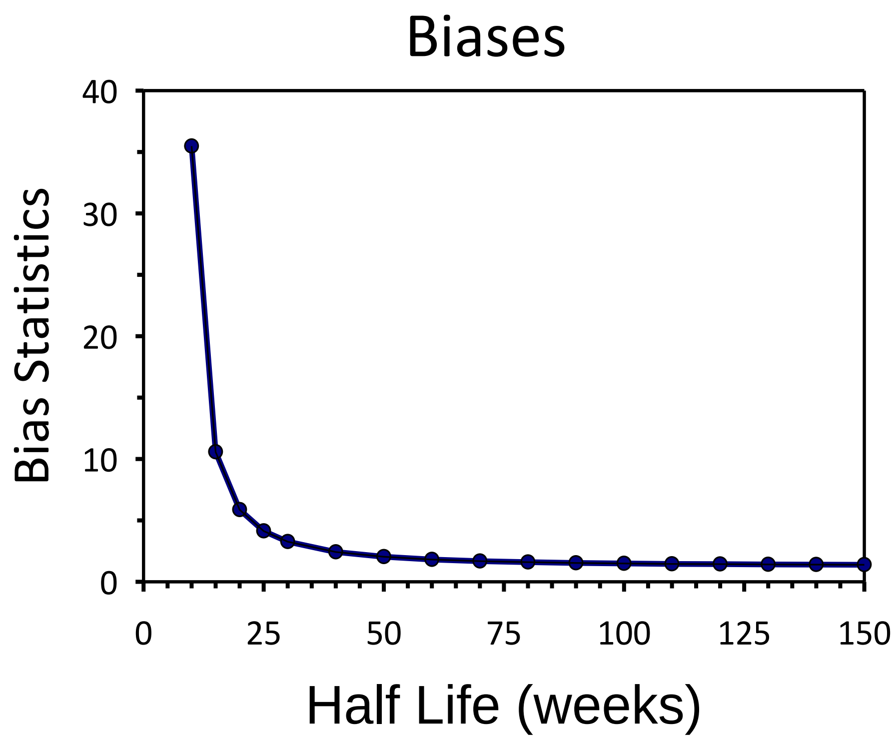
Case Study: The Perils of Non-Robust Models

- Take the largest 100 US equities as of 16-Sept-2015, with complete return history to 13-Jan-1999
- Estimate family of asset covariance matrices using EWMA with a variable half-life
- Each week, construct the minimum-volatility fully invested portfolio
- Bias statistic represents ratio of realized risk to forecast risk
- Risk forecasts become increasingly poor as the half-life is shortened



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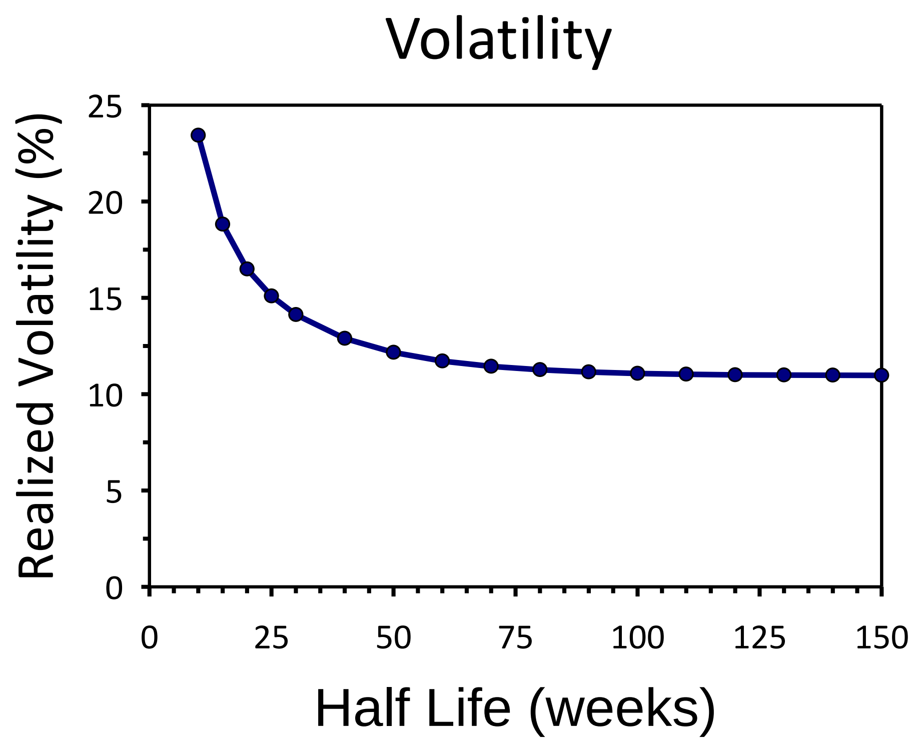
Out-of-Sample Volatility

- Mean-variance optimization produces a portfolio with the minimum *ex ante* volatility for a given level of factor exposure
- If all stocks have the same expected return, the minimum volatility fully invested portfolio has the maximum Sharpe ratio

- Realized volatility increases as half-life parameter shortens:
  - Equal-weighted portfolio had realized volatility of 16.5 percent
  - For HL<25w, optimization actually led to *increased* portfolio volatility



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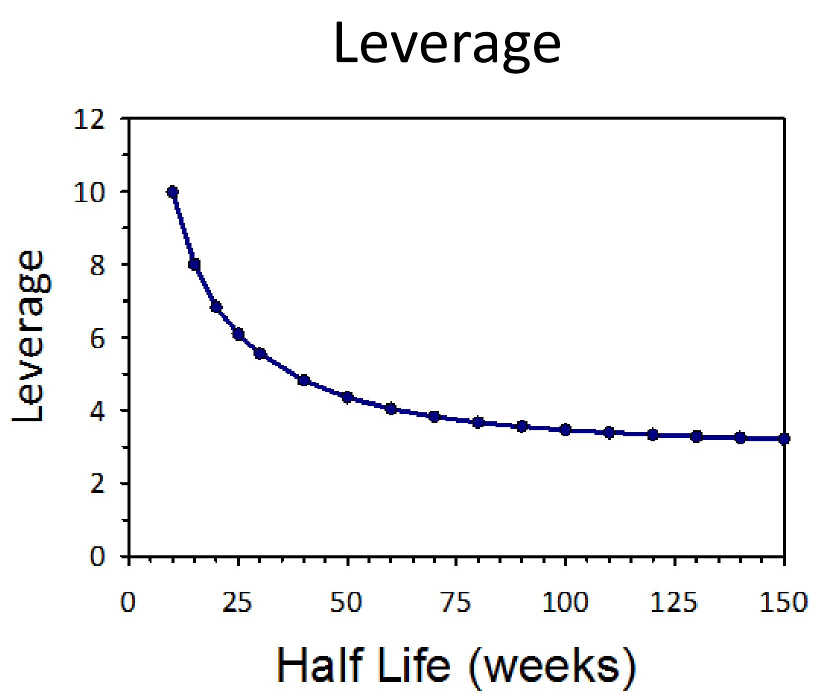
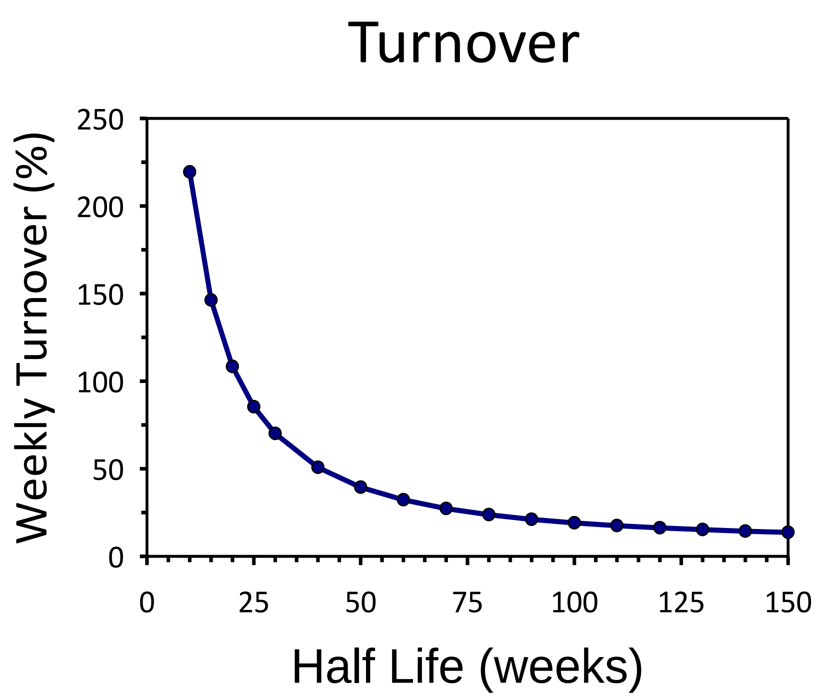
Leverage and Turnover

- Portfolio leverage increased sharply with shorter half-life
  - Half-life of 10 weeks produced a leverage of 10
  - Resulting portfolio is 550% long and 450% short
- Turnover rises dramatically with shorter half-life
  - 10-week half life produced more than 200% weekly turnover





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Candidate Models

Separating Volatilities and Correlations

- Divide the task of constructing a factor covariance matrix into two parts:
  - Estimate the factor volatilities
  - Estimate the factor correlation matrix
- Factor volatilities are typically estimated using a relatively short half-life parameter (i.e., responsive forecasts)
- Factor correlations typically use longer half-life parameters
  - Reduces noise in the correlation matrix
  - Produces accurate risk forecasts
- The factor covariance matrix is easily reconstructed:

$$F_{jk} = \rho_{jk}\sigma_j\sigma_k$$

- Present study focuses on comparing the model quality of *correlation matrices* for the equity block

Sample Correlation Matrix

- Sample correlation matrix possesses many attractive properties:
  - Provides arguably the best estimate for any pairwise correlation
    - Best Linear Unbiased Estimate (BLUE) under standard econometric assumptions
  - Gives intuitive and transparent estimates, since it is based on the "textbook" definition of correlation coefficient
  - Produces accurate risk forecasts for most portfolios (with the notable exception of optimized portfolios)

- Sample correlation matrix also possesses an "Achilles heel":
  - If there are K factors and T periods, then sample correlation matrix contains zero eigenvalues (i.e., rank-deficient matrix) whenever $T<K$
  - Rank-deficient matrices predict the existence of "phantom" riskless portfolios that do not exist in reality
  - Sample correlation is ***not*** robust for portfolio construction

Objectives: (a) correlation estimates should closely mimic the sample, and (b) provide robust forecasts for portfolio construction purposes

Other Techniques for Estimating Correlations

- Principal Component Analysis (PCA)
  - Statistical technique to extract global factors from the data
  - Assume a small number of global factors (principal components) fully capture correlations of local factors (i.e., uncorrelated residuals)
- Random Matrix Theory (RMT)
  - Statistical technique similar to PCA (factors extracted from data)
  - Eigenvalues beyond a cutoff point are simply averaged
- Time-series Approach
  - Specify "global" factor returns to explain "local" factor correlations
  - Estimate the exposures by time-series regression
- Eigen-adjustment Method
  - Eigenvalues of sample correlation matrix are systematically biased
  - Adjust the eigenvalues to remove biases

Menchero, Wang, and Orr. *Improving Risk Forecasts for Optimized Portfolios,* Financial Analysts Journal, May/June 2012, pp. 40-50

Blended Correlation Matrices

- Ledoit and Wolf (2003) showed that blending the sample covariance matrix with a one-factor model yielded optimized fully invested portfolios with lower out-of-sample volatility
- Blend sample correlation (using weight *w*) with PCA correlation using *J* principal components derived from *K* local factors
- Specify number of PCA factors by parameter μ , where $J=\mu K$
- Two-parameter model for correlation matrix:

$$\mathbf{C}_B(\mu, w) = w\mathbf{C}_0 + (1-w)\mathbf{C}_P(\mu)$$

Blended Matrix

- Optimal blending parameters are determined empirically
- Technique represents the new Bloomberg methodology

Ledoit and Wolf. *Improved Estimation of the Covariance matrix of Stock Returns,* Journal of Empirical Finance, December 2003, pp. 603-621

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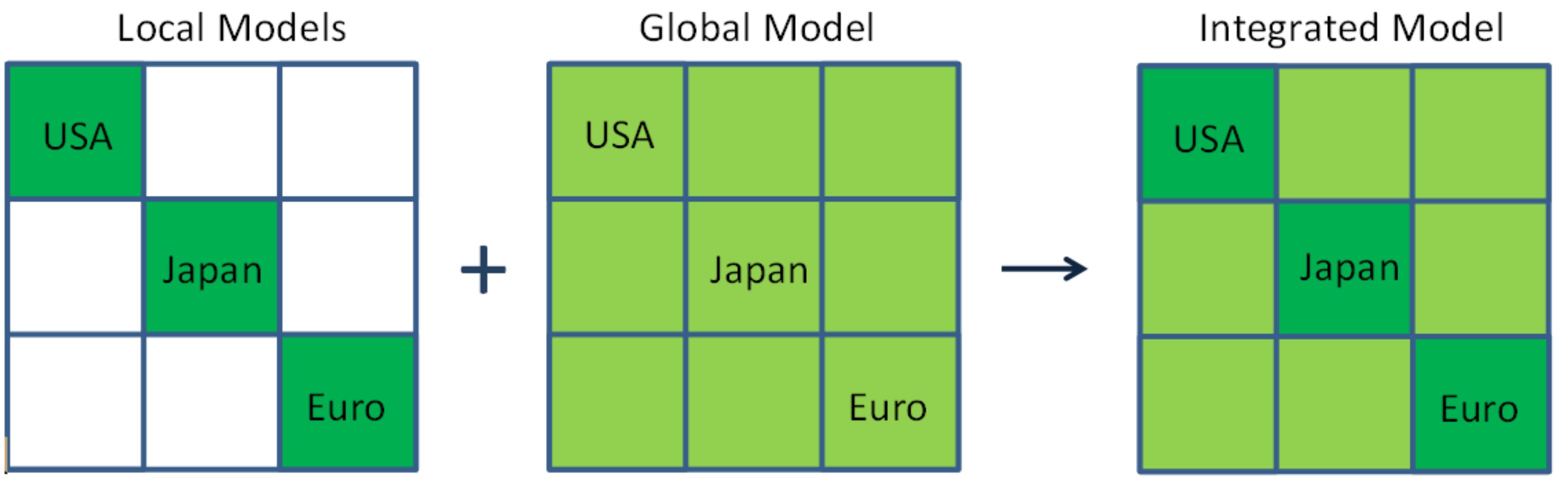
Adjusted Correlation Matrices

- Local models provide our "best" estimates of the correlation matrices for the diagonal blocks
- Global model is used to estimate the off-diagonal blocks
- Diagonal blocks of the global model differ from the correlation matrices obtained from the local models



- Integrated model is formed by "adjusting" the global model to replicate the local models along the diagonal blocks

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Evaluating the Accuracy of Correlation Forecasts

Measuring Biases in Risk Forecasts

- Bias statistic represents the ratio of forecast risk to realized risk

$$z_{nt} = \frac{r_{nt}}{\sigma_{nt}} \quad \rightarrow \quad B_n = \text{std}\left(z_{nt}\right) \quad \rightarrow \quad \bar{B} = \frac{1}{N}\sum_n B_n$$

Bias Statistic

- If the risk forecasts are *exactly correct*, the expected value of the bias statistic is precisely equal to 1
- If the risk forecasts are *unbiased* but noisy, the expected value of the bias statistic is *slightly greater* than 1
- Example: suppose we over-forecast volatility by 10% half of the time, and under-forecast by 10% half the time

$$E\left[B\right] = \sqrt{\frac{1}{2}\left(\frac{1}{0.9}\right)^2 + \frac{1}{2}\left(\frac{1}{1.1}\right)^2} = 1.02$$

- Typical bias statistic for unbiased risk forecasts is about 1.03

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Factor-Pair Portfolios

- Construct test portfolios capable of resolving minor differences in volatility forecasts due to differences in correlations
- Consider factor-pair portfolios

$$R = f_1 + w f_2 \quad \rightarrow \quad \sigma^2 = \sigma_1^2 + w^2 \sigma_2^2 + 2\rho w \sigma_1 \sigma_2$$

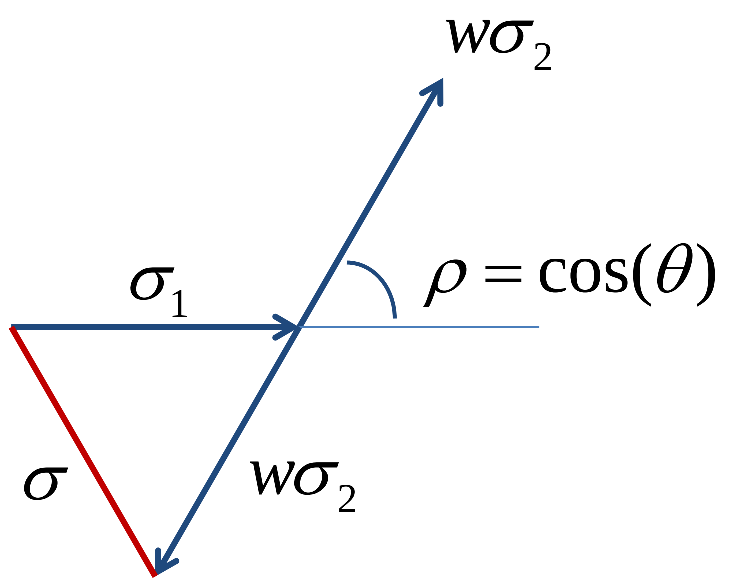
- Solve for the weight *w* that maximizes the percentage of risk due to the off-diagonal correlation

- Solution is given by

$$\boxed{w = \pm\left(\sigma_1 / \sigma_2\right)}$$

- Portfolio volatility

$$\sigma = \sqrt{2}\sigma_1 \left(1 - |\rho|\right)^{1/2}$$



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Description of Study

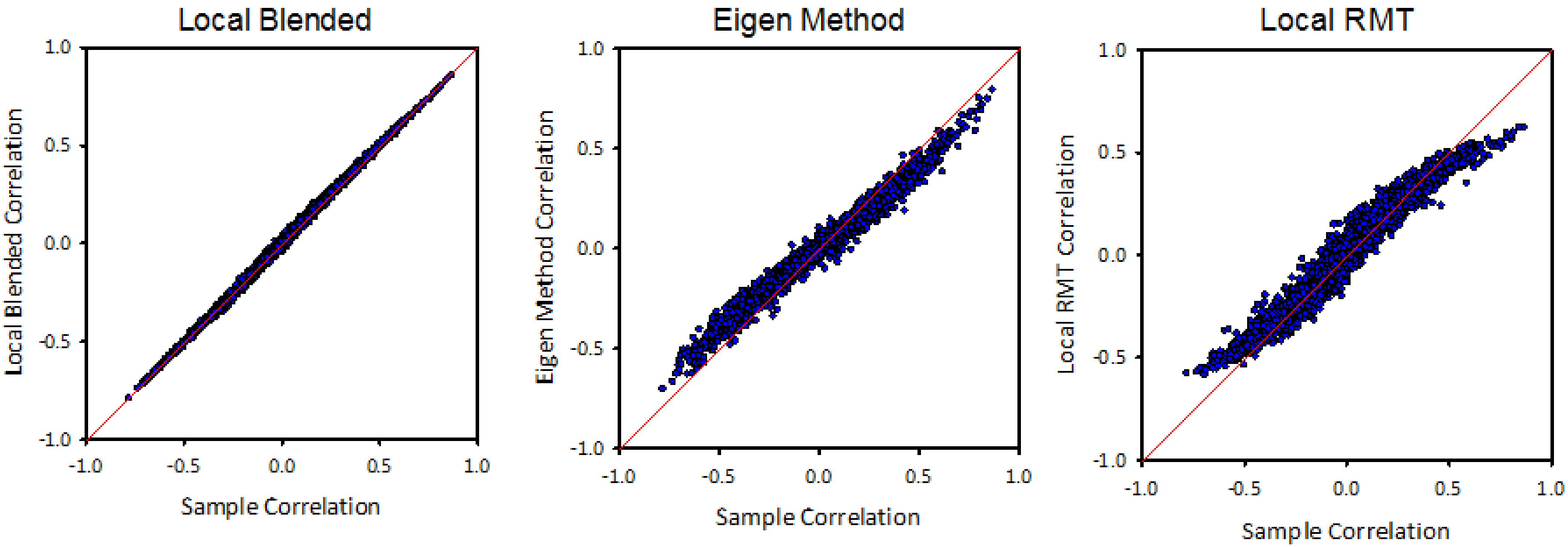
- Sample period contains 713 weeks (03-Jan-01 to 27-Aug-14)
- Model contains K=319 factors spanning nine equity blocks
- Evaluate accuracy of correlations using factor-pair portfolios

Parameters used in Study:

- Use T=200 weeks (equal weighted) as estimation window
- For PCA, RMT, and blended matrices
  - Use μ_L=0.25 for local blocks
  - Use μ_G =0.10 for global block
- For blended correlation matrices
  - Assign 80% weight to the sample (w=0.8) for local blocks
  - Assign 20% weight to the sample (w=0.2) for global blocks
- Blending parameter selection criteria:
  - Small deviation from the sample correlation
  - Low realized volatility of optimized portfolios

Correlation Scatterplots (Diagonal Blocks)

- Local Blended provides a near "perfect fit" to the sample
- Eigen method and Local RMT exhibit systematic biases



- Compute B-stats for all factor-pairs with mean sample correlation above 0.40 (292 portfolios)
- Eigen method and Local RMT exhibit biases
- Sample and Local Blended are near ideal value

Model	Bias Stats
Sample	1.034
Local Blended	1.036
Eigen method	0.949
Local RMT	0.946

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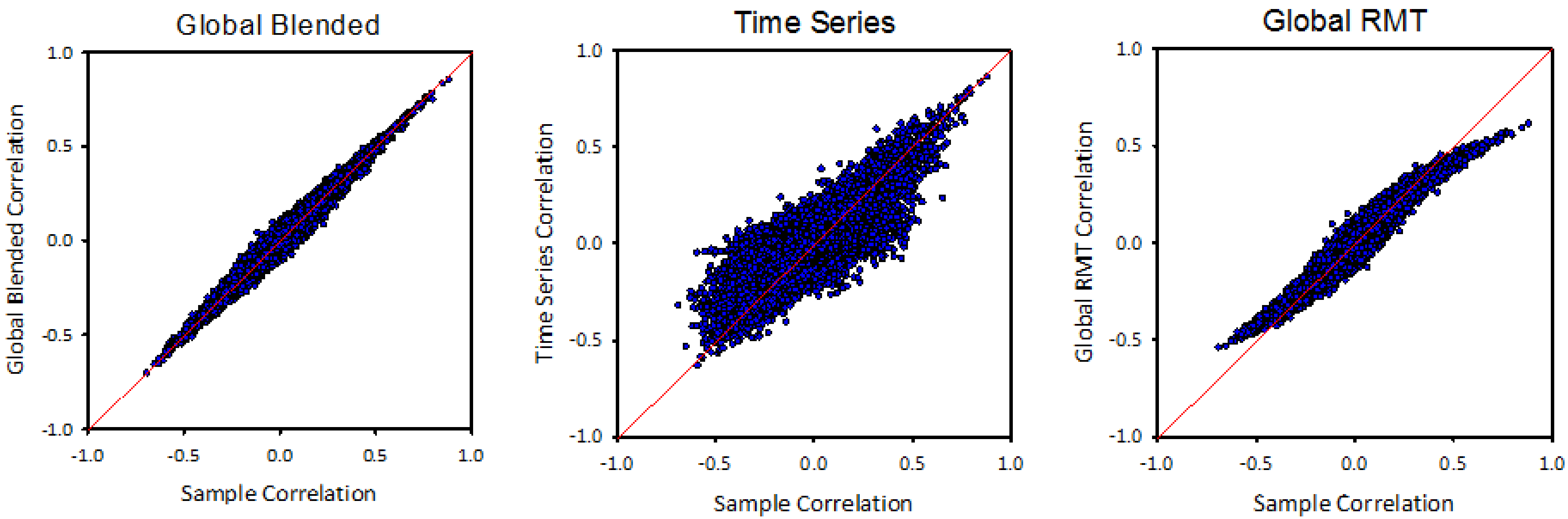
Correlation Scatterplots (Off-Diagonal Blocks)

- Global Blended provides an excellent fit to the sample
- Time Series and Global RMT exhibit systematic biases



- Compute B-stats for all factor-pairs with mean sample correlation above 0.50 (163 portfolios)
- Time Series and Global RMT exhibit biases
- Sample and Global Blended are near ideal value

Model	Bias Stats
Sample	1.034
Global Blended	1.030
Time Series	0.958
Global RMT	0.901

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Evaluating the Quality of Optimized Portfolios

Quality of Optimized Portfolios

- Portfolio optimization typically represents the most demanding task for any risk model (the “acid” test)
- Optimized portfolios have the maximum possible *ex ante* information ratio
- This implies that optimized unit-exposure portfolios have the minimum volatility

$$\mathbf{w}_k^A = \frac{\mathbf{\Omega}_A^{-1}\boldsymbol{\alpha}_k}{\boldsymbol{\alpha}_k'\mathbf{\Omega}_A^{-1}\boldsymbol{\alpha}_k}$$

Optimal portfolio (Model A)

- Define the mean volatility ratio for Model A

$$v_A = \frac{1}{K}\sum_k \frac{\sigma_k^A}{\sigma_k^{\text{Ref}}}$$

Model with lowest volatility ratio wins

- Construct optimal portfolios for each of *K*=319 factors and rebalance on a weekly basis (Jan-2001 to Aug-2014)

Portfolio Optimization (*Ex Ante*)

- Decompose optimal portfolio into alpha and hedge portfolios:

$$\mathbf{w} = \frac{\boldsymbol{\Omega}^{-1}\boldsymbol{\alpha}}{\boldsymbol{\alpha}'\boldsymbol{\Omega}^{-1}\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha} + \mathbf{h}$$

$$\mathbf{w} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_N \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_N \end{bmatrix}$$

Alpha Hedge

- Hedge portfolio is uncorrelated with the optimal portfolio

$$\mathbf{h}'\boldsymbol{\Omega}\mathbf{w} = 0$$ Property 1

- The hedge portfolio has zero alpha

$$\mathbf{h}'\boldsymbol{\alpha} = 0$$ Property 2

- Hedge portfolio is negatively correlated with the alpha portfolio

$$\frac{\mathbf{h}'\boldsymbol{\Omega}\boldsymbol{\alpha}}{\sigma_h \sigma_\alpha} < 0$$ Property 3

Hedge portfolio reduces portfolio risk without changing the expected return

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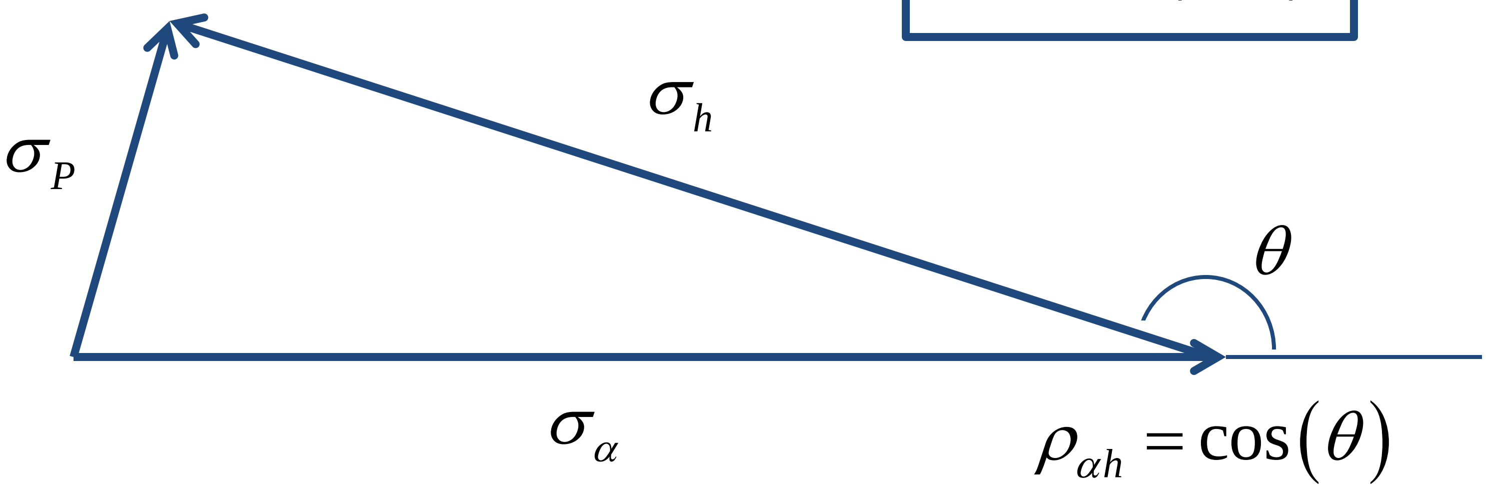
Geometry of Portfolio Optimization (*Ex Ante*)

- Hedge portfolio is uncorrelated with optimal portfolio

$$\rightarrow \quad \boxed{\sigma_P^2 = \sigma_\alpha^2 - \sigma_h^2}$$ Portfolio Variance

- Let $\rho_{\alpha h}$ denote the predicted correlation between $\boldsymbol{\alpha}$ and $\mathbf{h}$
- The magnitude of the correlation determines quality of hedge
- Optimal position in hedge portfolio: $$\boxed{\sigma_h = \sigma_\alpha \left|\rho_{\alpha h}\right|}$$



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Potential Pitfalls of Optimization

- Optimization leads to superior *ex ante* performance, but is no guarantee of improvement *ex post*
- Estimation error within the covariance matrix represents a potential pitfall in portfolio optimization

- Estimation error in the correlation:
  - Risk models “paint an overly rosy picture” of the correlation between the alpha and hedge portfolios

- Estimation error in the volatility:
  - Risk models may misestimate the volatility of the hedge portfolio

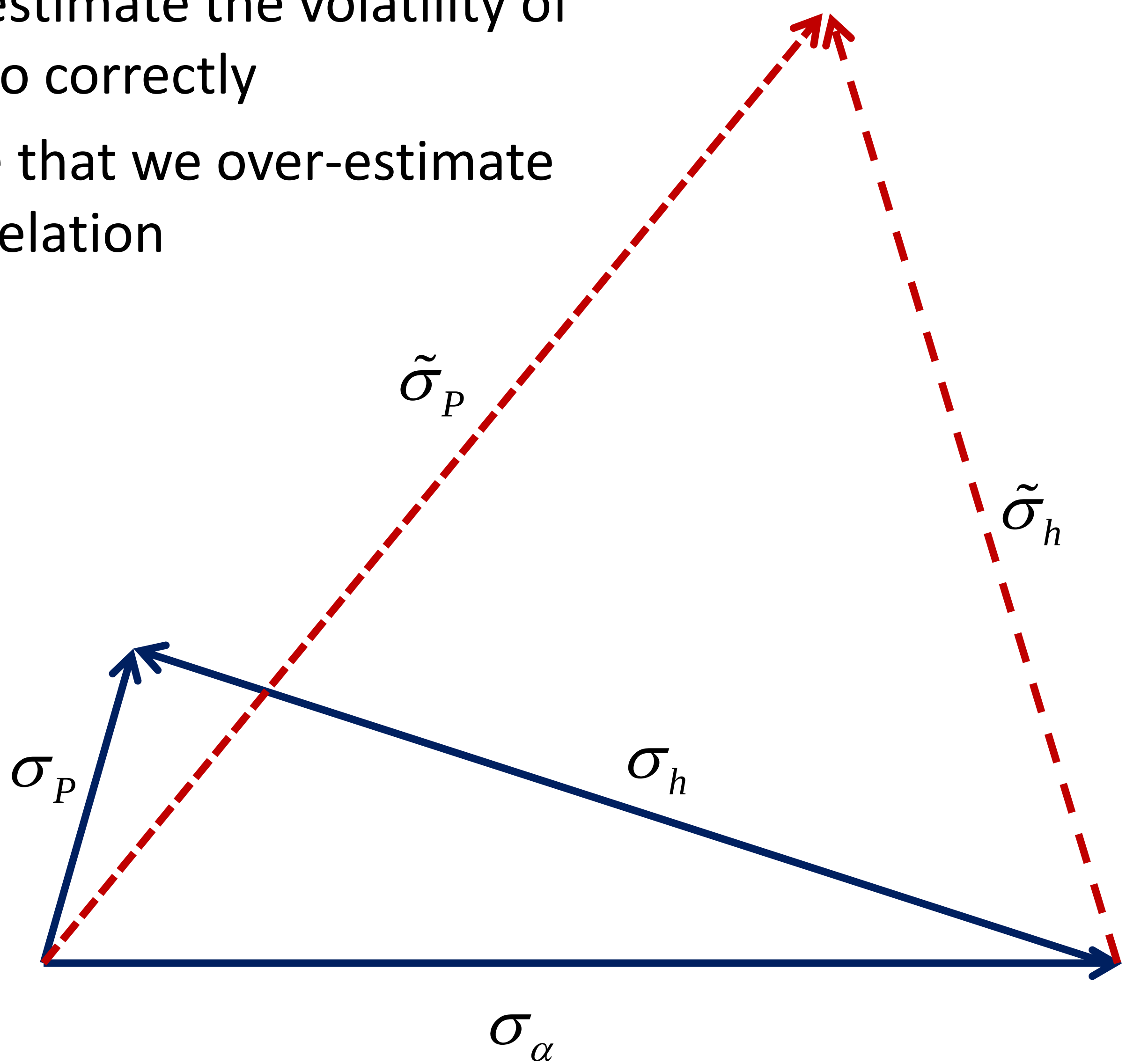
- Estimation error gives rise to several detrimental effects:
  - Underestimation of risk of optimized portfolios
  - Higher out-of-sample volatility of optimized portfolios
  - Positive realized correlation between optimized and hedge portfolios

Estimation Error in the Correlation

- Suppose that we estimate the volatility of the hedge portfolio correctly
- However, suppose that we over-estimate magnitude of correlation

<u>Side Effects</u>:

1) Optimized portfolio has high volatility out-of-sample
2) Hedge portfolio is positively correlated with optimized portfolio



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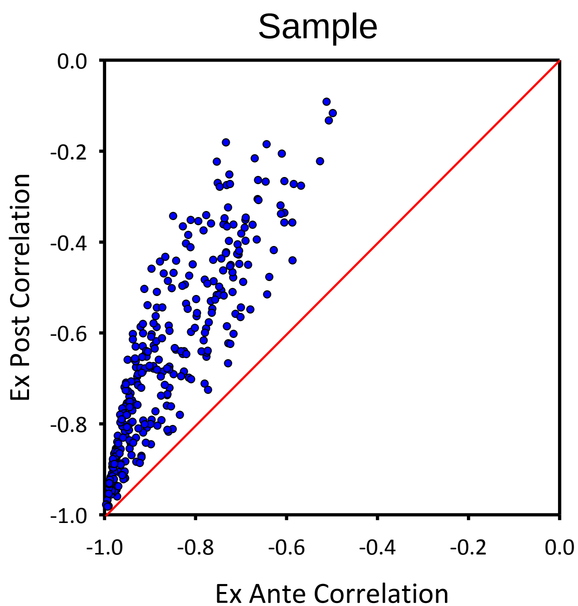
Biases in Correlation Forecasts (Within Block)

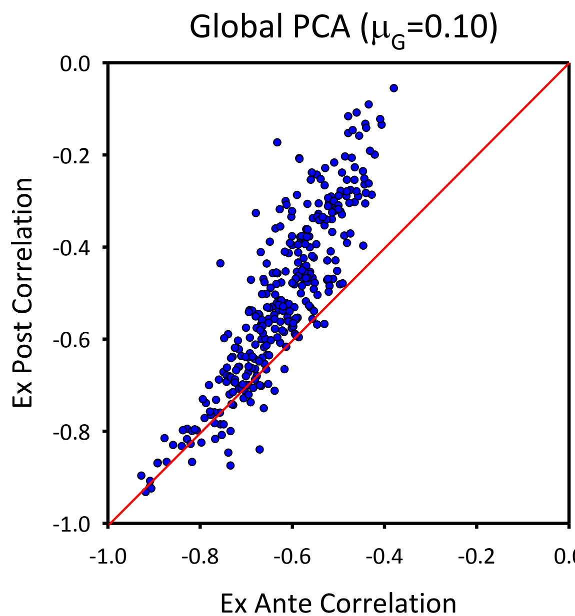
- Compute correlations between alpha and hedge portfolios
- All models systematically over-predicted correlations

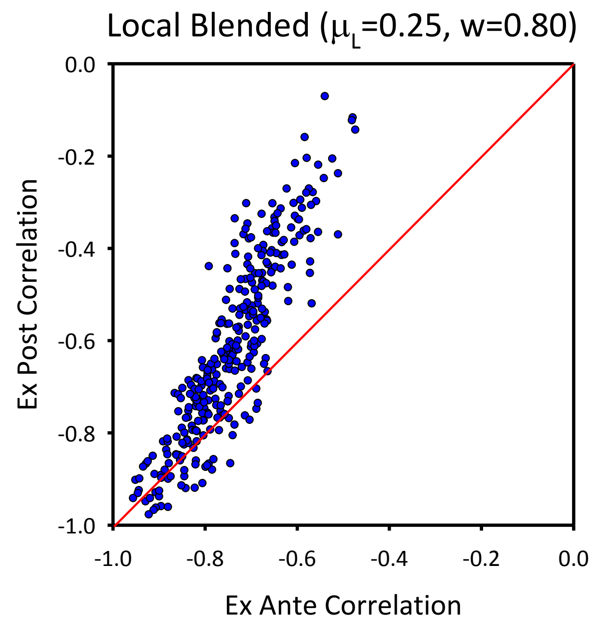
Correlations

Model	*Ex Ante*	*Ex Post*
Sample	-0.85	-0.65
Global PCA	-0.62	-0.51
Local Blended	-0.74	-0.62

- Local blended hedge portfolio nearly as effective as sample hedge portfolio (*ex post*)
- Bias was smaller for local blended model than for sample, allowing better sizing of position







Objectives in Portfolio Optimization

- Low out-of-sample volatility for optimized portfolios
  - Measure using volatility ratio
- Accurate risk forecasts for optimized portfolios
  - Measure using bias stats or Q-stats
- Efficient *ex post* allocation of the risk budget
  - Realized risk should align with expected returns
- Low factor leverage

$$L_t = \sum_k \left| X_{k,t} \right| \quad \rightarrow \quad \bar{L} = \frac{1}{T} \sum_t L_t$$

- Low factor turnover

$$TO_t = \sum_k \left| X_{k,t+1} - X_{k,t} \right| \quad \rightarrow \quad \overline{TO} = \frac{1}{T} \sum_t TO_t$$

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Optimized Factor Portfolios (Diagonal Blocks)

- Compute averages over all 319 optimized factor portfolios
- Allow hedging using only factors within the same block

	Optimized Factor Portfolios				
Model	B-stats	Q-stats	Vol Ratio	Leverage	Turnover
Sample	2.067	4.951	0.955	16.574	0.686
Local PCA	1.302	2.628	1.030	4.554	0.380
Global PCA	1.129	2.485	1.000	4.489	0.425
Local RMT	1.122	2.493	1.025	4.082	0.290
Global RMT	1.071	2.468	1.006	4.197	0.356
Time Series	1.009	2.471	1.072	2.583	0.088
Eigen-method	1.376	2.891	0.882	12.392	0.523
Local Blended	1.204	2.569	0.903	6.861	0.352

- Sample correlation underpredicted risk by factor of 2
- Sample and eigen-method had highest turnover and leverage
- Local blended model scored well across measures

Optimized Factor Portfolios (Across Blocks)

- Compute averages over all 319 optimized factor portfolios
- Allow hedging using local factors from different blocks

	Optimized Factor Portfolios				
Model	B-stats	Q-stats	Vol Ratio	Leverage	Turnover
Global PCA	1.367	2.687	1.000	7.842	1.095
Global RMT	1.271	2.600	1.005	7.727	1.028
Time Series	1.101	2.491	1.030	3.734	0.193
Global Blended	1.370	2.681	0.977	9.707	0.864
Global PCA (Adjusted)	1.386	2.746	0.884	9.691	1.125
Global RMT (Adjusted)	1.339	2.690	0.876	9.539	0.998
Time Series (Adjusted)	1.406	2.813	0.911	11.717	0.700
Global Blended (Adjusted)	1.417	2.782	0.890	11.031	1.023

- Matrix adjustment replicates local models for diagonal blocks:
  - Allows for more effective hedging using factors within the same block
  - Reduces out-of-sample portfolio volatility
  - Generally increases portfolio leverage and turnover

Parameter Selection (US Equities)

- Leverage/Turnover minimized by few PC and low sample weight
- STD is minimized by taking many PC and high sample weight
- Volatility is minimized at intermediate values

Leverage

Mu (local)	w=0	w=0.20	w=0.40	w=0.60	w=0.80	w=1.00
0.10	2.90	3.10	3.63	4.36	5.52	15.73
0.20	3.95	4.04	4.42	5.04	6.21	15.73
0.30	4.69	4.72	5.02	5.58	6.77	15.73
0.40	5.79	5.68	5.83	6.29	7.50	15.73
0.50	6.75	6.47	6.51	6.92	8.20	15.73
0.60	8.32	7.64	7.51	7.91	9.31	15.73
0.70	9.69	8.63	8.40	8.86	10.34	15.73
0.80	12.41	10.59	10.43	11.03	12.47	15.73
0.90	15.61	14.60	14.53	14.74	15.13	15.73
1.00	15.73	15.73	15.73	15.73	15.73	15.73

Turnover

Mu (local)	w=0	w=0.20	w=0.40	w=0.60	w=0.80	w=1.00
0.10	0.12	0.12	0.16	0.21	0.28	0.51
0.20	0.25	0.22	0.22	0.26	0.31	0.51
0.30	0.38	0.31	0.29	0.30	0.33	0.51
0.40	0.55	0.43	0.38	0.36	0.36	0.51
0.50	0.67	0.52	0.44	0.40	0.38	0.51
0.60	1.01	0.71	0.56	0.46	0.41	0.51
0.70	1.20	0.81	0.61	0.49	0.43	0.51
0.80	1.76	1.01	0.70	0.54	0.47	0.51
0.90	1.37	0.83	0.65	0.56	0.51	0.51
1.00	0.51	0.51	0.51	0.51	0.51	0.51

Volatility Ratio

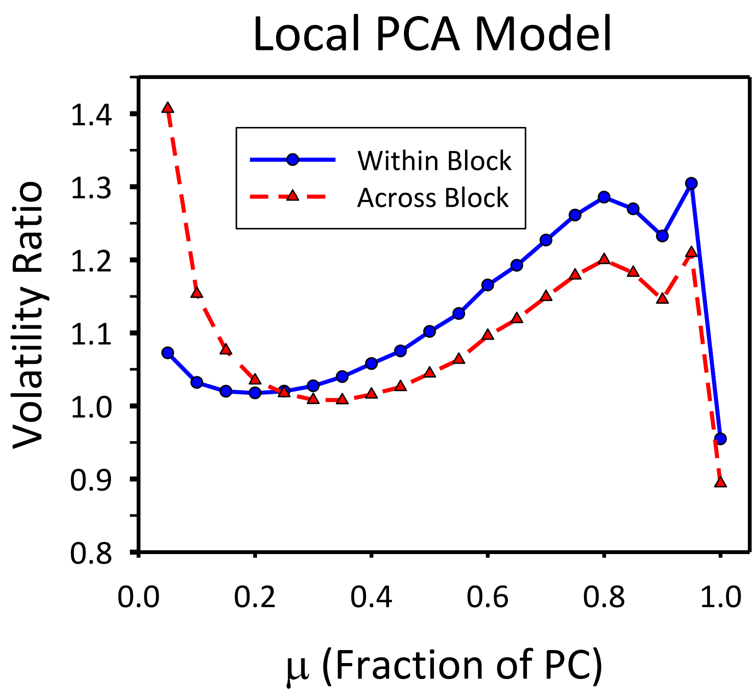
Mu (local)	w=0	w=0.20	w=0.40	w=0.60	w=0.80	w=1.00
0.10	1.087	1.020	0.989	0.971	0.957	1.118
0.20	1.040	1.004	0.984	0.970	0.953	1.118
0.30	1.045	1.010	0.989	0.971	0.948	1.118
0.40	1.078	1.034	1.004	0.977	0.948	1.118
0.50	1.119	1.057	1.015	0.979	0.949	1.118
0.60	1.199	1.096	1.032	0.984	0.958	1.118
0.70	1.289	1.133	1.044	0.989	0.973	1.118
0.80	1.438	1.157	1.051	1.011	1.021	1.118
0.90	1.376	1.157	1.103	1.091	1.097	1.118
1.00	1.118	1.118	1.118	1.118	1.118	1.118

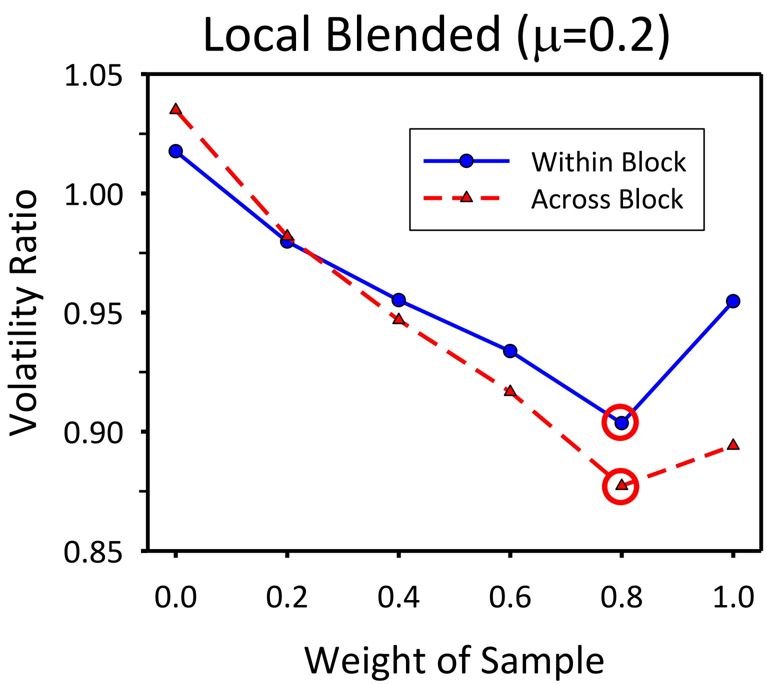
Standard Deviation

Mu (local)	w=0	w=0.20	w=0.40	w=0.60	w=0.80	w=1.00
0.10	0.083	0.067	0.050	0.033	0.017	0.000
0.20	0.054	0.043	0.032	0.021	0.011	0.000
0.30	0.045	0.036	0.027	0.018	0.009	0.000
0.40	0.035	0.028	0.021	0.014	0.007	0.000
0.50	0.029	0.023	0.017	0.012	0.006	0.000
0.60	0.022	0.017	0.013	0.009	0.004	0.000
0.70	0.016	0.013	0.010	0.007	0.003	0.000
0.80	0.010	0.008	0.006	0.004	0.002	0.000
0.90	0.005	0.004	0.003	0.002	0.001	0.000
1.00	0.000	0.000	0.000	0.000	0.000	0.000

Volatility Ratio

- Create a Local PCA and Blended model for every local block
  - For Local PCA, vary number of principal components (μ_L)
  - For Local Blended model, choose μ_L =0.2 and vary *w*
- Local PCA exhibited local minimum, but sample had even lower out-of-sample volatility
- Across block uses global PCA with local model as the target





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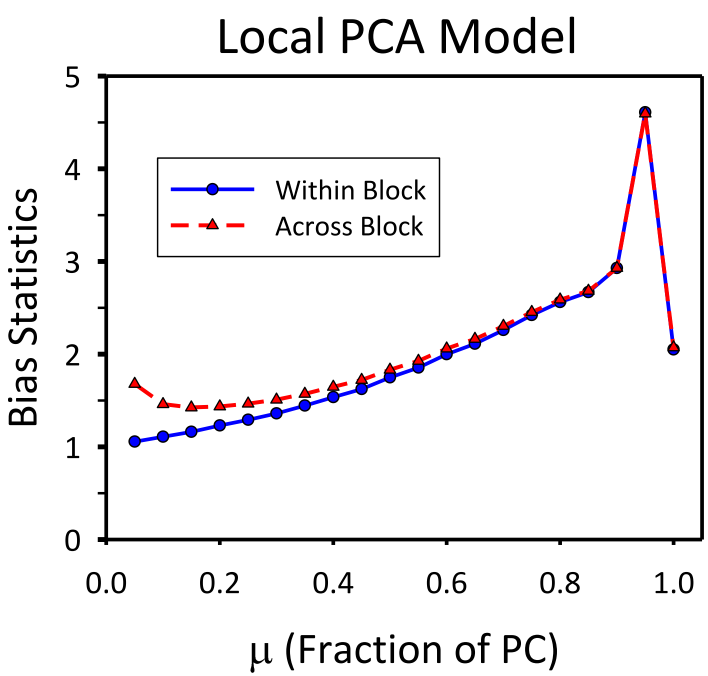
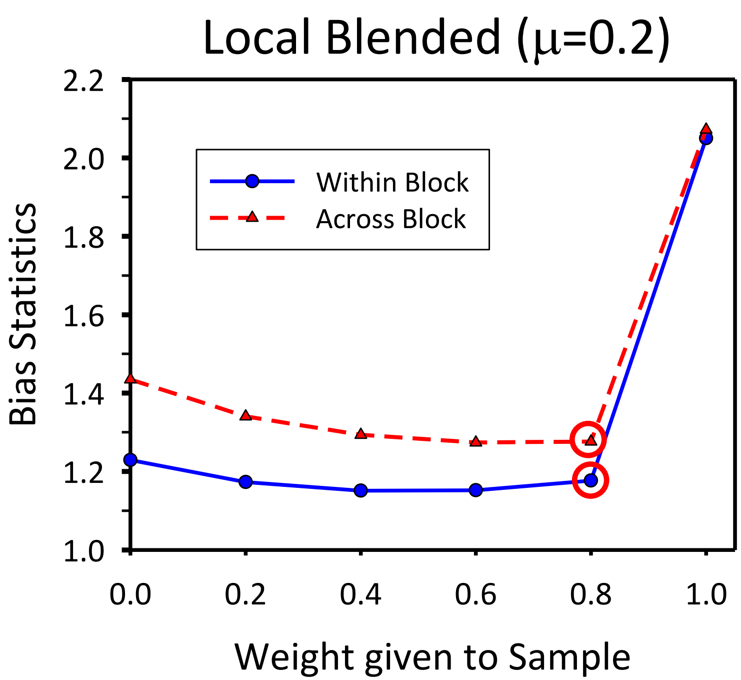
Bias Statistics

- Local PCA model vastly under-predicts risk as the number of principal components approaches the maximum (i.e., μ→1)
- Sample makes more accurate risk forecasts than the Local PCA model with many factors
- Even small blending (*w*=0.8) improves accuracy of risk forecasts for optimized portfolios



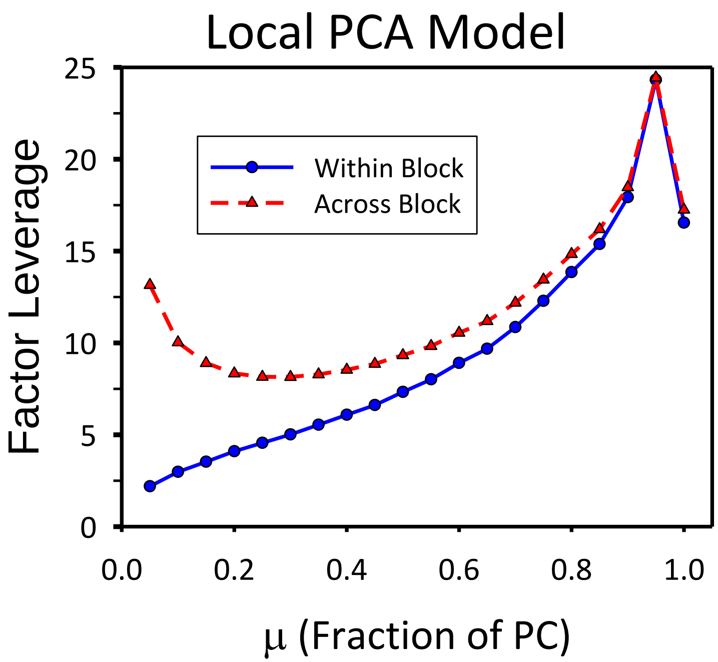


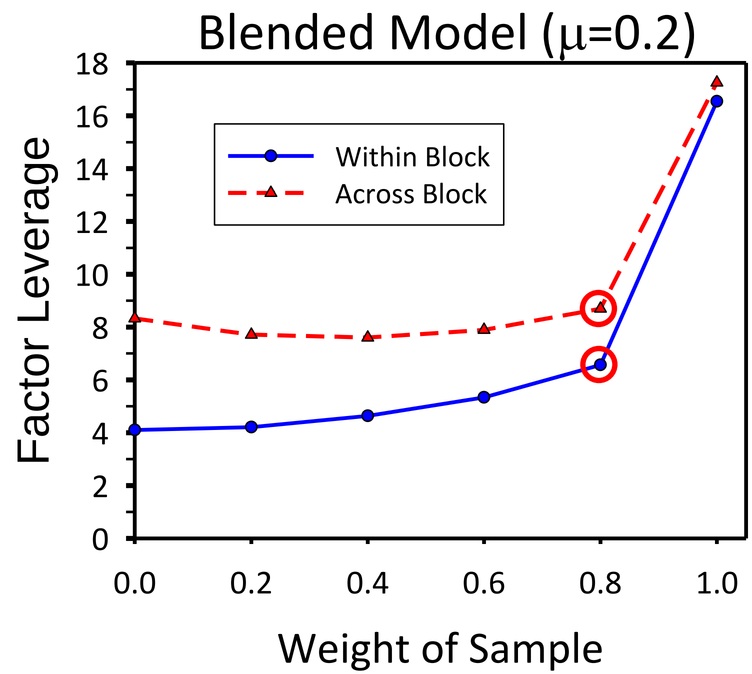
Factor Leverage

- Compute mean factor leverage across factors and time

$$L_t = \sum_k \left| X_{k,t} \right| \quad \rightarrow \quad \bar{L} = \frac{1}{T} \sum_t L_t$$

- For Local PCA, leverage increases sharply as μ→1
- For blended model, even small mixing (*w*=0.8) is sufficient to greatly reduce mean factor leverage





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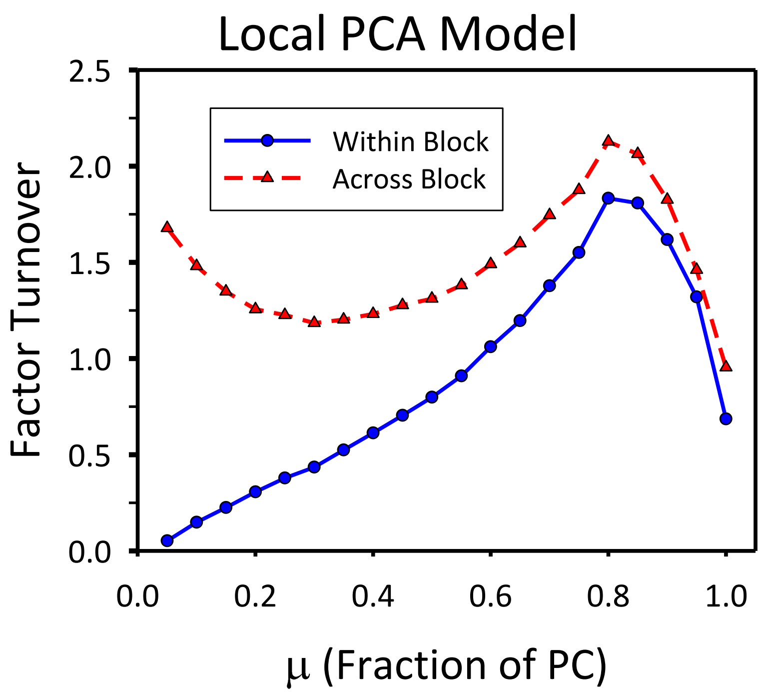
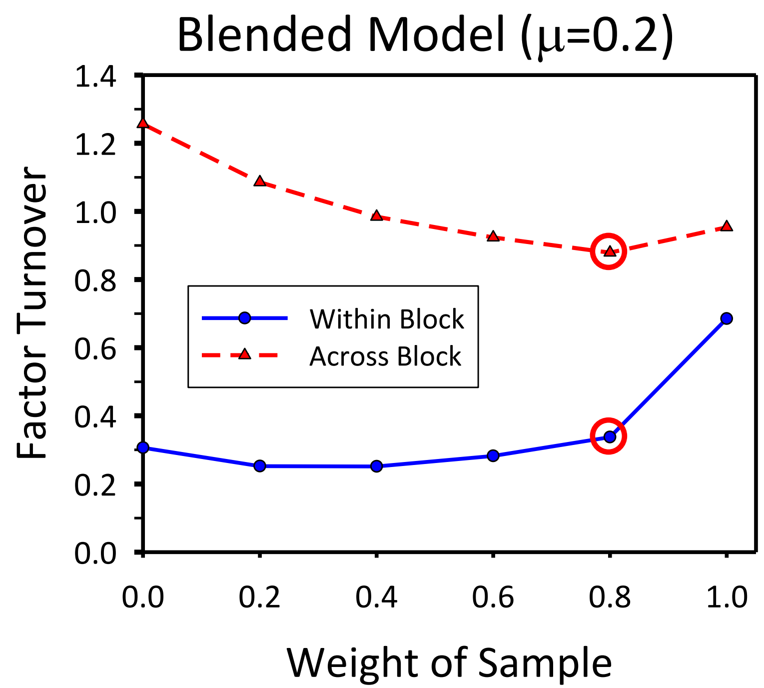
Factor Turnover

- Compute mean factor turnover

$$TO_t = \sum_k \left| X_{k,t+1} - X_{k,t} \right| \quad \rightarrow \quad \overline{TO} = \frac{1}{T} \sum_t TO_t$$

- For Local PCA, turnover rises sharply with increasing μ
- Blended model reduced factor turnover considerably





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Ranking the Candidate Models

Ranking the Models within Local Markets

- Use two measures for forecast accuracy
- Use two measures for quality of optimized portfolios
- Convert into z-scores (positive scores are above average)
- Form composite z-score (equally weight four components)

	Factor-Pairs		Optimized Factors		Composite
Model	Q-stats	Std	Vol Ratio	Turnover	z-score
Sample	0.536	1.104	0.451	-1.722	0.184
Local PCA	0.254	-0.057	-0.706	0.042	-0.234
Global PCA	0.635	0.222	-0.243	-0.213	0.200
Local RMT	0.022	0.051	-0.630	0.563	0.003
Global RMT	-0.002	0.006	-0.327	0.181	-0.071
Time Series	-2.405	-2.228	-1.336	1.728	-2.118
Eigen-method	0.443	0.029	1.559	-0.782	0.624
Local Blended	0.517	0.872	1.233	0.203	1.411

- New Bloomberg methodology performed above average on all four measures and earned the highest composite score

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Ranking the Models across Multiple Markets

- New Bloomberg methodology
  - Scored above average on three of four measures
  - Produced highest composite z-score

Model	Factor-Pairs		Optimized Factors		Composite
	Q-stats	Std	Vol Ratio	Turnover	z-score
Global PCA	0.767	0.474	-0.850	-0.702	-0.174
Global RMT	-1.998	0.334	-0.933	-0.485	-1.720
Time Series	-1.008	-2.146	-1.327	2.220	-1.261
Global Blended	0.820	0.717	-0.489	0.045	0.610
Global PCA (Adjusted)	0.415	0.400	1.000	-0.800	0.566
Global RMT (Adjusted)	0.463	0.405	1.125	-0.386	0.897
Time Series (Adjusted)	-0.044	-0.869	0.568	0.577	0.130
Global Blended (Adjusted)	0.585	0.684	0.905	-0.468	0.952

- Unified methodology applied throughout estimation process facilitates implementation and comprehension of model

MAC2 versus MAC1 Comparison

Bloomberg MAC2 and MAC1 Models

- MAC1 refers to the first-generation Bloomberg MAC model
  - Computes diagonal blocks using RMT method with shrinkage
  - Computes off-diagonal blocks using the time-series method
  - For equities, "core" factors taken from global equity model
    - e.g., Japan autos is regressed on Japan factor and global auto factor
  - For other blocks, "core" factors are weighted average of local factors
    - e.g., Core factor for oil commodities is weighted average of Brent and WTI "shift"
  - Apply integration matrix to recover the diagonal blocks
- MAC2 refers to the new Bloomberg MAC model:
  - Uses blended methodology for both diagonal and off-diagonal blocks
  - Applies integration matrix to recover local models on diagonal blocks
- MAC1 and MAC2 models use EWMA with same HL parameters:
  - 26 weeks for volatilities
  - 52 weeks for correlations

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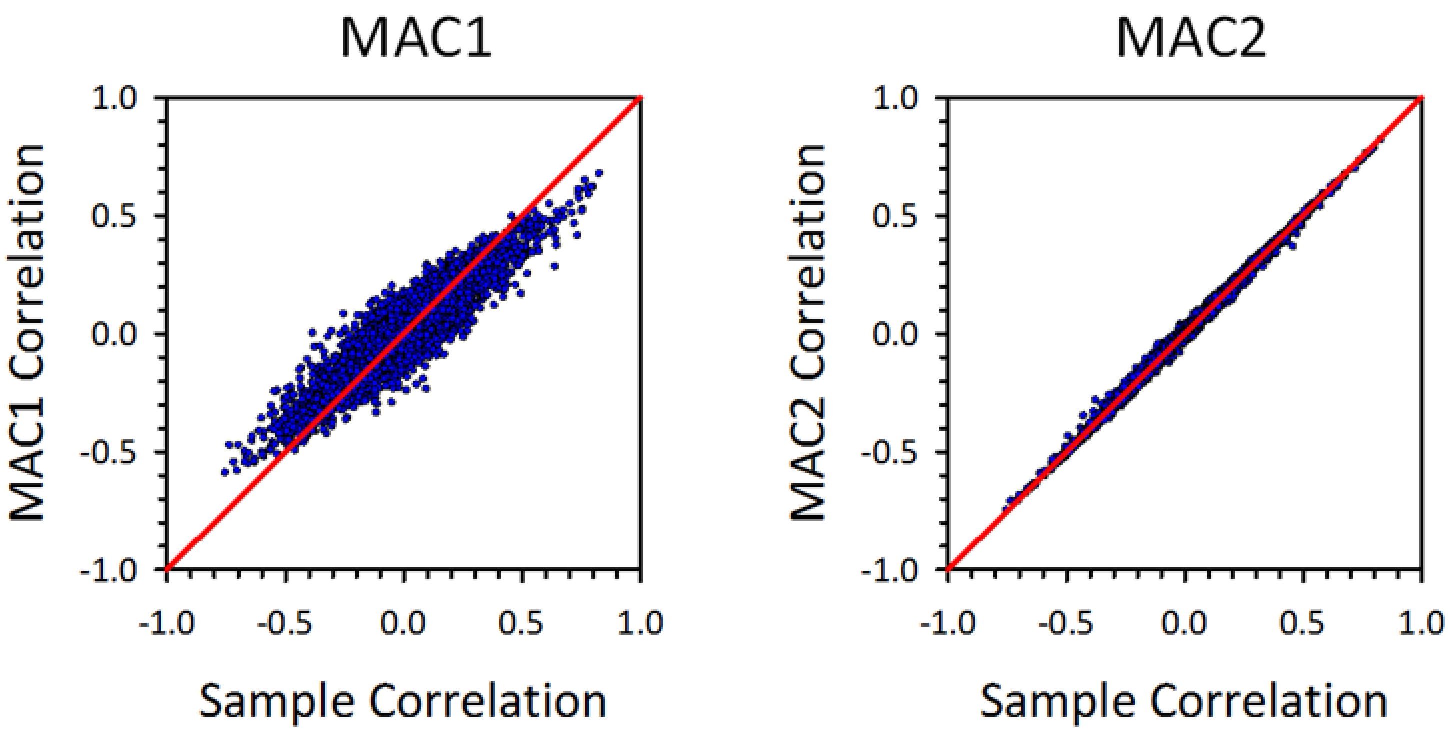
Diagonal Equity Blocks

- Make scatterplots of estimated correlations versus sample
- Example: US equity factors versus US equity factors

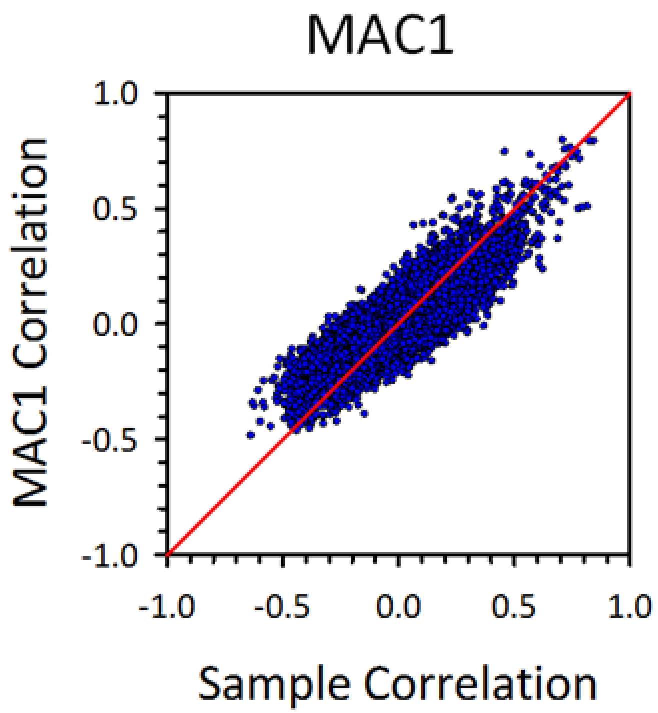


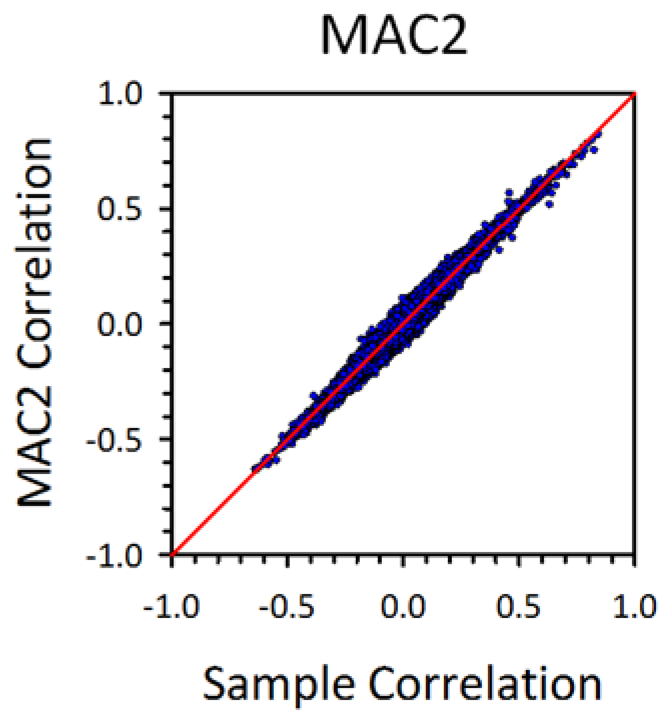
27-Aug-2014

- MAC2 estimates closely mimic the sample correlation
- MAC1 estimates exhibit more scatter and some biases

Off-Diagonal Equity Blocks

- Make scatterplots of estimated correlations versus sample
- Example: US equity factors versus Japan equity factors





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- MAC2 estimates closely mimic the sample correlation
- MAC1 estimates exhibit more scatter and some biases

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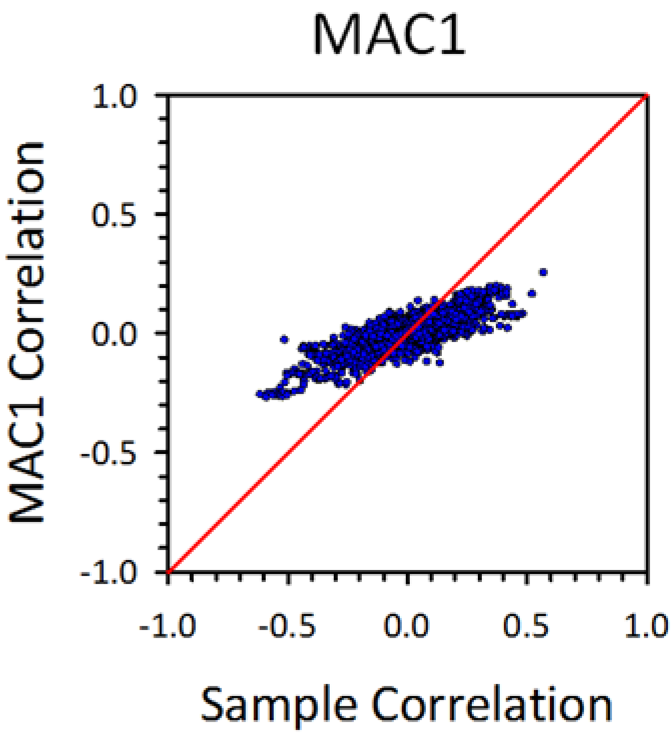
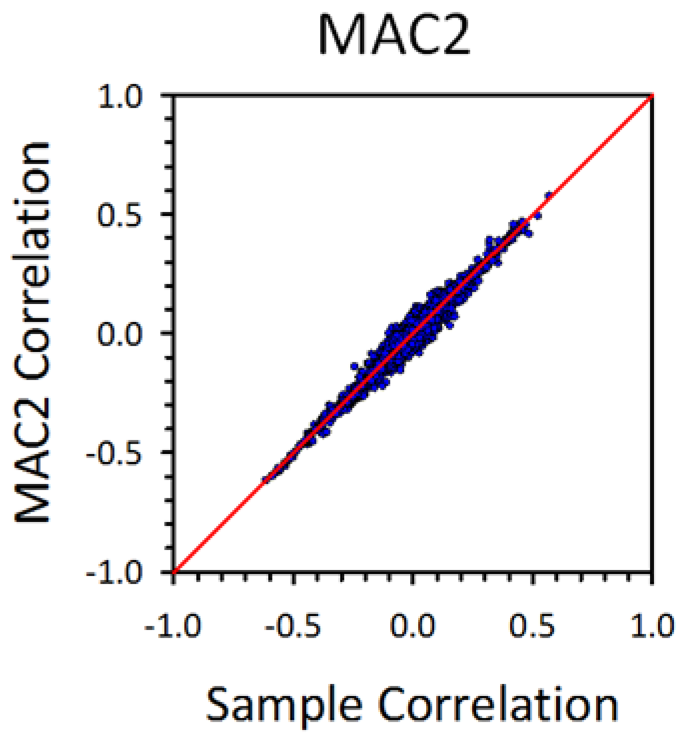
US Equity versus US Fixed Income

- MAC2 estimates closely mimic the sample correlation
- MAC1 correlations exhibit considerable biases





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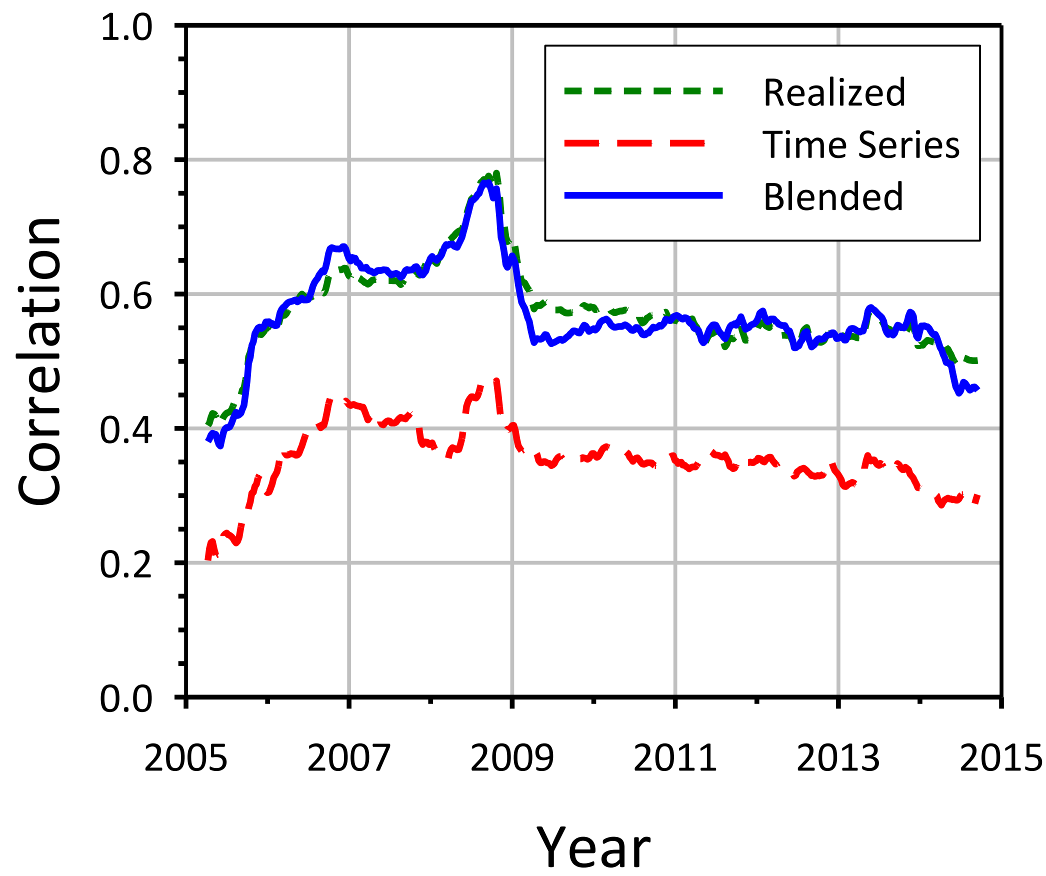
- Results suggest that the time-series method may not be effective at fully explaining correlations across asset classes

Cross Asset-Class Correlations versus Time

- Consider the correlation between the US energy factor (equity) and the crude-oil commodity factor (Brent shift)
- Plot predicted and realized correlations (52w HL)

- Blended approach captures the observed relationship very closely
- Time-series method systematically under-predicts correlation
- Suggests missing factors in time-series approach



Scenario Analysis

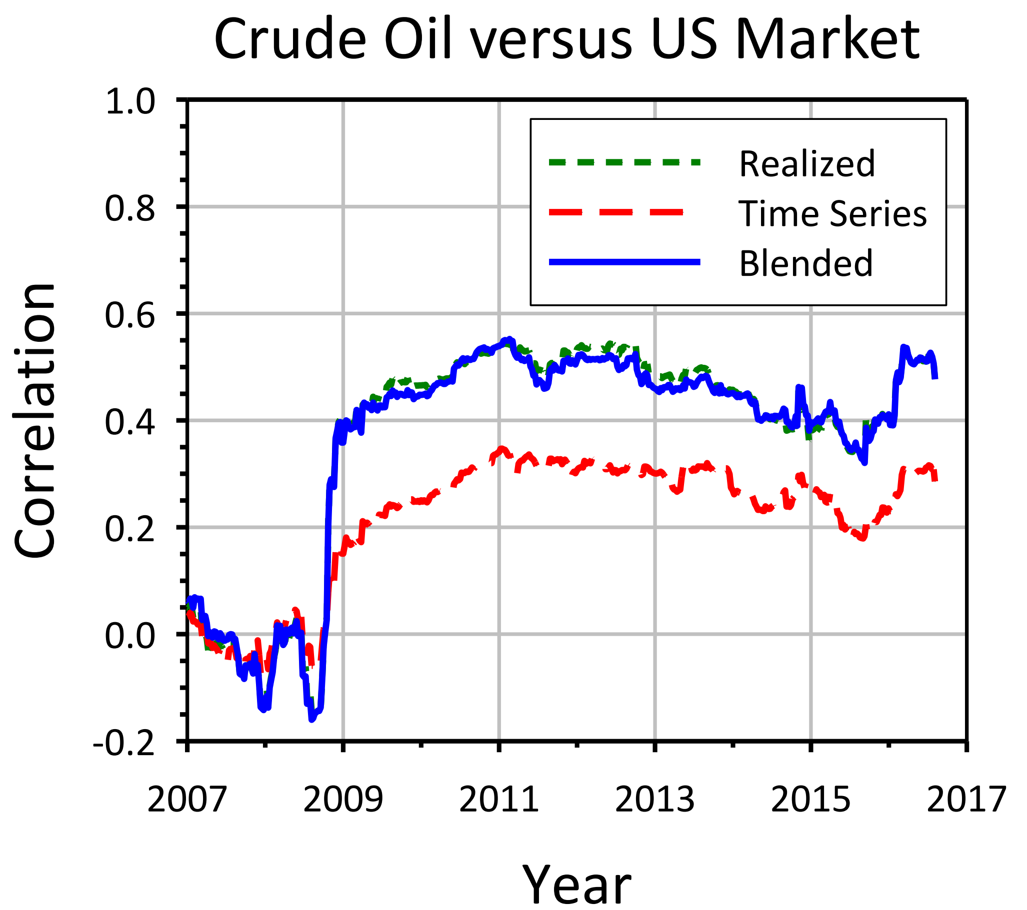
- What is the expected impact of a 20% drop in crude oil on the return of the S&P 500?

$$E[R_P] = \sum_k X_k^P E[f_k]$$

- Shocked variables are propagated to other factors

$$E[f_k] = \left(\frac{\rho_{kS} \sigma_k}{\sigma_S} \right) R_S$$

- Differences in correlations lead to differences in the expected returns of the propagated variables



Scenario Analysis Attribution

- MAC1 predicts drop of 2.36% for S&P 500
- MAC2 predicts drop of 3.61% for S&P 500
- MAC2 betas were generally larger due to higher correlations

MAC1 Model

Factor	Exposure	Beta	Propagation	Contribution
US Market	1.000	0.120	-2.40%	-2.40%
Energy	0.073	0.246	-4.92%	-0.36%
Utilities	0.036	-0.091	1.81%	0.07%
Other Factors	◦◦◦	◦◦◦	◦◦◦	0.33%
Total				-2.36%

MAC2 Model

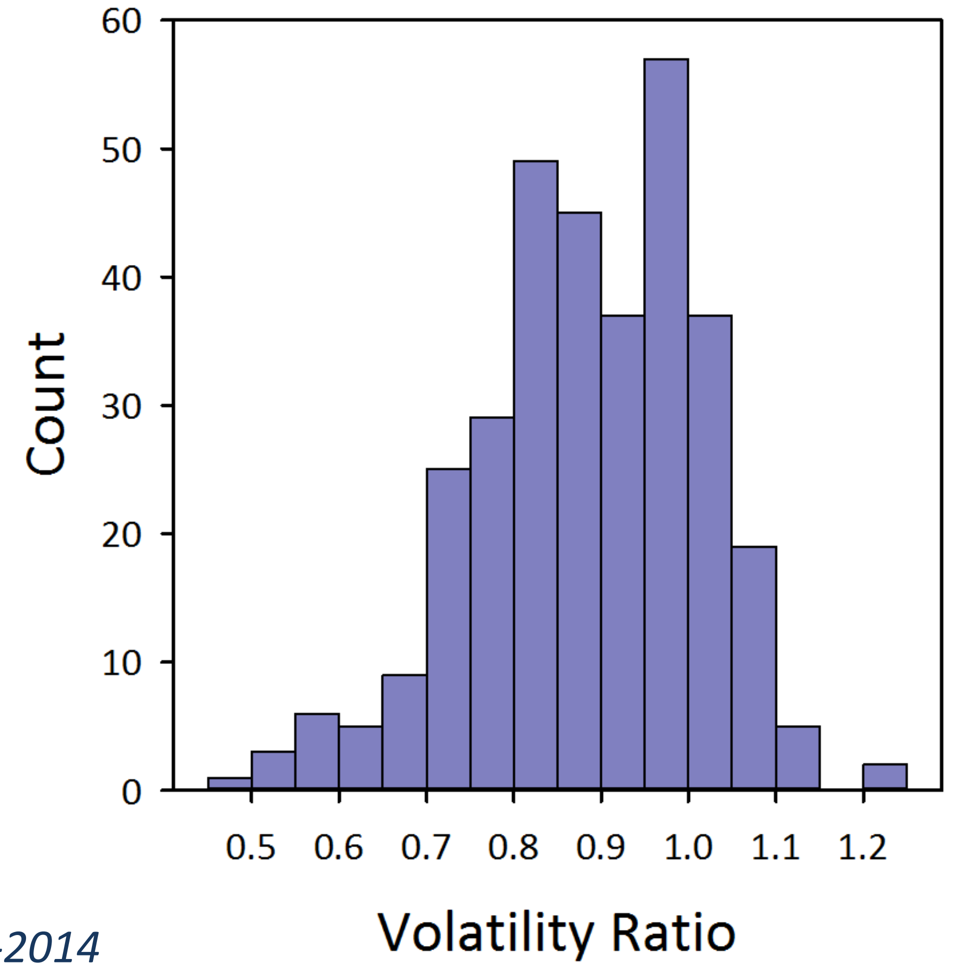
Factor	Exposure	Beta	Propagation	Contribution
US Market	1.000	0.192	-3.83%	-3.83%
Oil Exploration	0.053	0.473	-9.46%	-0.50%
Utilities	0.036	-0.181	3.60%	0.13%
Other Factors	◦◦◦	◦◦◦	◦◦◦	0.59%
Total				-3.61%

Optimized Factor Portfolios (Across Blocks)

- For each of the 319 local equity factors, compute the volatility ratio between MAC2 and MAC1 for optimized factor portfolios

$$v = \frac{1}{K}\sum_{k}\frac{\sigma_k^{\mathrm{MAC2}}}{\sigma_k^{\mathrm{MAC1}}}$$

- MAC2 Model produced lower volatility in more than 80 percent of portfolios
- The average volatility ratio was 0.89
- Similar results hold for within-block optimizations



Sample Period: 30-Mar-2005 to 27-Aug-2014

Summary

- Introduction of second-generation Bloomberg model (MAC2)
- Adopted "blank-slate" approach to select the best model among a broad set of candidate models
- New methodology:
  - Two-parameter model uses blended correlations at all estimation levels
  - Parameters are empirically determined
  - Generally, larger blocks assign smaller weight to sample correlation
  - Integration matrix is applied to recover local models on diagonal blocks
- New model closely mimics the sample correlation even across different asset classes (e.g., equity versus fixed income)
- New model guarantees full-rank covariance matrix to provide reliable forecasts for portfolio construction

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Technical Appendix

Sample Correlation Matrix

- Sample period contains 713 weeks (01-03-01 to 8-27-14)
- Model contains *K*=319 local factors (for nine equity blocks)
- Compute sample covariance matrix ($\mathbf{F}_0$) over *T*=200 weeks

$$F_{jk} = \frac{1}{T-1} \sum_t \left(f_{jt} - \bar{f}_j \right)\left(f_{kt} - \bar{f}_k \right)$$

- Let $\mathbf{S}_0$ be a diagonal matrix of factor volatilities from $\mathbf{F}_0$

$$\mathbf{C}_0 = \mathbf{S}_0^{-1} \mathbf{F}_0 \mathbf{S}_0^{-1}$$

Sample Correlation Matrix

- $\mathbf{C}_0$ provides an unbiased estimate of pairwise correlation
- However, $\mathbf{C}_0$ is rank deficient (119 zero eigenvalues)
- $\mathbf{C}_0$ falsely implies the existence of "riskless portfolios"
- $\mathbf{C}_0$ is not suitable for portfolio optimization purposes

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PCA Correlation Matrices (Global and Local)

- Transform the sample correlation matrix to diagonal basis

$$\mathbf{D}_0 = \mathbf{U}'\mathbf{C}_0\mathbf{U}$$ Columns of **U** are eigenvectors of $\mathbf{C}_0$

- Keep only the first *J* components, where *J*<*T* and *J*<*K*

$$\tilde{\mathbf{C}} = \tilde{\mathbf{U}}\tilde{\mathbf{D}}\tilde{\mathbf{U}}'$$ $\tilde{\mathbf{U}}$ is a *KxJ* matrix

- Compute the "idiosyncratic" variance

$$\Delta_{kk} = 1 - \mathrm{diag}_k\left(\tilde{\mathbf{C}}\right) \rightarrow \boxed{\mathbf{C}_P = \tilde{\mathbf{U}}\tilde{\mathbf{D}}\tilde{\mathbf{U}}' + \mathbf{\Delta}}$$ PCA matrix

- Scale PCA correlation matrix with official factor volatilities

$$\boxed{\mathbf{F}_P = \mathbf{S}\mathbf{C}_P\mathbf{S}}$$ PCA covariance matrix

- Global PCA refers to PCA technique on all local factors (K=319)
- Local PCA refers to applying PCA on the diagonal blocks

Random Matrix Theory (Global and Local)

- Consider the diagonal matrix $\hat{\mathbf{D}}$:
  - First *J* elements are the largest eigenvalues of sample correlation matrix
  - Remaining *K-J* elements are the average of remaining eigenvalues
- Rotate back to the original basis

$$\hat{\mathbf{C}} = \mathbf{U}\hat{\mathbf{D}}\mathbf{U}'$$

Note: diagonal elements not equal to 1

- Scale rows and columns to recover 1 along the diagonals

$$\mathbf{C}_R = \hat{\mathbf{S}}^{-1}\hat{\mathbf{C}}\hat{\mathbf{S}}^{-1} \quad \text{where} \quad \hat{S}_{kk} = \sqrt{\hat{C}_{kk}}$$

- Scale RMT correlation matrix with official factor volatilities

$$\boxed{\mathbf{F}_R = \mathbf{S}\mathbf{C}_R\mathbf{S}}$$

RMT covariance matrix

- Global RMT refers to RMT technique on all local factors (K=319)
- Local RMT refers to applying RMT on the diagonal blocks

Time-Series Methods (Full Factor Set)

- Assume local factors are driven by a small set of global factors

$$\mathbf{f} = \mathbf{g}\mathbf{B} + \mathbf{e}$$

$\mathbf{f}$ is TxK, (local factors) $\mathbf{g}$ is TxJ, (global factors) $\mathbf{B}$ is JxK, (factor loadings) $\mathbf{e}$ is TxK (purely local)

- For equities, the full set of explanatory variables is given by the factor returns of a global equity multi-factor model
- Factor loadings are estimated by time-series regression

$$\mathbf{B} = \left(\mathbf{g}'\mathbf{g}\right)^{-1} \mathbf{g}'\mathbf{f}$$

- Define factor covariance matrices

$$\mathbf{G} = \frac{\mathbf{g}'\mathbf{g}}{T-1} \qquad \mathbf{E} = \frac{\mathbf{e}'\mathbf{e}}{T-1} \qquad \mathbf{D} = \mathrm{diag}\left(\mathbf{E}\right)$$

- Local factor correlation matrix

$$\mathbf{F}_{FS} = \mathbf{B}'\mathbf{G}\mathbf{B} + \mathbf{D} \quad \rightarrow \quad \mathbf{C}_{FS} = \mathbf{S}_{FS}^{-1}\mathbf{F}_{FS}\mathbf{S}_{FS}^{-1}$$

Time Series (Full Set)

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Time-Series Methods (Partial Factor Set)

- Partial-set method mirrors full-set method, except each local factor is regressed on a small subset of global factors
- For instance, the Japan Automobile factor might only be regressed on two global factors: Japan and Automobiles
- This results in a sparse factor loadings matrix, $\tilde{\mathbf{B}}$
- Local factor covariance matrix is given by

$$\mathbf{F}_{PS} = \tilde{\mathbf{B}}'\mathbf{G}\tilde{\mathbf{B}} + \mathbf{D}$$

- The correlation matrix is given by

$$\boxed{\mathbf{C}_{PS} = \mathbf{S}_{PS}^{-1}\mathbf{F}_{PS}\mathbf{S}_{PS}^{-1}}$$

Time Series (Partial Set)

- Selection of relevant global factors:
  - May contain a significant subjective element
  - Omission of important factors may lead to misestimation of risk

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Eigen-Adjusted Correlation Matrices (Local)

- Menchero, Wang, and Orr (2012) showed that eigenvalues of sample covariance matrix are systematically biased

$$\mathbf{D}_0 = \mathbf{U}'\mathbf{C}_0\mathbf{U}$$ Columns of **U** are eigenvectors of $\mathbf{C}_0$

- Let $\tilde{\mathbf{D}}_0$ denote the diagonal matrix of de-biased eigenvalues
- Perform reverse rotation to original basis:

$$\tilde{\mathbf{C}} = \mathbf{U}\tilde{\mathbf{D}}_0\mathbf{U}'$$ Note: diagonal elements not equal to 1

- Scale rows and columns to recover 1 along the diagonals

$$\boxed{\mathbf{C}_E = \tilde{\mathbf{S}}^{-1}\tilde{\mathbf{C}}\tilde{\mathbf{S}}^{-1}}$$ Eigen-adjusted correlation matrix

- Eigen-adjusted method is only applicable for the local blocks

Menchero, Wang, and Orr. *Improving Risk Forecasts for Optimized Portfolios,* Financial Analysts Journal, May/June 2012, pp. 40-50

Blended Correlation Matrices (Global and Local)

- Ledoit and Wolf (2003) showed that blending the sample covariance matrix with a factor model yielded optimized portfolios with lower out-of-sample volatility
- We blend the sample correlation matrix (using weight *w*) with the PCA correlation matrix (using *J* factors)
- Specify number of PCA factors by parameter μ, where $J = \mu K$
- Two-parameter model for correlation matrix:

$$\mathbf{C}_B(\mu, w) = w\mathbf{C}_0 + (1-w)\mathbf{C}_P(\mu)$$

Blended Matrix

- Blending can be applied at either global or local level

Ledoit and Wolf. *Improved Estimation of the Covariance matrix of Stock Returns,* Journal of Empirical Finance, December 2003, pp. 603-621

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Adjusted Correlation Matrices

- Local portfolio managers (e.g., US equities) want the "best" correlation matrix, known as the *target* correlation matrix
- Factor correlation matrix may be adjusted so that diagonal blocks agree with the target correlation matrix

$$\mathbf{C}_T = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}$$ Target correlation matrix

$$\hat{\mathbf{C}} = \begin{bmatrix} \hat{\mathbf{C}}_{11} & \hat{\mathbf{C}}_{12} \\ \hat{\mathbf{C}}_{21} & \hat{\mathbf{C}}_{22} \end{bmatrix}$$ Estimated correlation matrix

- Define adjustment matrix **A**

$$\mathbf{A} = \begin{bmatrix} \mathbf{C}_{11}^{1/2}\hat{\mathbf{C}}_{11}^{-1/2} & 0 \\ 0 & \mathbf{C}_{22}^{1/2}\hat{\mathbf{C}}_{22}^{-1/2} \end{bmatrix} \rightarrow \boxed{\hat{\mathbf{C}}_A = \mathbf{A}\hat{\mathbf{C}}\mathbf{A}'}$$ Adjusted matrix

- Diagonal blocks now agree with $\mathbf{C}_T$,
- Off-diagonal blocks given by: $\hat{\mathbf{C}}_A(2,1) = \mathbf{C}_{22}^{1/2}\hat{\mathbf{C}}_{22}^{-1/2}\hat{\mathbf{C}}_{21}\hat{\mathbf{C}}_{11}^{-1/2}\mathbf{C}_{11}^{1/2}$