A Quarterly Transactions-Based Index of Institutional Real Estate Investment Performance and Movements in Supply and Demand

by

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(Please note that the results presented in this paper are preliminary, representing work still in progress.)

Abstract:

The purpose of this paper is to describe and hone a methodology by which a quarterly transactions-based index of property-level investment performance may be regularly produced for the U.S. institutional real estate investment industry. The objective is to produce a series of indices of investment periodic total returns and capital appreciation (or price-changes) for the major property types included in the NCREIF Property Index. These indices are based on transaction prices so as to avoid appraisal-based sources of index "smoothing" and lagging bias. In addition to producing variable-liquidity indices, the approach developed in this paper employs the Fisher-Gatzlaff-Geltner-Haurin (*REE* 2003) methodology to produce separate indices tracking movements on the demand and supply sides of the investment market, including a "constant-liquidity" (demand side) index. Extensions of Bayesian noise filtering techniques developed by Gatzlaff & Geltner (*REF* 1998) and Geltner & Goetzmann (*JREFE* 2000) are employed to allow development of quarterly frequency, market segment specific indices. The hedonic price model used in the indices is based on an extension of the Clapp & Giacotto (*JASA* 1992) "assessed value method", using a NCREIF-reported recent appraised value of each transacting property as the composite "hedonic" variable, thus allowing time-dummy coefficients to represent the difference each period between the (lagged) appraisals and the transaction prices. The index may be used to produce a *mass appraisal* of the NCREIF property database each quarter, a byproduct of which would be the ability to provide transactions price based "automated valuation model" estimates of property value for each NCREIF property each quarter.

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> *"In summary, we argue that the NCREIF Index is ready to evolve into two more specialized successor families of index products: one tailored for fundamental asset class research support, and the other tailored for investment performance evaluation benchmarking and performance attribution."*
> -- From: D.Geltner & D.Ling, Benchmarks & Index Needs in the U.S. Private Real Estate Investment Industry: Trying to Close the Gap (A RERI Study for the Pension Real Estate Association), October 17, 2000.

This paper describes an initiative by the MIT Center for Real Estate to attempt to address the need described in the industry white paper quoted above for a "fundamental asset class research" index of real estate investment performance and market conditions. The vision contained in that report was for a state-of-the-art, transactions-based index of commercial real estate. The idea was not to replace the appraisal-based NCREIF Property Index (NPI), but to complement it.[1] The new transactions-based index would be designed to tap the capabilities of modern econometrics to distill information from property transaction prices. The result would be an index that would provide the academic and industry investment research communities with certain useful characteristics that the appraisal-based NPI lacks. The present plan is for the MIT/CREDL to produce this research index quarterly and make it available as a service to the investment research community. This paper describes the prototype of the index product that is under development.

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[1] See Geltner & Ling (2001, 2005).

Since the dawn of the modern investments industry with the advent of modern portfolio theory and rigorous investment management and analysis almost 50 years ago, asset classes in the core of the institutional investment portfolio have required indices of periodic total returns that accurately track investment performance and the state of the market for the asset class. The NPI was developed over a quarter century ago to address this need for real estate.

While the NPI is quite useful, and most appropriate for many functions (e.g., as a benchmark for investment manager performance), the research community has never been entirely satisfied with it in some respects. Because the NPI is based on appraised values of the properties in the index, given the nature of the appraisal process, and also because most properties in the index are not fully or independently reappraised every quarter, the index exhibits a degree of "smoothing" and "lagging" relative to the underlying real estate market.[2] This can be problematic for some research and analysis purposes, such as some types of multi-asset class studies and comparisons (including portfolio optimization), and studies of market turning points or historical market conditions. Although techniques have been developed to "unsmooth" or "reverse-engineer" the NPI to eliminate the smoothing and lagging, these techniques are inevitably somewhat ad hoc or mathematically complex, and difficult for the broader investment community to understand.[3]

The bottom line is that studies of the fundamental nature and characteristics of the real estate asset market would greatly benefit from an accurate and transparent,

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[2] See for example: Geltner & Miller (2001), Chapter 25; and for a literature review: Geltner, MacGregor & Schwann (2003).

[3] See for example: Brown (1985), Blundell & Ward (1987), Quan & Quigley (1989, 1991); Geltner (1991); Giacotto & Clapp (1992); Geltner(1993); Fisher, Geltner & Webb (1994); Lai & Wang (1998); Fisher & Geltner (2000), Fu (2003).

transactions-based index, that avoids the smoothing and lagging in the NPI. Meanwhile, as the NCREIF Index has matured, its database has grown to include a sufficiently large number of property transactions, so that in combination with recent developments in econometric methodology, it is possible to produce a useful transactions-based index from the NCREIF database. The index described in this paper is characterized by the following features:

- It is transactions-based index, calibrated directly on the transaction prices of properties sold each quarter from the NPI database, though it also makes use of all the information available in the appraisal-based officially-reported values of all of the properties in the NPI.

- It is capable of on-going, regular production at the quarterly frequency, reporting total investment return as well as the capital appreciation return component each quarter, at the all-property level and at the level of the four major property sectors: office, industrial, retail, and apartment.

- It allows for the "mass appraisal" of all properties in the NPI database every quarter, enabling an up-to-date, transactions price based estimate of the value of each property (though such property-level valuation cannot be reported publicly as it would violate NCREIF's masking guidelines).

- It is based on state-of-the-art econometric techniques honed recently in the real estate economics academic community, including correction for possible sample selection bias in the sold properties and noise filtering at the quarterly frequency.

- In addition to a standard transactions price based index that reflects the pro-cyclical variable liquidity in the real estate asset market, the index described here allows separate estimation of movements on the demand side and on the supply side of the institutional property market. As described in recent prior research, the demand side index can be interpreted as a "constant liquidity index" (CLI), which collapses both price and trading volume measures of changes in market conditions into a single metric, the percentage change in price that would allow a constant expected time on the market or constant turnover ratio of trading volume in the market.

The index described in this paper exhibits some of the major characteristics that we would expect from a transactions-based index. It shows evidence of leading the NPI in time based on the timing of the turning points of the major historical cycle in the asset market, and it exhibits greater volatility and less autocorrelation (less inertia), including less seasonality. Furthermore, the additional volatility seems to "make sense", including quarterly down-ticks during notable historical moments when we would expect the property market to have fallen at least temporarily (but when the NPI does not register losses), such as the tax act of 1986 (unfavorable to real estate), the stock market crash of 1987, the Gulf War of 1991, the financial crisis of 1998, the September 2001 terrorist attack, and the start of the Iraq War in 2003.

The remainder of this paper is organized as follows. Section 1 presents the basic theory and methodology on which the transactions based index is based, including its extension to include demand and supply indices. Section 2 describes the data and the specific estimation and index construction techniques used in the current prototype index. Section 3 presents the index development results, and some basic analysis of the index returns, including a simple portfolio optimization analysis. A conclusion section summarizes and reports on next steps.

1. Theory and Methodology

To facilitate understanding not only of the variable liquidity transactions index but also of the demand and supply indices, we must begin with a very fundamental model of what underlies both the observed transaction prices and the observed volume of transactions each period within the NCREIF population of properties. The model we use was developed by Fisher, Gatzlaff, Geltner, and Haurin (2003), referred to hereafter as FGGH. The indices presented in this paper are based on this model, with some enhancements to the specific estimation methodology, which we will describe here.

The FGGH model represents a double-sided search market with heterogeneous participants and heterogeneous properties. Observable transaction prices and observable transaction volume both derive from interaction between two populations of market participants: potential buyers (non-owners) on the demand side, and potential sellers (owners) on the supply side. The model is depicted graphically in Exhibit 1, with the three panels showing three successive points in time. The horizontal axis depicts reservation prices, and the bell-shaped curves show the frequency distributions of potential buyers' (the left-hand curve) and potential sellers' (the right-hand curve) reservation prices. The dispersion depicted in these reservation price distributions reflects the heterogeneity of individual market participants' perceptions of values of the properties (as well as their differing search costs, etc). The overlap between the distributions allows for profitable trading of properties, as reflected in observed transaction volume. As time passes and news arrives, both the buyer and seller populations revise their reservation prices, but not necessarily in identical ways. The result is that the overlap region varies over time, corresponding to variation in the trading volume (the turnover ratio or "liquidity") within the population of properties. Pro-cyclical variable liquidity, that is, greater transaction volume during "up" markets (which is a striking empirical fact in real estate markets), suggests that the demand side (potential buyers) reservation price distribution moves quicker and/or farther than the supply side (potential sellers) reservation price distribution, in response to the arrival of news relevant to value.

Insert Exhibit 1 about here.

Hedonic modeling controls for heterogeneity across properties, and Heckman's procedure controls for sample selection bias in the transacted properties by modeling both transaction price and transaction sales propensity. By modeling both price and sale probability it is possible to identify property value (i.e., reservation price) equations separately for both the buyer population and the seller population. The buyers' valuations provide the demand side valuations and the constant-liquidity index, while the sellers' valuations provide the supply side index. The specifics of the methodology are presented below, which is an extension of FGGH.

On the demand side of the market is a population of potential buyers whose reservation prices are modeled by equation (1):

$$RP_{it}^{b} = \sum \alpha_{j}^{b} X_{ijt} + \sum \beta_{t}^{b} Z_{t} + \varepsilon_{it}^{b} \qquad (1)$$

Similarly, on the supply side of the market is a population of potential sellers (owners) whose reservation prices are modeled by equation (2)

$$RP_{it}^{s} = \sum \alpha_{j}^{s} X_{ijt} + \sum \beta_{t}^{s} Z_{t} + \varepsilon_{it}^{s} \qquad (2)$$

In these equations, the variables are described below:

RP_{it}^{b}, RP_{it}^{s} = the natural logarithm of a buyer's (seller's) reservation price for asset i as of time t (the price at which agents will stop searching or negotiating and agree to an immediate transaction);

ε_{it}^{b}, ε_{it}^{s} = normally distributed mean zero random errors (reflecting heterogeneity within the buyer and seller populations, respectively);

X_{ijt} = a vector of j asset-specific characteristics of the properties relevant to valuation (the *"hedonic"* variables);

Z_{t} = a vector of zero/one time-dummy variables (Z_t =1 in quarter t).

In (1) and (2), the $\sum \alpha_{j}^{b} X_{ijt}$ and $\sum \alpha_{j}^{s} X_{ijt}$ components reflect systematic asset-specific values common to *all* potential buyers and all potential sellers, respectively. Temporal variation is possible in the X_{ijt} (hence the *t* in the subscript), reflecting variation over time in the perceived hedonic quality of the property. In typical applications of real estate hedonic value modeling the X_{ijt} vector consists of a number of qualitative and quantitative dimensions of property utility, such as size, age, location, etc. In the case of commercial investment property valuation, many of these hedonic dimensions of utility would be summarized quantitatively in the rent that the property can charge (which, of course, also relates directly to the financial valuation of the asset).

Within the NCREIF database, an even more complete summary of the value of the property is the most recent appraised value of the property. In the spirit of the Clapp & Giacotto (1992) "assessed value method", the most recent appraised value of each property in the database may be used as a summary statistic collapsing the entire X_{ijt} vector into a single scalar value for each property in each time period. We will label this variable A_{it} and note that it clearly reflects both cross-sectional and temporal dispersion. Thus, the $\sum \alpha_j^b X_{ijt}$ and $\sum \alpha_j^s X_{ijt}$ components are simplified to: $\alpha^b A_{it}$ and $\alpha^s A_{it}$.[4]

The dispersion within the buyer reservation price distribution is governed by the dispersion in ε_{it}^b, while the dispersion within the seller distribution is governed by ε_{it}^s. These error terms are random, varying across the individual potential buyers and across individual potential sellers, reflecting unobservable characteristics of the parties and their perceptions of the properties.

In contrast, the β_t^b and β_t^s coefficients represent systematic and common factors across all buyers and all owners (respectively), within each period of time. β_t^b and β_t^s are also common across all assets (*i*) within each period of time (like a time-varying

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[4] Note that since the reservation price model is in log values, we would also take the log of the appraised value.

"intercept"), reflecting the population as a whole during period *t*. The combined effect of the differences between the α^b and α^s coefficients (given the current values of A_{it}), and between the β_t^b and β_t^s coefficients is therefore what distinguishes the buyer and seller reservation price distributions systematically from each other, each period. These population-specific responses govern the central tendency within each population, in each period of time.

Movements over time in the valuations' central tendencies are reflected in the changes over time in the $\left(\alpha^b A_{it} + \sum \beta_t^b Z_t\right)$ or $\left(\alpha^s A_{it} + \sum \beta_t^s Z_t\right)$ components, for the buyers and sellers respectively. Such value changes over time may be due either to changes over time in the values of the A_{it} summary hedonic variables (which reflect both cross-sectional and longitudinal dispersion), or to the periodic variation in the β_t^b and β_t^s parameters (which reflect purely longitudinal changes in the "intercepts", or valuation components not otherwise captured in the A_{it} variables). In the present NCREIF application in which we are using each property's most recent appraisal as the catch-all hedonic variable, the β_t intercepts will reflect primarily only the difference each period between the central tendency of the appraisals and the central tendency of the transaction prices, for period *t*.

Transactions are consummated when and only when the buyer's reservation price exceeds the seller's: $RP^b_{it} \geq RP^s_{it}$. Only under this condition do we observe a transaction price, P_{it}. In other words, consistent with rational investment decision-making (NPV maximization):

$$P_{it} = \begin{cases} observed, & \text{if } RP_{it}^{b} - RP_{it}^{s} \geq 0 \\ unobserved, & \text{if } RP_{it}^{b} - RP_{it}^{s} < 0. \end{cases} \qquad (3)$$

The observed transaction price must lie in the range between the buyer's and seller's reservation prices, both of which are unobserved. The exact price depends on the outcome of a negotiation, and depends on the strategies and bargaining power of the two parties. To produce demand and supply indices, we follow FGGH and assume that the transaction price will equal the midpoint between the buyer's and seller's reservation prices.[5]

Using (1) through (3) and our midpoint price assumption, we see that among sold assets the expected transaction price (for asset *i* as of time *t*) is:

$$E\left[P_{it}\right] = \frac{1}{2}\left(\alpha_j^b + \alpha_j^s\right)A_{it} + \frac{1}{2}\sum_t \left(\beta_t^b + \beta_t^s\right)Z_t + \frac{1}{2}E\left[\left(\varepsilon_{it}^b + \varepsilon_{it}^s\right)\middle| RP_{it}^b \geq RP_{it}^s\right]. \qquad (4)$$

The expectation of the sale price consists of three components: the expected midpoint between the asset-specific buyer and seller perceptions of value, the midpoint between the market-wide buyer and seller period-specific intercepts, and the expected value of the

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[5] There is no reason to assume that either side of the negotiation will systematically have greater bargaining power or negotiating ability. Our assumption of trades at the midpoint is more realistic and more general than the assumption used in many previous studies in the real estate literature that all trades are at the buyer's offer price, and the midpoint price assumption is consistent with Wheaton's (1990) model of the housing market as a double-sided search market. However, within the framework developed in this section it is technically straightforward to replace the midpoint assumption with other specific assumptions (for example, allowing variable pricing across the cycle). Analysis available from the authors suggests that alternative assumptions yield results either similar to, or empirically less plausible than, the results obtained from the midpoint price assumption.

random error, which is itself the midpoint between the buyer's and seller's random components *among the parties that consummate transactions*. This last term is, in general, nonzero, because of the condition that the buyer's reservation price must exceed the seller's reservation price in any observable consummated transaction.

We can measure E[P_{it}] by estimating (4) via the following regression based on observed transaction prices within the NCREIF population:

$$P_{it} = a\ A_{it} + \sum_t \beta_t Z_t + (\varepsilon_{it}\ \left| RP^b_{it} \geq RP^s_{it} \right.) \qquad (5)$$

where: $a = \frac{1}{2}\left(\alpha^b + \alpha^s\right)$, $\beta_t = \frac{1}{2}\left(\beta^b_t + \beta^s_t\right)$, and $\varepsilon_{it} = \frac{1}{2}\left(\varepsilon^b_{it} + \varepsilon^s_{it}\right)$ (and recall that Z_t is a zero/one time-dummy). Such a model will predict an estimated value, $\hat{P}_{it}$, for each property i in each period t within the NCREIF population.

As noted, the stochastic error term in (5) may have a nonzero mean because the observed transaction sample consists only of selected assets, namely, those for which $RP^b_{it} \geq RP^s_{it}$. If $E\left[\left(\varepsilon^b_{it} + \varepsilon^s_{it}\right) \left| RP^b_{it} \geq RP^s_{it} \right.\right] \neq 0$, this will cause simple OLS estimation of (5) to have biased coefficients. As described in FGGH, this sample selection bias problem can be corrected by the well known Heckman procedure which involves estimation of a separate probit model of property sale probability.

In our context, this sales model is useful not only in the Heckman procedure to correct for sample selection bias in the value model, but also to enable separate identification of the buyers (demand side) and sellers (supply side) valuation models, the former of which presents the constant liquidity valuation, as described in FGGH.

The probit model of property sale probability is based fundamentally on the decision of whether to sell an asset or not. The latent variable describing the decision for the i-th asset in period t is S_{it}^{*}:

$$S_{it}^{*} = RP_{it}^{b} - RP_{it}^{s}. \tag{6}$$

S_{it}^{*} is not observable, only the outcome S_{it} is observed:

$$S_{it} = \begin{cases} 1, & if\ \ S_{it}^{*} \geq 0 \\ 0, & \text{if otherwise.} \end{cases} \tag{7}$$

In other words, a sale occurs if and only if $RP^{b}_{it} \geq RP^{s}_{it}$, in which case $S_{it} = 1$, otherwise $S_{it} = 0$.

Equation (6) defines S_{it}^{*} to equal the difference between the buyer's and seller's reservation prices for the asset. Subtracting (2) from (1) as in (6) yields:

$$S_{it}^{*} = (\alpha^{b} - \alpha^{s})A_{it} + \sum(\beta_{t}^{b} - \beta_{t}^{s})Z_{t} + (\varepsilon_{it}^{b} - \varepsilon_{it}^{s}). \tag{8}$$

Following FGGH, define: $\omega = \alpha^{b} - \alpha^{s}$, $\gamma_{t} = \beta_{t}^{b} - \beta_{t}^{s}$, and $\eta_{it} = \varepsilon_{it}^{b} - \varepsilon_{it}^{s}$. The Z_t variable here is the same as that in (1), (2), and (5), a zero/one time-dummy variable. Equations (7) and (8) can be estimated as a probit model:

$$\Pr[S_{it} = 1] = \Phi[\omega\ A_{it} + \sum \gamma_{t} Z_{t}] \tag{9}$$

where $\Phi[\ ]$ is the cumulative density function (cdf) of the normal probability distribution evaluated at the value inside the brackets, based on A_{it} and Z_t. The probit model estimates the coefficients and residuals only up to a scale factor. The estimated coefficients in (9) are ω / σ and γ_t / σ, and the estimated error is η_{it} / σ, where $\sigma^2 = Var(\varepsilon_{it}^b - \varepsilon_{it}^s)$. Label the estimated probit coefficients $\hat{\omega}_t$ and $\hat{\gamma}_t$, so that: $\hat{\omega} = \omega / \hat{\sigma} = (\hat{\alpha}^b - \hat{\alpha}^s) / \hat{\sigma}$, and $\hat{\gamma}_t = \gamma_t / \hat{\sigma} = (\hat{\beta}_t^b - \hat{\beta}_t^s) / \hat{\sigma}$.

This allows unbiased and consistent estimation of the price model, which is thusly modified from (5) to include the inverse Mills ratio, λ_{it}, as indicated in equation (10) below.[6]

$$P_{it} = a\ A_{it} + \sum \beta_t Z_t + \sigma_{\varepsilon\eta} \lambda_{it} + \upsilon_{it}\,. \qquad (10)$$

As equation (10) is estimated based on a sample of transaction prices, this model allows the construction of a transaction-based index of the NCREIF population of properties. This can be done in at least two ways, both of which begin with the price model's predicted value of each property, each period:

$$\hat{P}_{it} = \hat{a} A_{it} + \sum \hat{\beta}_t Z_t + \hat{\sigma}_{\varepsilon\eta} \lambda_{it} \qquad (11)$$

We can construct an index by defining a "representative property", call it property "*p*". Property *p* is characterized by a typical or average value of A_{it} and of λ_{it} each period, and also by a typical income flow (call it CF_{pt}). Then, the index returns are based on the

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[6] As described in the FGGH (2003) appendix, $\hat{\sigma}$ is a standard output of econometric software packages that implement the Heckman procedure. Such packages also correct for heteroskedasticity in the procedure.

predicted value of property *p* each period and property *p*'s cash flow each period. Thus, in period *t* the capital return for Property *p* (and by construction, for the index as well) is:[7]

$$g_{pt} = \left(\exp[\hat{P}_{pt}] - \exp[\hat{P}_{pt-1}]\right) \Big/ \exp[\hat{P}_{pt-1}] \qquad (12a)$$

and the income return is:

$$y_{pt} = \left(CF_{pt}\right) \Big/ \exp[\hat{P}_{pt-1}] \qquad (12b)$$

and the total return is:

$$r_{pt} = g_{pt} + y_{pt} \qquad (13)$$

The second way to construct an index is "mass appraisal". In this approach equation (11) is used to produce an estimated value of each property in the NPI database, each period: $\hat{P}_{it}$. The total return and capital return is then computed for each property, each period, in the same manner as above for the representative property:

$$r_{it} = \frac{CF_{it} + \exp[\hat{P}_{it}] - \exp[\hat{P}_{it-1}]}{\exp[\hat{P}_{it-1}]} = \frac{CF_{it}}{\exp[\hat{P}_{it-1}]} + \frac{\exp[\hat{P}_{it}] - \exp[\hat{P}_{it-1}]}{\exp[\hat{P}_{it-1}]} = y_{it} + g_{it} \qquad (14)$$

Then these individual property returns are aggregated across all properties in the NPI each period. The aggregation may be by equal-weighting across the properties, or value-weighting (as in the official NPI). In the case of the latter the index return is computed as:

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[7] Recall that $\hat{P}_{pt}$ is in log levels. Exponentiation is required to convert from log levels to straight levels to define a simple periodic geometric return index instead of a continuously-compounded return index.

$$r_t = \sum_i \left[\left(\frac{\exp[\hat{P}_{it-1}]}{\sum_i \exp[\hat{P}_{it-1}]} \right) r_{it} \right] \qquad (15a)$$

In the former case (equal weighting), it is simply:

$$r_t = \sum_{i=1}^{N_t} \frac{r_{it}}{N_t} \qquad (15b)$$

where N_t is the total number of properties in the NPI in period t.

Because the underlying hedonic value model (10) is a log value model, the above-described mass appraisal procedure will result in a slight bias in the estimated straight level values obtained from exponentiating the predicted log values of (11), and this bias will induce a slight error (but no bias) in the return index.[8] These effects are very minor and may be corrected through well known mathematical adjustments (Neyman and Scott, 1060; Goldberger, 1968; Miller, 1983).

Note that the estimation of each individual property's value as of each period via equation (11) not only enables the construction of a mass appraisal index, but also allows provision of the transactions-based estimated value of each property each period, a value that might be of interest to the property owners.

The above described procedures, based on the price model in equation (10), provide a transactions-based version of the NCREIF Index. As the hedonic variable is represented by the current appraised value of each property each period, A_{it} , it is easy to

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[8] The mathematical rule known as "Jensen's Inequality", combined with the concavity of the log function, causes the average of the logs to always be ***<u>less than</u>*** the log of the average. This results in a slight downward bias in the estimated log value level $\hat{P}_{it}$ in equation (11).

see how this model incorporates all of the information available in the appraisals, and adds to that any additional information conveyed by the current transaction prices of properties sold from the NPI during period *t*. The estimated value of each property is simply its current appraised value plus the coefficient on the time dummy variable corresponding to the current quarter *t*. (Recall that the time dummies equal zero for any quarter other than the current quarter *t*.) The time-dummy coefficient reflects the difference between the value indication implied by current transactions minus that implied by the current appraisal. To the extent that transaction prices are more current than appraised values, the value model will capture that difference.[9]

It is important to note that the result up to here provides what can accurately be described as a *variable liquidity* index. That is, while the index accurately represents typical transaction prices prevailing among consummated deals in the market each quarter, such prices reflect varying ease or ability to sell properties across time. In other words, the index reflects varying transaction volume or turnover, and hence, varying "liquidity" over time (as thusly defined). This is because liquidity, as indicated by trading volume or transaction frequency, varies over time in the commercial real estate investment market. Furthermore, this variation is systematic and pro-cyclical, with greater liquidity during "up" markets, and less during "down" markets.[10] Elaborating from FGGH, the above-described variable-liquidity valuation and returns estimates can

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[9] It should be noted that when estimated on a pooled database this model specification cannot avoid a potential danger of collinearity between the appraised value variable and some of the time-dummy variables. Such collinearity could cause an under-estimation of some of the time-dummy coefficients, which could cause the resulting index to understate the difference between the transaction price based valuations and the appraisal-based NPI valuations. This point will be discussed further later in this paper.

[10] One cause of such variable liquidity in the NPI could be a type of "self-fulfilling prophecy" of transactions occurring at or near appraised values, first suggested by Fisher, Geltner, & Webb (1994). If NCREIF members are under pressure not to sell properties at prices below appraised value, and if appraised values lag behind market values, then it will be difficult to sell properties during down markets.

be adjusted to reflect constant liquidity over time (that is, constant "ease of selling", or constant expected time-on-the-market). As described below, this procedure also allows the separate identification of indices of demand side and supply side valuations and market movements over time. Indeed, the index of movements on the demand side of the market is the "constant liquidity" index.

We begin by recalling that equation (10) provides a model of observed equilibrium transaction prices in the relevant property market while equation (9) provides a model of observed equilibrium transaction volume in that market as reflected in the sale probability of a given asset. Each of these equations reflects the movements in the demand and supply sides of the property market, but in different ways. This enables these two models to be treated simultaneously to identify explicit demand and supply side indices for the market, as follows.

First consider the demand side of the market. Based on equation (1), the central tendency of the buyers' valuations is given by

$$V_{it}^{b} = \sum \alpha_{j}^{b} X_{ijt}^{P} + \beta_{t}^{b} = \alpha^{b} A_{it} + \beta_{t}^{b} \tag{16}$$

and changes in demand are determined by movements in the buyers' reservation price distribution. In log differences, these changes (capital returns) are given by:[11]

$$V_{it}^{b} - V_{it-1}^{b} = \alpha^{b}\left(A_{it} - A_{it-1}\right) + \beta_{t}^{b} - \beta_{t-1}^{b} \tag{17}$$

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[11] Recall that Z_t is a zero/one time-dummy variable, so the change in the market value between period *t-1* and period *t* simply equals the difference between the two time-dummy coefficients.

Estimates of the buyers' coefficients, α^b and β_t^b can be derived as follows. First, estimation of (10) yields $\hat{a}_j$ and $\hat{\beta}_t$, and from (4) we see that:

$$\begin{aligned} \hat{a} &= (1/2)\left(\hat{\alpha}^b + \hat{\alpha}^s\right) \\ \Rightarrow \hat{\alpha}^b &= 2\hat{a} - \hat{\alpha}^s \end{aligned}$$

and: (18)

$$\begin{aligned} \hat{\beta}_t &= (1/2)\left(\hat{\beta}_t^b + \hat{\beta}_t^s\right) \\ \Rightarrow \hat{\beta}_t^b &= 2\hat{\beta}_t - \hat{\beta}_t^s \end{aligned}$$

From the probit estimation (9) and its underlying equation (8) we have:

$$\hat{\omega} = (\hat{\alpha}^b - \hat{\alpha}^s)\big/\hat{\sigma}$$

and: (19)

$$\hat{\gamma}_t = (\hat{\beta}_t^b - \hat{\beta}_t^s)\big/\hat{\sigma}$$

Thus, we can solve (18) and (19) simultaneously to obtain[12]:

$$\hat{\alpha}^b = \hat{a} + \tfrac{1}{2}\hat{\sigma}\hat{\omega} \ .$$

And: (20)

$$\hat{\beta}_t^b = \hat{\beta}_t + \tfrac{1}{2}\hat{\sigma}\hat{\gamma}_t$$

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[12] Note that $\hat{\sigma}$ equals two times the "probit sigma" parameter that is automatically output standard software in probit estimation routines. (See FGGH Appendix.) Thus, the adjustments in equation (20) simply equal the probit sigma times the probit coefficient estimates.

Thus, an estimate of the buyers' valuation each period can be obtained from (20) and (16):

$$\hat{V}_{it}^{b} = \hat{\alpha}^{b} A_{it} + \hat{\beta}_{t}^{b} \tag{21}$$

As described in FGGH, such an estimate of buyers' valuations can be interpreted as a *constant liquidity* (that is, constant ease of selling, or constant expected time-on-the-market) value estimate for property i . The demand side valuation estimate in (21) can be used to produce a constant-liquidity transaction-based index of capital value changes or of total returns, using the same procedure described above in equations (11)-(15), only for constant-liquidity values and returns instead of variable-liquidity values and returns, based on $\hat{V}_{it}^{b}$ instead of $\hat{P}_{it}$.[13]

To produce the supply side index the same type of simultaneous solution of (18) and (19) reveals that:

$$\hat{\alpha}^{s} = \hat{a} - \tfrac{1}{2}\hat{\sigma}\hat{\omega}$$

and: (22)

$$\hat{\beta}_{t}^{s} = \hat{\beta}_{t} - \tfrac{1}{2}\hat{\sigma}\hat{\gamma}_{t}.$$

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[13] It should be noted that buyers' side valuations will have a lower average value than the equilibrium transaction prices estimated in equation (11), as the central tendency of non-owners' valuations will lie below that of owners (previous selection causes owners, that is, previously successful buyers, having higher average valuations than non-owners), and therefore below the average transaction prices, which lie between potential buyers' and potential sellers' valuations. This will cause demand side (constant liquidity) total returns to have a tendency to be higher than the variable liquidity total returns, on average over the long run. (Recall that total returns include the income component, the cash flow as a fraction of property value. If the denominator, property valuation, is smaller, then this fraction will be larger, given that the annual income flow is an objective, exogenous value.)

The supply side reservation price value estimate for property *i* in period *t* is then:

$$\hat{V}_{it}^{s} = \hat{\alpha}^{s} A_{it} + \hat{\beta}_{t}^{s} \tag{23}$$

2. NCREIF Data and Index Estimation Procedure

Section 1 has laid out the fundamental theory and the general index construction methodology that underlies the variable liquidity transactions based index, including the extension to create demand and supply indices. In this section we will describe at a more detailed level the NCREIF database and the specific estimation and index construction procedures we have employed in the prototype index. We begin with a description of the database.

Since its inception in 1982 the National Council of Real Estate Investment Fiduciaries (NCREIF) has been collecting quarterly income and value reports (in addition to other data, and starting with historical data since the end of 1977) for all the properties held for tax-exempt investors on the part of NCREIF’s data-contributing member firms, which include almost all of the “core” real estate investment managers for pension funds in the U.S. This database is used to construct the NCREIF Property Index (NPI), the only property-level “benchmark” index of regular institutional commercial real estate investment performance in the U.S. The index reports quarterly total returns and capital appreciation and income return components. When the index begins in 1978 it includes 233 properties worth a total of $581,000,000. By 1984, the starting date of the transactions index, the NPI includes 1000 properties worth almost $10 billion. By the end of 2003 (the end date of the current prototype transactions index) the NPI covers over 4000 properties worth in the aggregate some $130 billion. The database is well diversified by property type, and property type sub-indices are reported. The four major property types include office (29%), industrial (37%), apartment (20%), and retail (13%).[14]

In general, properties enter the index when they are at least 60% leased, and then remain in the index until they are sold.[15] Properties are generally reappraised at least once per year, on a staggered basis, so that some properties are reappraised every quarter. Property values are reported into the database every quarter for every property, but commonly value reports between reappraisals simply carry over the previous valuation (or else add only the book value of any capital improvements completed during the quarter). When properties are sold their last value reported in the database is the disposition sales transaction price.[16]

Our index begins in 1984 because prior to then there was insufficient transaction frequency to form a reliable transactions-based index.[17] Since that time the NPI database has included over 9000 different properties, of which 3898 have been sold. Of these, we

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[14] Hotel properties make up less than 2% of the all-property index. The percentages reported here are calculated by number of properties, as represented in the 2003 database used in our index estimation.

[15] The index is meant to represent the investment performance of stabilized investment property operations, not development investments. Note also that the index is at the property level, excluding any effects of financing or fund management.

[16] Properties enter the database when they are acquired, or when their investment manager joins NCREIF. Often a property’s first reported value in the database may be its acquisition transaction price, but necessarily and not always, and it is impossible to know whether or not a first reported value is a transaction price or an appraisal. Until recently, when a property was sold out of the database, its disposition transaction price was entered in the index in the quarter prior to its disposition. In constructing the transactions based index we control for this consideration so as to register transaction prices in the quarters in which the transactions were actually consummated (closed).

[17] the property type specific sub-indices must begin even later (for the same data sufficiency reason), in 1994.

are able to use 3628 sale transactions in estimating the hedonic price model. (Some sales must be dropped because they were of properties that were not held in the database long enough to obtain an independent appraisal estimate of their value, the primary explanatory variable in the hedonic price model.) Altogether, we have observations of 121,353 property-quarters, counting each property times each quarter it is in the database, including properties in quarters when they are not sold. This pooled database is the source of our estimation of the probit sales model, as well as the source of the mass appraisal and representative property forms of construction of the transactions based indices.

The first step in building the transactions index is to estimate the selection-corrected hedonic price model specified in equation (10) of Section 1, based on the sold property sample in the NPI database. We estimate this model first at the annual frequency, where we have on average almost 200 price observations per period. The exact specification of this model and the estimation results are shown in Exhibit 2. Note that the model is estimated simultaneously for all properties and for each of the four property types using a "stacked" specification with property-type dummy variables estimated on all 3628 transactions. Based on experience from previous studies, the dependent variable has been defined as the log price per square foot of building area. As noted in Section 1, the anchor explanatory variable is based on an extension of the Clapp & Giacotto (1992) "assessed value method." However, unlike Clapp and Giacotto's "assessed values", our "appraised values" are updated regularly, such that we are able to use appraisals just prior to the transaction sales as our composite hedonic variable. In particular, we use the log of the value per square foot reported by NCREIF *two quarters* *prior* to the transaction sale. This was found to be necessary to ensure that the explanatory variable is independent of the dependent variable (transaction price). As noted in Section 1, the result is that the time dummy coefficients in the model represent the difference each period between the (lagged) appraisals and the transaction prices.[18]

Insert Exhibit 2 about here.

It will be noted that the price model specification in Exhibit 2 includes some additional "hedonic" type explanatory variables besides the appraised value. The variable "jointven" is a dummy variable indicating whether the property is held in a joint venture structure as opposed to simple sole-party ownership. This variable was included because previous research has indicated that it can affect either property price or sale propensity, and as it is not a property characteristic but rather a characteristic of property ownership, it would not normally be considered in the appraised value. However, it is seen not be significant here.

In addition, the price model includes seven geographical location dummy variables, corresponding to seven of the eight multi-state regions defined in the NPI. (The omitted "base case" eighth region is East North Central.) Also included are property type

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[18] In order to reduce temporal aggregation bias that results from averaging sale prices over the calendar year (see Geltner 1993, 1997), in the case of annual frequency estimation of the price model we have modified the Bryan & Colwell (1982) definition of time-dummy variables (to apply to a hedonic model instead of a repeat-sale model). Thus, at the annual frequency our time-dummy variables are defined as follows: For a sale in the *q-th* calendar quarter of year *t*, the time-dummy for year *t* equals 1 – (4-*q*)/4 and the time-dummy for year *t-1* equals (4-*q*)/4. No modification is made for quarterly frequency estimation, as we have no information on when, within each quarter, the sale takes place. It should also be noted that, in principle colinearity between the time dummy variables and the appraised values could affect the index. However, this appears not to be a problem. We found little correlation between the time-dummies and the appraised values, and separate estimation of the annual frequency index on each individual year's transactions produced a result very similar to the pooled estimation.

dummy variables for eight sub-categories within three of the four major property types: office, industrial, and retail. (The omitted fourth property type is apartment.) Keep in mind that in principle there is no reason why additional property-specific location and property characteristic variables beyond the composite hedonic variable labeled A_{it} (recent appraisal) cannot be incorporated into the hedonic price model. Going back to the underlying reservation price models in equations (1) and (2), such additional hedonic variables would be components of the j-dimensional X_{ijt} hedonic vector that are not adequately captured in the composite hedonic variable A_{it}. In the present case, the selected location and property-type dummy variables have been included because prior research (notably FGGH) indicated that they could be important in the price or sales model. As noted, however, some of these variables are not significant here, and they may be dropped in future refinements of the prototype index presented here.

The results presented in Exhibit 2 are corrected for transaction sample selection bias using the standard Heckman (1979) two-step procedure described in Section 1. The specification and results of the 1st-stage probit selection model (corresponding to equation (9) in Section 1) are presented in Exhibit 3. Based on prior research, this model of property sale probability includes as explanatory variables, in addition to the appraised value composite hedonic variable and the time dummy variables necessary for constructing the constant liquidity index [the A_{it} and Z_t variables of equation (9)], two other variables: building size (square feet), and the "jointven" indicator variable previously noted. (A constant is also included.)

Insert Exhibit 3 about here.

As can be seen in Exhibit 3, while the selection model appears to be a fine model of property sales probability (as indicated by the strong t-statistics on most coefficients), the selection bias indicator variable, “lambda”, is not significantly different from zero. Indeed, when we compare the representative property index based on the selection-corrected price model of Exhibit 2 with a similar representative property index based on the simple OLS price model without sample selection bias correction, the two indices are almost identical. Thus, in contrast to findings in the previous literature on commercial property transactions based indices, sample selection bias does not appear to be an issue with our model specification.[19] On the other hand, the probit model contains some interesting results regarding sales characteristics in the NCREIF database. The strongly significant and negative coefficients on both the appraised value/SF and the square foot variables suggests that not only do larger properties sell less frequently, but also “higher quality” properties (as indicated by higher appraised value per square foot).

The next step in transaction price index development is to construct a longitudinal price index based on the hedonic price model in Exhibit 2. Here we use the “representative property” method defined in equation (12a) of Section 1. We define the “representative property” as a property characterized by having the population mean

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[19] See Munneke & Slade (2000, 2001), and FGGH (*op.cit.*). Apparently, the appraised value composite hedonic explanatory variable is able to capture the effect of most differences between the sold and unsold property samples much more effectively than the specifications used in the previous research. Some insight into this result may be suggested by the finding in Fisher, Gatzlaff, Geltner, & Haurin (2004) that a property’s current appraised value relative to NPI growth since acquisition (their “WINS” variable) was a predictor of sale likelihood.

values of all of the time-invariant explanatory variables in the price model of Exhibit 2. (This includes the location and property-type dummy variables, and the "jointven" dummy variable, and the resulting constant is: *dummyeffect* = -.00498.) Then the A_{pt} log lagged appraised value composite hedonic variable is constructed for the representative property *p* from the appraisal-based NPI annual capital return index, as follows.

An initial value of $10,000,000 is assumed, approximately equal to the average property appraised value in the NPI database as of the time of our index inception at the end of 1983. This value is divided by a constant 235,000 SF, the average size of properties in the NPI database. Thus, the "representative" NCREIF property had an appraised value of: $10,000,000 / 235,000 = $42.6/SF as of the beginning of 1984. This value is then grown according to the NPI annual appreciation returns (equal-weighted, cash flow based version of the NPI[20]). The result is a history of annual appraised value per square foot of a "representative property" of the NPI. The log of this value is used in the price model. In order to reflect the previously-noted two-quarter lag in the representative property appraised value, we define: $A_{pt} = NPI_{t-1}(1+g_{NPI,t})^{1/2}$, where NPI_{t-1} is the cumulative compound level of the NPI price appreciation index, and $g_{NPI,t}$ is the NPI price appreciation return for year *t*. Thus, we reflect only geometrically half of the

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[20] We use the equal-weighted version of the NPI to define the "representative property" as the "mean" or "average" property in the index. We use the cash flow based definition of appreciation return so as to include the effect of capital improvement expenditures in the capital appreciation of the index. This makes the NPI a property *value change* index (where value changes reflect both capital improvements as well as market changes). Later, in constructing a total return index, we must be consistent and use the cash flow based NPI income return component (net of capital improvement expenditures) to define the representative property's income.

current year's annual appreciation return in the 2-quarter lagged representative property appraisal for year *t*.[21]

Thus, considering that the NPI equal-weighted cash flow based appreciation return in 1984 was 6.84%, the representative property appraised value used in constructing the 1984 (end of year) index value is: \$42.6(1.0684)$^{1/2}$ = \$44. With reference to Exhibit 2 and equation (11), the transaction-based estimated log value of the representative property as of the end of 1984 is therefore obtained as follows:[22]

$$\begin{aligned}\hat{P}_{p1984} &= \_cons + year\_1984 + dummyeffect + LaggedLogA\left(A_{p1984}\right) \\ &= .08173 - .07145 - .00498 + 1.009514\left(\log[\$44]\right) \\ &= 3.826\end{aligned} \tag{24a}$$

These estimated log price values for each year are exponentiated to produce level values, and then geometric returns are calculated as per equation (12a) as:

$$\left(1 + g_{pt}\right) = \exp\left[\hat{P}_{pt}\right] / \exp\left[\hat{P}_{pt-1}\right] \tag{24b}$$

For example, in the case of values as of the ends of 1984 and 1983:

$$\begin{aligned}\left(1 + g_{p1984}\right) &= \exp[3.826]/\exp[3.836] \\ &= \$45.88/\$46.34 = 0.99 \\ \Rightarrow g_{p1984} &= -1\%.\end{aligned}$$

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[21] This type of fractional compounding of the NPI capital growth is not necessary in constructing the series of representative property appraised values at the quarterly frequency, where we simply take the representative valuation from two quarters prior to period *t*. At the annual frequency an alternative approach would be to use the quarterly NPI appreciation returns to construct mid-year to mid-year annual appreciation returns.

[22] The inverse Mills ratio "lambda" term is omitted here.

Thus, the transaction based index indicates a price decline of 1% in 1984 when the NPI registered a positive 6.84% appreciation return.

A cumulative appreciation (or capital growth) value level index can then be constructed by compounding the annual appreciation returns, starting from an arbitrary initial value. This is shown in Exhibit 4, together with the NPI appreciation value index (equal-weighted, cash flow based) over the same period.[23] It can be seen that the transaction based index is slightly more volatile than the NPI, and appears to slightly lead the NPI in time, with major turning points occurring one to three years earlier.

Insert Exhibit 4 about here.

It is important to note that the annual frequency index estimated in Exhibit 2 and depicted in Exhibit 4 does not show any evidence of random estimation error "noise". The index has low annual return volatility (5.7%), reasonable first-order autocorrelation in the returns (+17%), and a relatively "smooth" appearance in levels. All of these are characteristics of an absence of noise.

The next step in the index development procedure is to move from the annual frequency model of Exhibits 2 and 4 to the quarterly frequency. This step, of course, results in a reduction by a factor of four in the average number of sales transaction observations per period, to less than 50 transactions on average per quarter. This results in a problem of estimation error "noise" in the index. To show this, Exhibit 5 graphs a

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[23] As the starting value of each index is arbitrary, the indices depicted here (and in general in this paper) are set so that they have equal average value levels across the entire history.

quarterly index estimated by applying the model specification of Exhibit 2 at the quarterly instead of annual frequency, and comparing the resulting index to the annual-frequency index of Exhibit 4. Note the "spiky" appearance of the quarterly index, especially during the earlier history when there were fewer transaction observations.

Insert Exhibit 5 about here.

To address the noise problem at the quarterly frequency, we employ an extension of the Bayesian noise filtering technique developed by Goetzmann (1992), Gatzlaff and Geltner (1998), and Geltner and Goetzmann (2000). This technique involves the use of a ridge regression as a Method of Moments estimator. The estimator minimizes the squared errors of the predicted values (property prices) subject to moment restrictions in the results. The moment restrictions, characterizing the return time series statistics of the resulting estimated index, are based on *a priori* information about the nature of the results that should obtain. In the present case, the moment restrictions are employed as a "noise filter". The ridge procedure eliminates noise in the estimated index without inducing a temporal lag in the index returns. In the present context the moment restrictions are defined to produce a quarterly index whose annual end-of-year return time-series characteristics approach those of the manifestly noise-free annual index which we have just presented in Exhibits 2 and 4, which was estimated at the annual frequency, classically, without the Bayesian filter.

The ridge regression procedure works mechanically by adding "synthetic data" to the estimation database. Specifically, we add one "observation" for each of the 84 quarters. As noted, the synthetic data is based on the annual frequency version of the price model. The effect of the synthetic data is to “pull” the quarterly results toward the smoother (presumably noise-free) annual results. The strength of this “pull” which dampens random noise is inversely related to the number of actual price observations in the real data for each period of time. The ridge effect is adjusted by means a parameter, labeled *“k”*, which governs the strength of the synthetic data in the estimation process. Each of the 84 rows of synthetic data is multiplied by *k*. The higher the k, the greater the influence the added observations have on the regression results.

For each quarter, a row of synthetic data is constructed as follows. The LHS dependent variable price observations are taken directly from the annual frequency transaction index depicted in Exhibit 4, with quarterly values linearly interpolated between the annual end-of-year levels. The RHS synthetic A_{it} composite hedonic variable values are similarly constructed from the NPI appreciation index shown in Exhibit 4, only lagged two quarters. Each row of synthetic data corresponds to one quarter of calendar time, and therefore has one time dummy variable equal to unity, corresponding to the quarter represented by the row. Thus, the time dummies in the synthetic data make a diagonal square matrix of ones. (The constant and time-invariant dummy variables are also included in the ridge at their population mean levels.)

As noted, all the values in each row of synthetic data are multiplied by the ridge parameter *k*, which is adjusted until the resulting estimated index conforms to the moment restrictions noted above, which indicate a lack of noise. In the present case, we have used three criteria in deciding when the moment restrictions are met. The first two criteria are quantitative moment comparisons between the quarterly index and the index estimated at the annual frequency shown in Exhibit 4. First, we compare the annual volatility of the quarterly index (based on its end-of-year returns) to that of the annual index. Second, we compare the annual first-order autocorrelation of the two indices (again basing this on end-of-year annual returns for the quarterly index). Our third criterion is qualitative. We look at the resulting annualized (based on ends of years) quarterly index and compare it visually to the annual index shown in Exhibit 4. We select the lowest value of *k* for which all three of these criteria show a close similarity between the annualized quarterly index and the noise-free (and ridge-free) annual index shown in Exhibit 4.[24]

To the best of our knowledge, the ridge regression technique has not previously been used simultaneously with the Heckman selection correction procedure. The complication involved becomes apparent when you consider that from the point of view of the Heckman selection procedure, there are 84 "extra" observations in the second-stage price equation (as a result of using a ridge regression technique). We proceed as follows: First, the probit probability of sale model is estimated. These results are used to construct the inverse Mills ratio for use in the price equation (instead of simply running a packaged two-stage Heckman procedure). For each of the 84 synthetic quarterly observations in the price equation, we use the mean of all values of the inverse Mills ratio vector that fall in that respective quarter. This allows us to estimate the price equation with a value of the inverse Mills ratio for each observation.

The final step in the construction of the transactions based index is the inclusion of income to quantify the total return each period. This is done in a manner analogous to

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[24] The same procedure is applied separately to each of the property sector sub-indices.

the construction of the representative property capital returns from the NPI as in equation (24), only now we use the NPI income returns as well. For example, recall that the representative property had an appraised value of \$42.6/SF as of the end of 1983. As the NPI (equal-weighted, cash flow based) income return in 1984 was 5.85%, the net cash flow for the representative property in 1984 is presumed to be: \$42.6(.0585) = \$2.49 = CF_{p1984}. The general formula for computing the representative property transaction based total return, r_{pt} , is:

$$\left(1+r_{pt}\right)=\left(\exp\left[\hat{P}_{pt}\right]+CF_{pt}\right)/\exp\left[\hat{P}_{pt-1}\right] \qquad (25)$$

So in the case of 1984 it would be:

$$\begin{aligned}\left(1+r_{p1984}\right)&=\left(\exp[3.826]+\$2.49\right)/\exp[3.836]\\ &=\left(\$45.88+\$2.49\right)/\$46.34=\$48.37/\$46.34=1.0438\\ \Rightarrow r_{p1984} &= 4.38\%.\end{aligned}$$

Construction of transaction based representative property demand (constant liquidity) and supply side indices proceeds exactly as above, only based on $\hat{V}^{b}_{pt}$ and $\hat{V}^{s}_{pt}$ as described in equations (21) and (23) in Section 1. For example, continuing our previous 1984 example, and referencing the probit coefficient estimates reported in Exhibit 3, we have:

$$\hat{V}^{b}_{p1984} = \_cons + year\_1984 + dummyeffect + LaggedLogA\left(A_{p1984}\right)$$
$$+\,\text{sigmaProbit}\begin{pmatrix}\text{\_consProbit} + \hat{\gamma}_{1984} + \text{sqftProbit}(235000) \\ + \text{ jointvenProbit(jointvenMean)} + \text{LaggedLogA\_FProbit}\left(A_{p1984}\right)\end{pmatrix}$$

$$= .08173 - .07145 - .00498 + 1.009514\left(\log[\$44]\right) \qquad (26)$$
$$+\,.184257\begin{pmatrix}-1.419 + .150714 + .000000246(235000) \\ + .038464(.058263) - .19921\left(\log[\$44]\right)\end{pmatrix}$$

$$= 3.826 - 1.962 = 1.864.$$

Exponentiating, we have the 1984 buyer valuation (reservation price) for the representative property equal to: *exp*[1.864] = \$6.45/SF.[25]

Most of the difference in the returns between the variable-liquidity transactions based index and the demand and supply side indices will result from the probit time-dummy coefficients, $\hat{\gamma}_t$. These coefficients mirror the transaction frequency in the NCREIF property population. Unfortunately, this transaction frequency appears to be excessively random at the quarterly frequency. (Notice the "spiky" appearance in Exhibit 6.) Conversation with NCREIF members suggests that the specific quarterly timing of the recording of sales transactions is somewhat random, following a due-diligence and administrative process of scheduling the transaction closing, some time after the deal has been essentially agreed upon. The random and lagged nature of quarterly transaction report timing may be a source of noise in the quarterly price model, and may also result in a lagging phenomenon within the transaction price index. In constructing the demand

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[25] Estimated buyer valuations $\hat{V}^{b}_{pt}$ are typically below estimated transaction prices $\hat{P}_{pt}$, because the buyers' reservation price distribution is centered below the sellers'. (Recall Exhibit 1.) This difference in mean value level does not affect the longitudinal price-change percentages that track the demand side movements over time. (The same may be said for the supply side, except the mean seller valuations are above the transaction prices, again per Exhibit 1.)

and supply side indices at the quarterly frequency we have endeavored to mitigate this problem to some extent by employing a semi-annual averaging of the probit time-dummy coefficients. Exhibit 7 portrays the thusly-averaged coefficients superimposed on the variable-liquidity transaction price log levels.

Insert Exhibits 6 & 7 about here.

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3. Results and Analysis

The preceding section has walked the reader through the detailed steps of the index construction procedures. In this section we present the prototype indices we have developed to date.

Application of the procedures described in Section 2 results in the noise-filtered transactions based representative property cumulative quarterly appreciation index shown in Exhibit 8 together with the quarterly NPI.[26] (The index in Exhibit 8 is labeled "VL", for variable-liquidity, to distinguish it from the constant-liquidity version we will present shortly.) Note that the transactions based index exhibits greater volatility than the NPI, and like the annual index appears to slightly lead the NPI in major turning points. There is evidence that the volatility is real, in that particular historical events that would be

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[26] The detailed model estimation results corresponding to Equations (9) & (11) are presented in the Appendix. Note that the price model has an R^2 over 99.8%, while the probit sales model has a pseudo-R^2 of only 0.04. However, it must be recognized that we have N=121,353 observations, with only 3.628 sales transactions, making it difficult to obtain a high pseudo-R^2 in a selection model. (By way of comparison, with a much larger sales proportion in their annual-frequency data, Fisher *et al* (2004) obtain a maximum pseudo-R^2 of only slightly over 0.12 in a model that was focused explicitly on optimizing the sales prediction.)

expected to have negatively affected real estate markets are indeed reflected in depressions or down-ticks in the transactions based index (as pointed out in the exhibit). These historical events do not much appear in the NPI.

Insert Exhibits 8-12 about here.

The quarterly property-type sub-indices are presented Exhibits 9-12, for office, industrial, apartment, and retail, again in the form of cumulative appreciation level indices superimposed on the corresponding NCREIF property-type sub-indices. Due to transaction data scarcity at the property-type level, these indices begin only in the early 1990s, even using the ridge regression noise filter described previously. The office and industrial property indices, the two for which the most data is available, look fairly reasonable, including evidence of registering the two major indicator historical events: the 1998 financial crisis and the 9/11 terrorist attack. The apartment and retail indices do not show as clear a response.

Exhibit 13 returns us to the 20-year, all-property sample, and depicts the demand side (constant liquidity) and supply side transaction based indices at the quarterly frequency. While both indices appear rather “spiky” (likely due to the transaction frequency timing noise noted previously), the demand side index shows some appearance of tending to move a bit farther or quicker than the supply side index, consistent with pro-cyclical variable liquidity.

Insert Exhibit 13 about here.

To begin to explore the investment policy significance of the transaction based indices developed here, we have examined the quarterly total return statistics at the all-property level in comparison with those of other major asset classes. Exhibit 14 presents a summary of the major quarterly total return time series statistics for the NPI and the variable-liquidity transactions based index, along with several other major investment asset classes and indicators. Included are: (i) The NAREIT Equity REIT Index; (ii) the S&P500 Large Cap Stock Index; (iii) The Ibbotson Small Cap Stock Index; (iv) The Ibbotson Long-Term U.S. Government Bond Index; (v) The Ibbotson 30-Day Treasury Bill Index; and (vi) The Ibbotson Inflation (CPI) Index. The table reports the quarterly arithmetic mean total returns, quarterly volatility, Sharpe Ratio, and 1st-order autocorrelation coefficients for each asset class or series, as well as the cross-correlation among the series.

Insert Exhibit 14 about here.

It is interesting to note that while the transaction based index has notably higher volatility at the quarterly frequency and lower autocorrelation than the appraisal-based NPI, its volatility is still less than that of the stock and bond asset classes and its 1st-order autocorrelation is comparable. Also, while the transaction based index has higher correlation with both REITs and the stock market asset classes than the NPI does, its correlations with stocks is still low in absolute terms as well as relative to other securities based asset classes.

The result is that even when we use the transaction based index to represent private real estate, the role of private real estate is still prominent in a classical Markowitz mean-variance portfolio optimization, or a Sharpe-Maximizing (CAPM “Market Portfolio” type) efficient frontier analysis, based on historical investment performance statistics over the 1984-2003 period covered in our analysis. Exhibits 15 and 16 present area charts for the efficient frontier of risky assets as a function of target return (on the horizontal axis), with real estate measured either by the NPI (Exhibit 15) or the variable-liquidity transactions based index (Exhibit 16). We see that even using the transactions based index, private real estate plays a large role in the optimal portfolio, especially in the more conservative (lower return target, lower risk) range of investment policy. The difference in the optimal portfolio allocations shown in the area charts is small between the NPI and the VL transactions index. Exhibit 17 shows that the Sharpe-Maximizing portfolio allocation gives a large role to private real estate, though considerably less based on the transactions index than based on the NPI.[27]

Insert Exhibits 15-17 about here.

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[27] The riskfree interest rate is defined as the historical quarterly return earned by 30-Day Treasury Bonds during the period in question: 1984-2003. It should be noted that the mean return to the NPI during the historical period used in this analysis, 1.86%, was substantially below that of the broader period since the NPI inception in 1978 through 2004, which is 2.33%.

4. Conclusion

This paper has presented a new type of institutional investment real estate index, based on transaction prices and designed to support research on investment performance and asset market movements. The indices presented here are prototypes, still under development. Nevertheless, they suggest that there is considerable potential for transaction based versions of the NCREIF Index to provide interesting and useful information to the academic and industry research communities. It is the intention of the MIT Center for Real Estate to continue development of these and related index products, and to commence regular publication and dissemination of them, in cooperation with NCREIF, in the near future. The overall objective is to improve the level and quality of understanding and decision making in the real estate investment industry.

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About CREDL:

CREDL is an initiative of the MIT Center for Real Estate. The objective is to provide a “space” (virtual and real) for the development, refinement, and dissemination of quantitative measures of commercial property performance. This includes any dimension of performance: investment, operational, economic, engineering, environmental… CREDL is meant to provide a place to gather data, develop methodologies and tools, and provide a forum for analysis and perspective.

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Exhibit 1: Evolution of Buyer & Seller Reservation Price Distributions reflecting Variable Turnover.

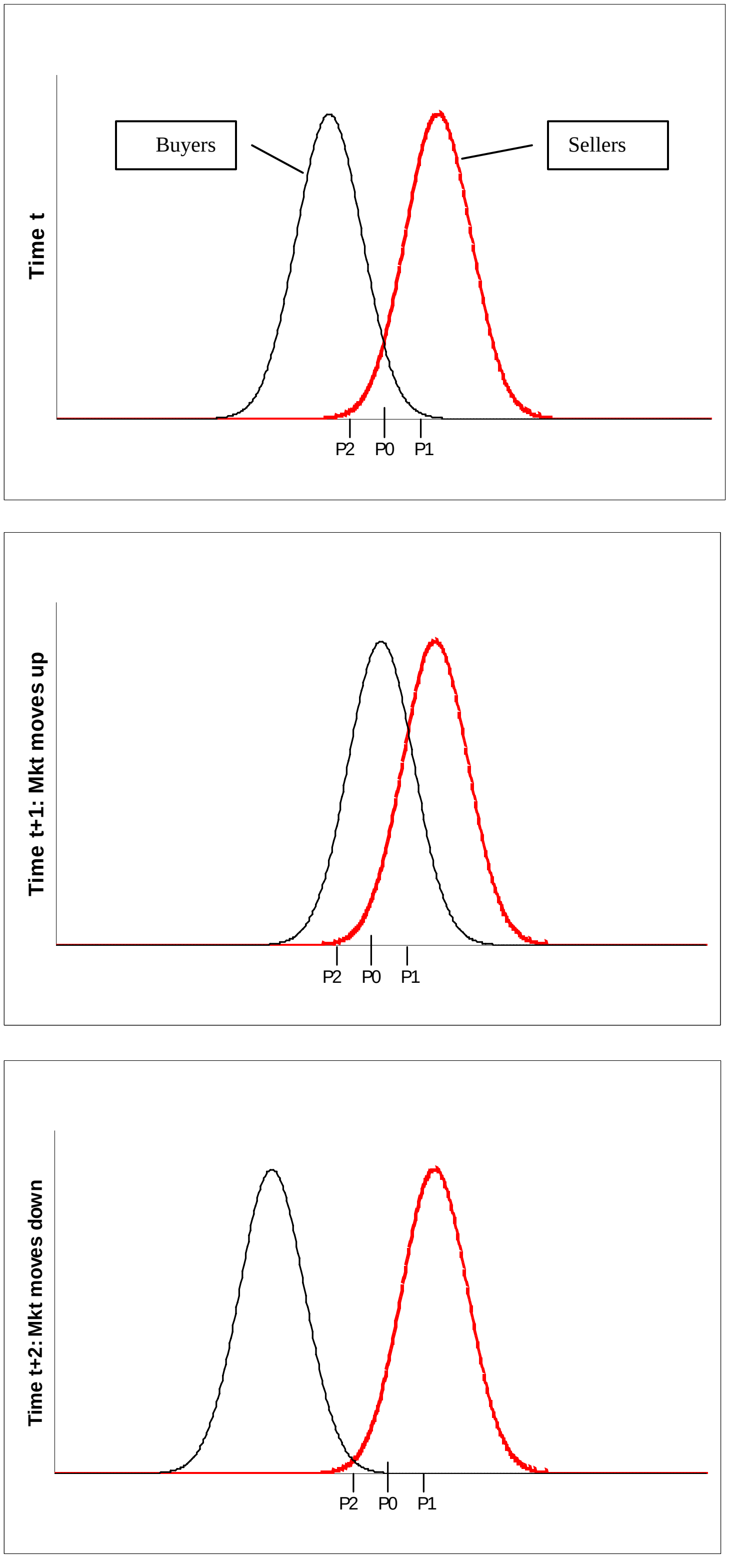


Exhibit 2: Annual Frequency Selection-Corrected Hedonic Price Model (Eqn.10), regressing log of sale price onto log of appraised value (lagged 2 qtrs), weighted annual time-dummies, and other variables as described in the text…

Heckman 2nd Step Results						
	Coef.	**Std. Err.**	**z**	**P>z**	**[95% Conf.**	**Interval]**
LogSalePri~F						
year_1984	-0.07145	0.045687	-1.56	0.118	-0.1609911	0.0180995
year_1985	-0.03777	0.045847	-0.82	0.41	-0.1276264	0.0520904
year_1986	-0.08036	0.046122	-1.74	0.081	-0.1707613	0.0100319
year_1987	-0.06644	0.045169	-1.47	0.141	-0.1549689	0.0220883
year_1988	-0.11389	0.045559	-2.5	0.012	-0.2031833	-0.0245955
year_1989	-0.14803	0.046522	-3.18	0.001	-0.2392149	-0.0568507
year_1990	-0.15252	0.044776	-3.41	0.001	-0.2402788	-0.0647604
year_1991	-0.19542	0.044355	-4.41	0	-0.2823591	-0.1084889
year_1992	-0.13488	0.044388	-3.04	0.002	-0.2218776	-0.047879
year_1993	-0.07022	0.043689	-1.61	0.108	-0.1558527	0.0154031
year_1994	-0.05322	0.044367	-1.2	0.23	-0.1401781	0.0337389
year_1995	-0.04726	0.044727	-1.06	0.291	-0.134926	0.0404011
year_1996	-0.05072	0.051894	-0.98	0.328	-0.1524275	0.0509928
year_1997	-0.00244	0.055763	-0.04	0.965	-0.1117373	0.1068482
year_1998	0.019129	0.0537	0.36	0.722	-0.08612	0.1243784
year_1999	-0.03184	0.051449	-0.62	0.536	-0.1326814	0.0689963
year_2000	-0.05885	0.049423	-1.19	0.234	-0.1557161	0.03802
year_2001	-0.06349	0.046456	-1.37	0.172	-0.1545426	0.0275626
year_2002	-0.0782	0.049372	-1.58	0.113	-0.1749678	0.0185647
year_2003	-0.05425	0.047892	-1.13	0.257	-0.1481194	0.0396153
en_div	-0.02822	0.013599	-2.08	0.038	-0.0548744	-0.0015691
me_div	-0.00684	0.014027	-0.49	0.626	-0.0343266	0.0206561
se_div	-0.00276	0.013193	-0.21	0.834	-0.0286173	0.0230996
sw_div	-0.00533	0.013566	-0.39	0.695	-0.0319142	0.021264
wn_div	-0.03499	0.016118	-2.17	0.03	-0.0665789	-0.0033981
wp_div	0.034075	0.012477	2.73	0.006	0.0096203	0.0585294
ne_div	-0.00367	0.015284	-0.24	0.81	-0.0336297	0.0262822
regionalma~m	0.086089	0.031935	2.7	0.007	0.0234979	0.1486804
retailmall~m	-0.06933	0.027995	-2.48	0.013	-0.1242046	-0.014465
retailsing~m	0.055619	0.022938	2.42	0.015	0.0106614	0.1005763
offcbd_dum	0.016624	0.015301	1.09	0.277	-0.0133665	0.0466138
offsub_dum	-0.00621	0.009048	-0.69	0.493	-0.0239426	0.0115237
warehouse_~m	-0.0118	0.009594	-1.23	0.219	-0.030602	0.0070067
indrd_dum	-0.01848	0.012209	-1.51	0.13	-0.0424082	0.0054487
indflex_dum	-0.04217	0.02518	-1.67	0.094	-0.0915235	0.0071787
jointven	0.008361	0.013723	0.61	0.542	-0.0185359	0.035258
LaggedLogA~F	1.009514	0.012377	81.57	0	0.985256	1.033772
_cons	0.08173	0.13515	0.6	0.545	-0.1831589	0.3466187

Exhibit 3: Heckman 1st-Stage (Probit Selection) Sales Probability Model, Annual Frequency:

	Heckman 1st Step Results						
		Coef.	**Std. Err.**	**z**	**P>z**	**[95% Conf.**	**Interval]**
select							
sqft	sqftselect	-2.46E-07	2.92E-08	-8.44	0	-3.04E-07	-1.89E-07
jointven	jointvenselect	0.038464	0.032007	1.2	0.229	-0.02427	0.101197
year_1984	year_1984select	0.150714	0.097945	1.54	0.124	-0.04126	0.342684
year_1985	year_1985select	0.251765	0.095705	2.63	0.009	0.064187	0.439342
year_1986	year_1986select	0.264572	0.095696	2.76	0.006	0.077011	0.452133
year_1987	year_1987select	0.168304	0.096473	1.74	0.081	-0.02078	0.357387
year_1988	year_1988select	0.291346	0.093735	3.11	0.002	0.107629	0.475063
year_1989	year_1989select	0.348413	0.092709	3.76	0	0.166707	0.53012
year_1990	year_1990select	0.152212	0.095346	1.6	0.11	-0.03466	0.339087
year_1991	year_1991select	0.13416	0.094976	1.41	0.158	-0.05199	0.320309
year_1992	year_1992select	-0.00471	0.09543	-0.05	0.961	-0.19175	0.182328
year_1993	year_1993select	0.163995	0.092879	1.77	0.077	-0.01805	0.346035
year_1994	year_1994select	0.288181	0.091241	3.16	0.002	0.109352	0.46701
year_1995	year_1995select	0.29028	0.091638	3.17	0.002	0.110674	0.469887
year_1996	year_1996select	0.590234	0.08912	6.62	0	0.415562	0.764905
year_1997	year_1997select	0.707038	0.088585	7.98	0	0.533414	0.880661
year_1998	year_1998select	0.645646	0.089302	7.23	0	0.470617	0.820675
year_1999	year_1999select	0.584216	0.089612	6.52	0	0.408579	0.759852
year_2000	year_2000select	0.520956	0.089737	5.81	0	0.345074	0.696837
year_2001	year_2001select	0.407951	0.090237	4.52	0	0.23109	0.584811
year_2002	year_2002select	0.512468	0.090407	5.67	0	0.335273	0.689663
year_2003	year_2003select	0.493384	0.088926	5.55	0	0.319093	0.667675
LaggedLogA~F	LaggedLogA~Fselect	-0.19921	0.011832	-16.84	0	-0.2224	-0.17602
_cons	_consselect	-1.41906	0.094507	-15.02	0	-1.60429	-1.23383
	lambda	-0.01989	0.065727	-0.3	0.762	-0.14872	0.10893
	rho	-0.10796					
	sigma	0.184257					

Exhibit 4: Representative Property Transaction Based Index (Annual Frequency Estimation) vs NPI:

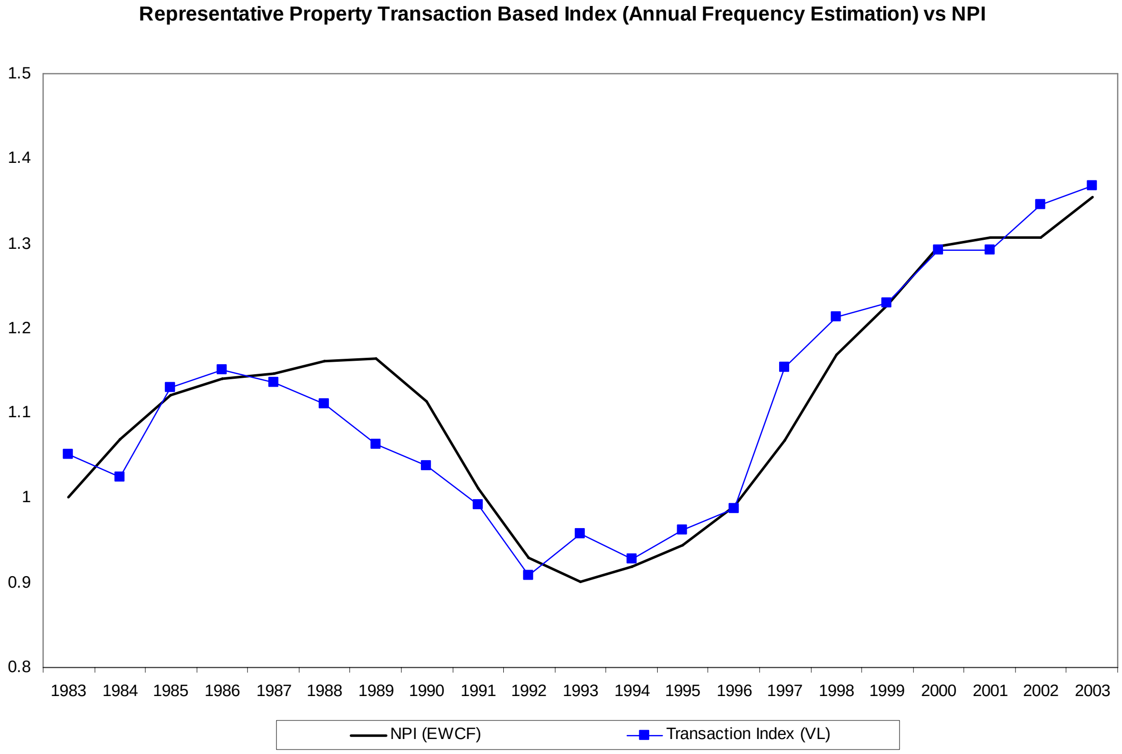


Exhibit 5: Representative Property Transaction Price Index: Annual vs Quarterly Estimation

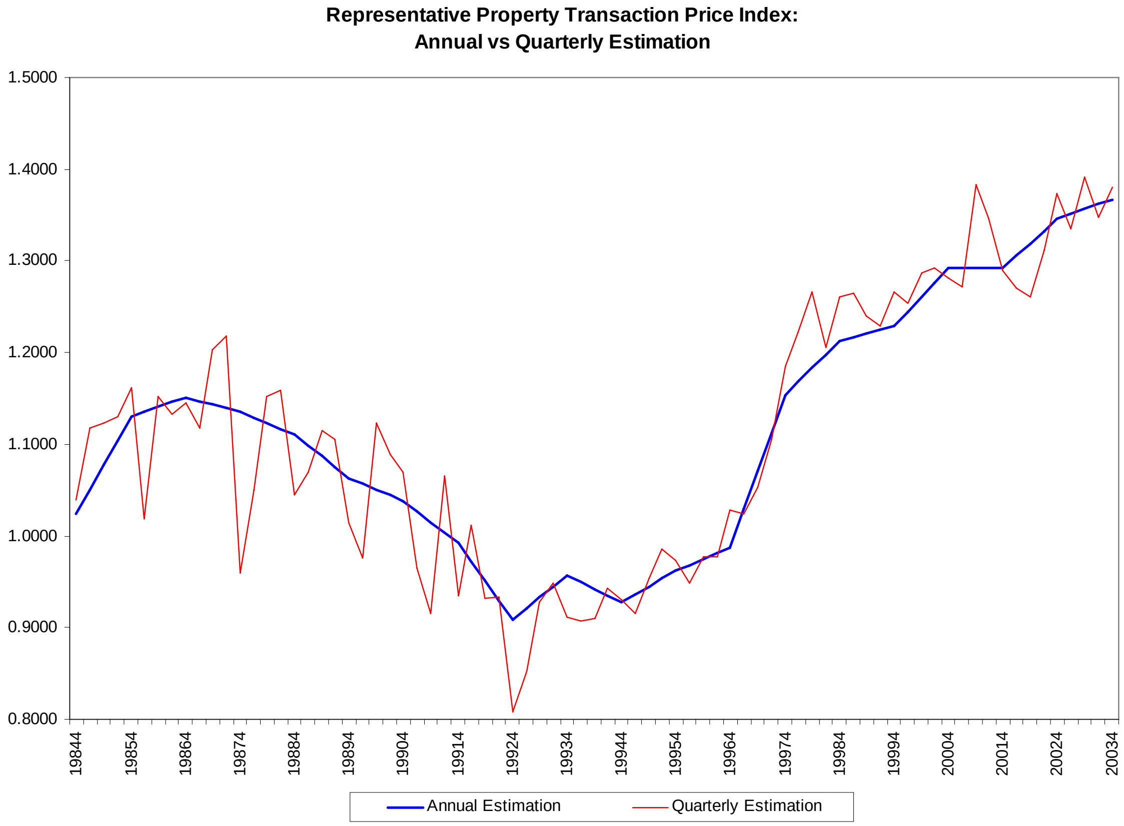


Exhibit 6: Quarterly All-property Probit Time-Dummy Coefficients (relative to average), Tracing relative frequency of property sales transactions in the NCREIF database…

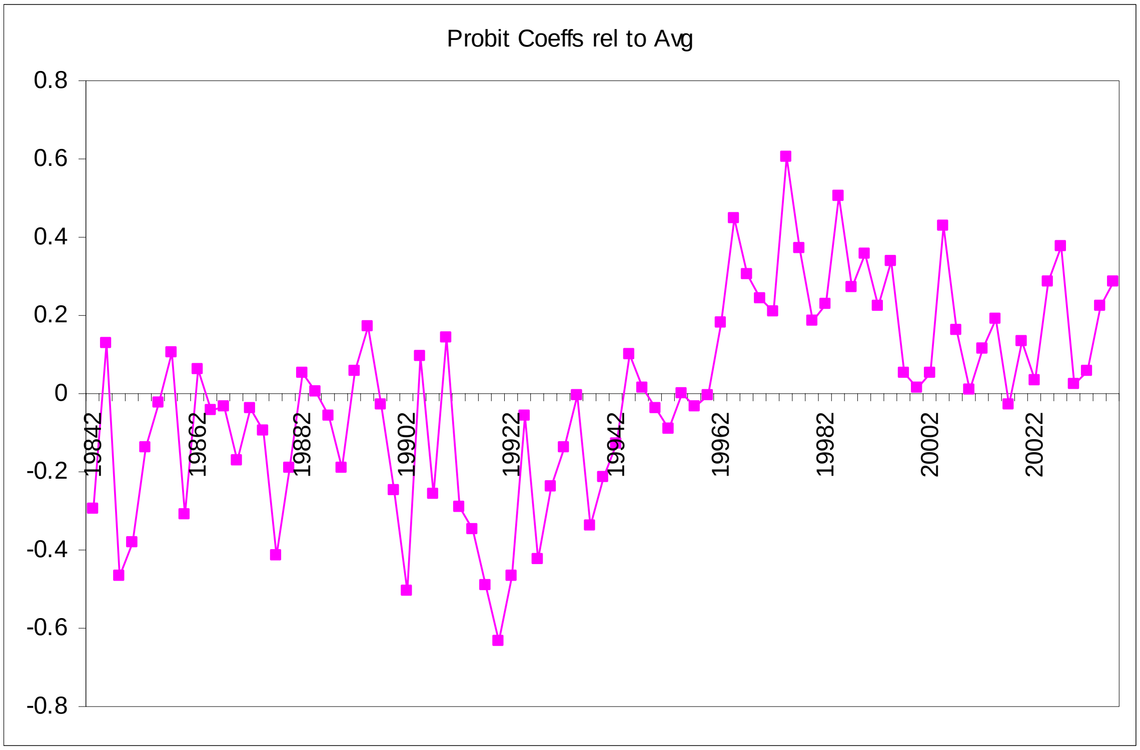


Exhibit 7: Semi-Annual Averaged Probit Time-Dummy Coefficients Superimposed on NCREIF Transaction Price Log Levels…

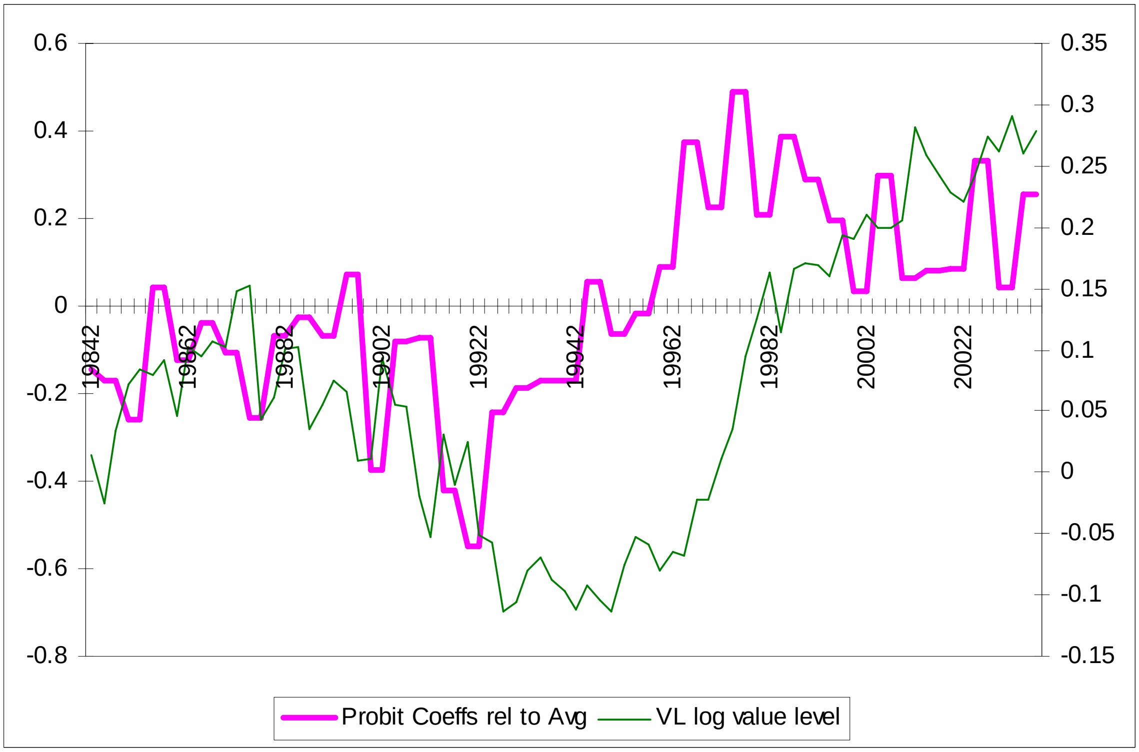


Exhibit 8: Quarterly Appreciation Levels, Transactions Based vs NPI:

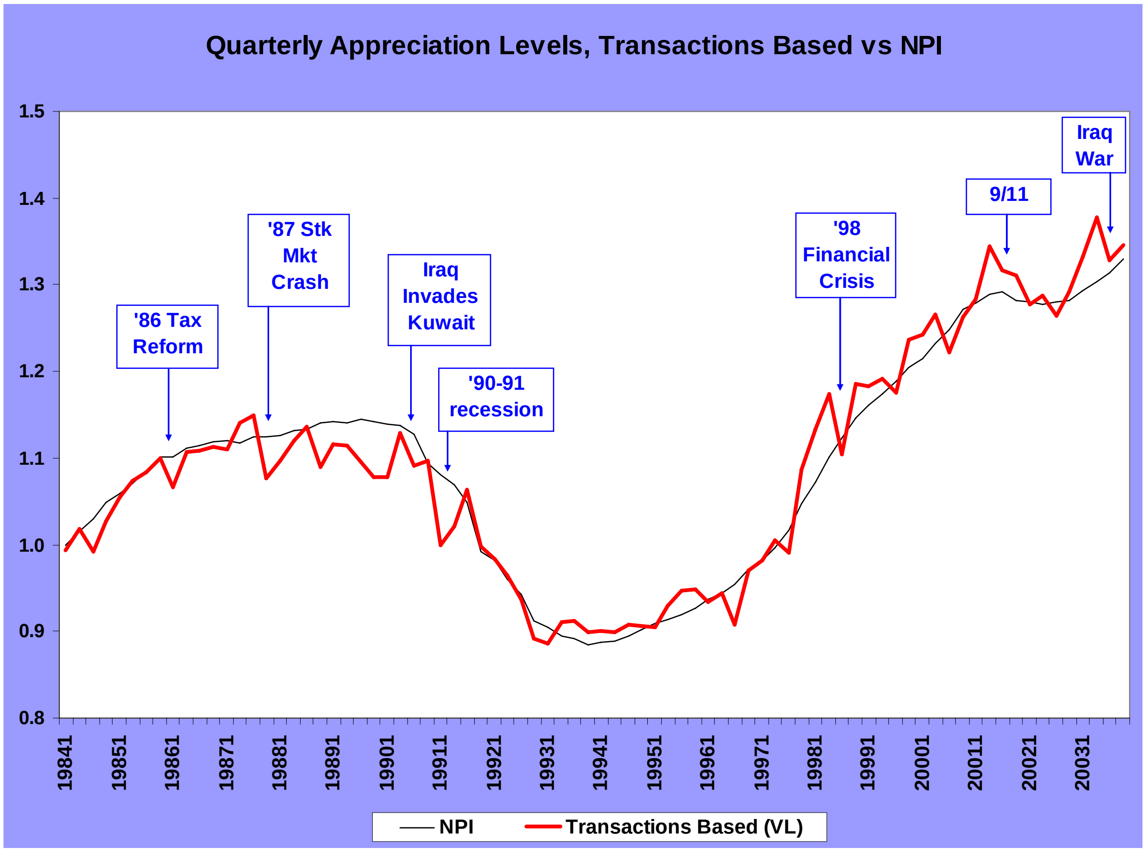


Exhibit 9:

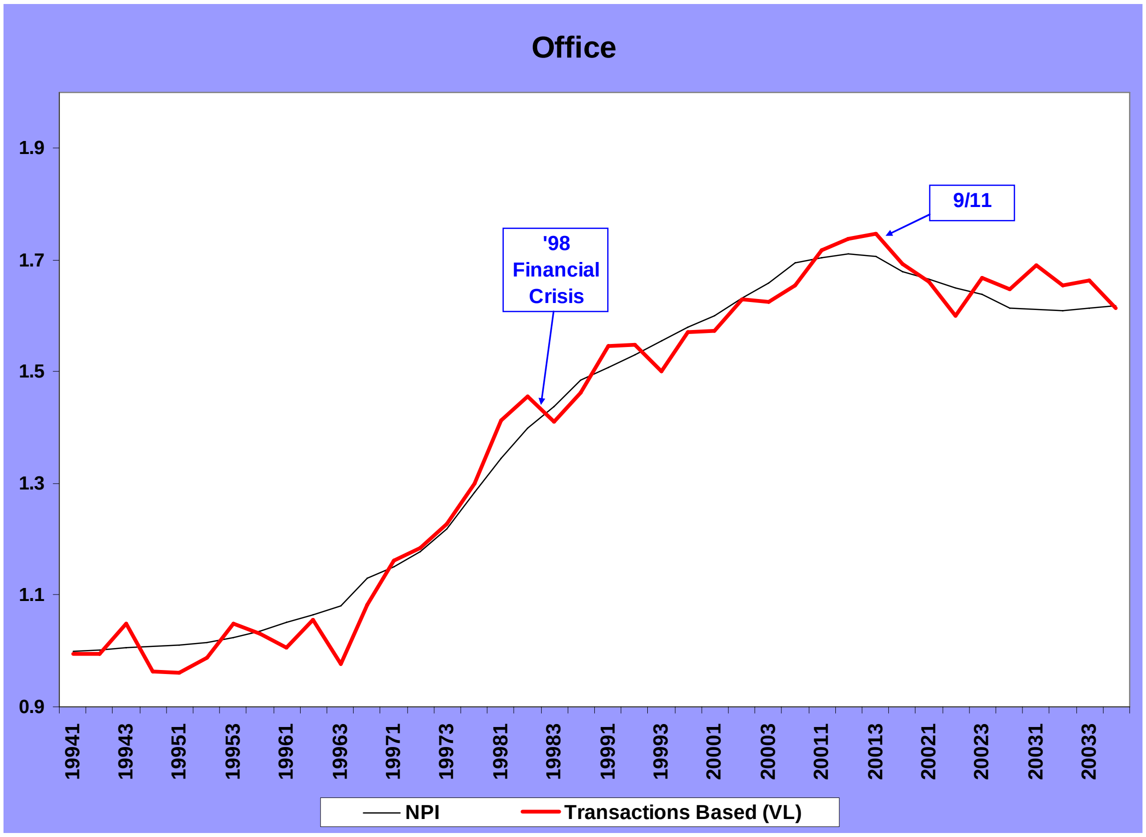

Office
1.9
1.7
1.5
1.3
1.1
0.9
'98 Financial Crisis
9/11
19941
19943
19951
19953
19961
19963
19971
19973
19981
19983
19991
19993
20001
20003
20011
20013
20021
20023
20031
20033
NPI
Transactions Based (VL)

Exhibit 10:

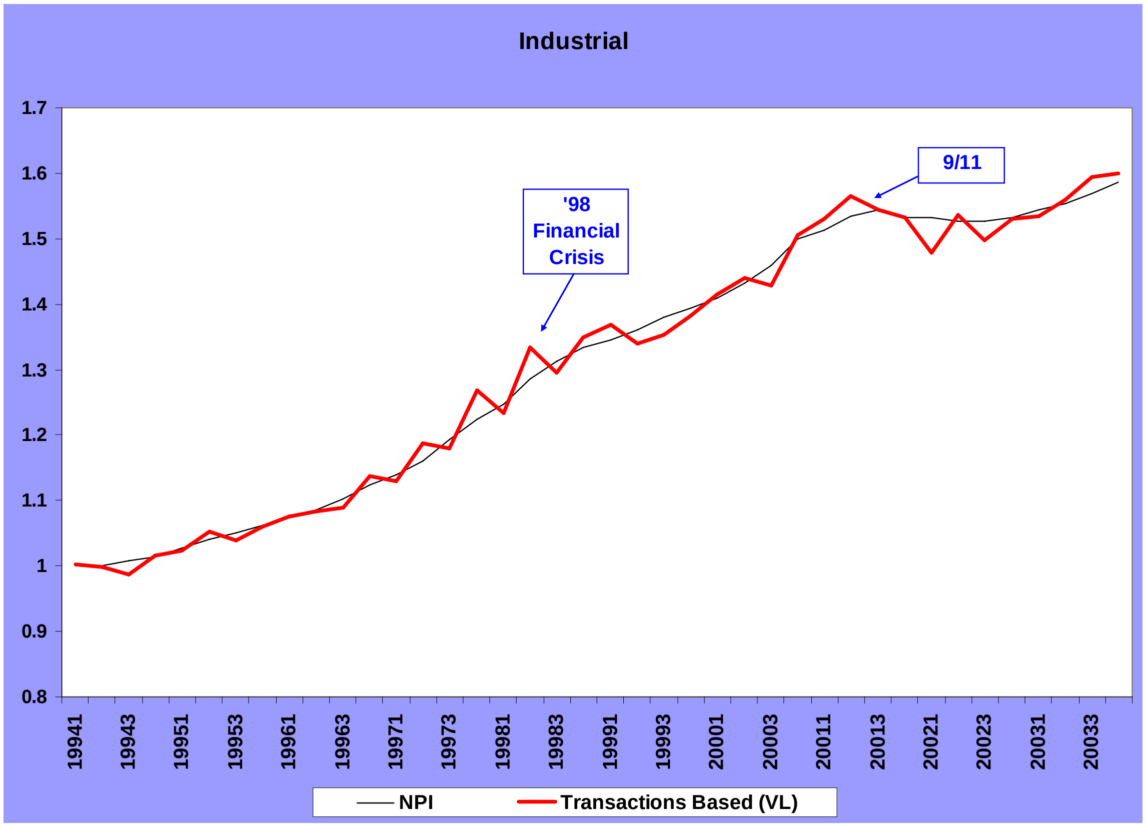

Industrial
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
19941
19943
19951
19953
19961
19963
19971
19973
19981
19983
19991
19993
20001
20003
20011
20013
20021
20023
20031
20033
'98 Financial Crisis
9/11
NPI
Transactions Based (VL)

Exhibit 11:

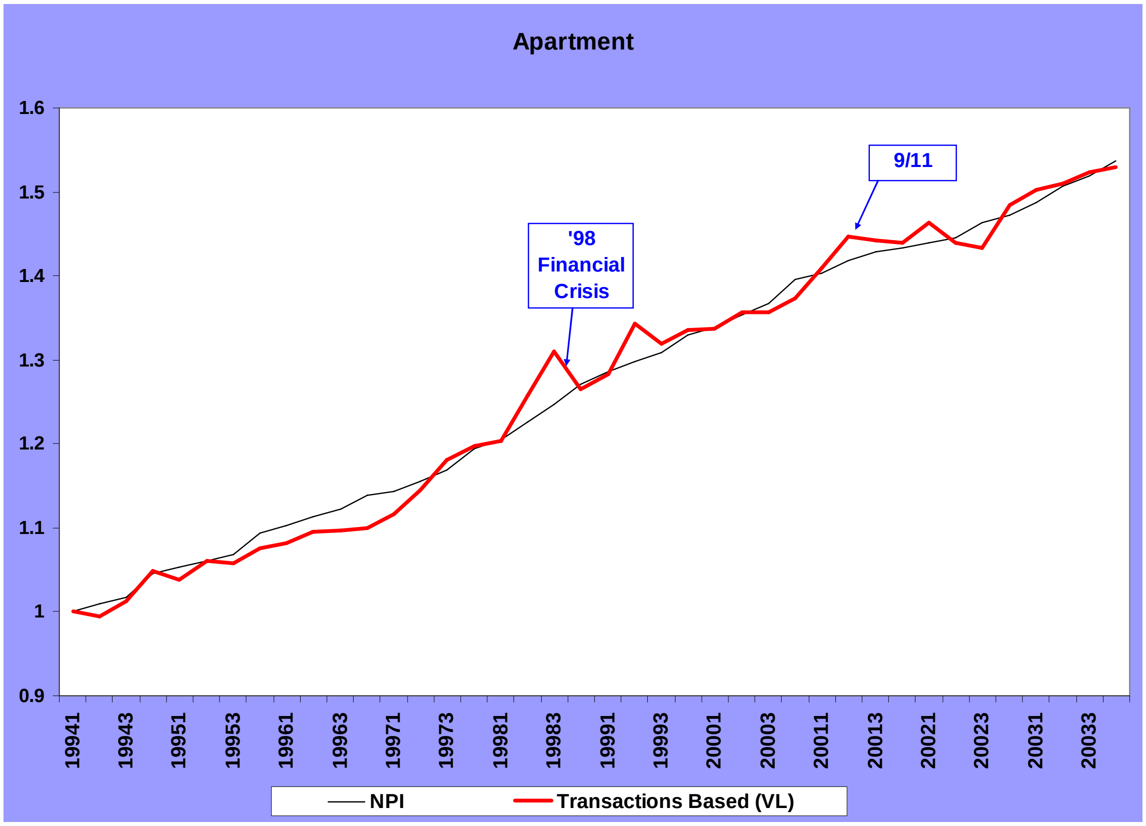

Apartment
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
'98 Financial Crisis
9/11
19941
19943
19951
19953
19961
19963
19971
19973
19981
19983
19991
19993
20001
20003
20011
20013
20021
20023
20031
20033
NPI
Transactions Based (VL)

Exhibit 12:

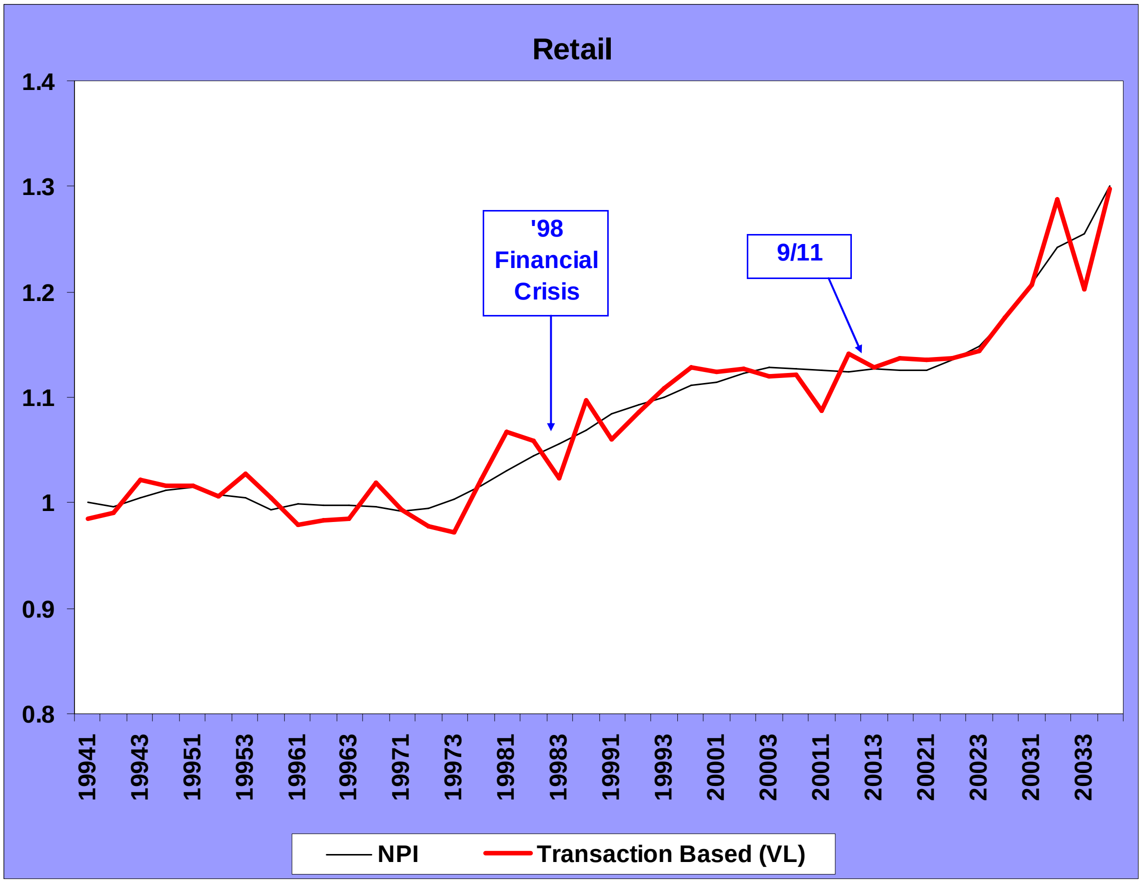

Retail
1.4
1.3
1.2
1.1
1
0.9
0.8
'98 Financial Crisis
9/11
19941
19943
19951
19953
19961
19963
19971
19973
19981
19983
19991
19993
20001
20003
20011
20013
20021
20023
20031
20033
NPI
Transaction Based (VL)

Exhibit 13:

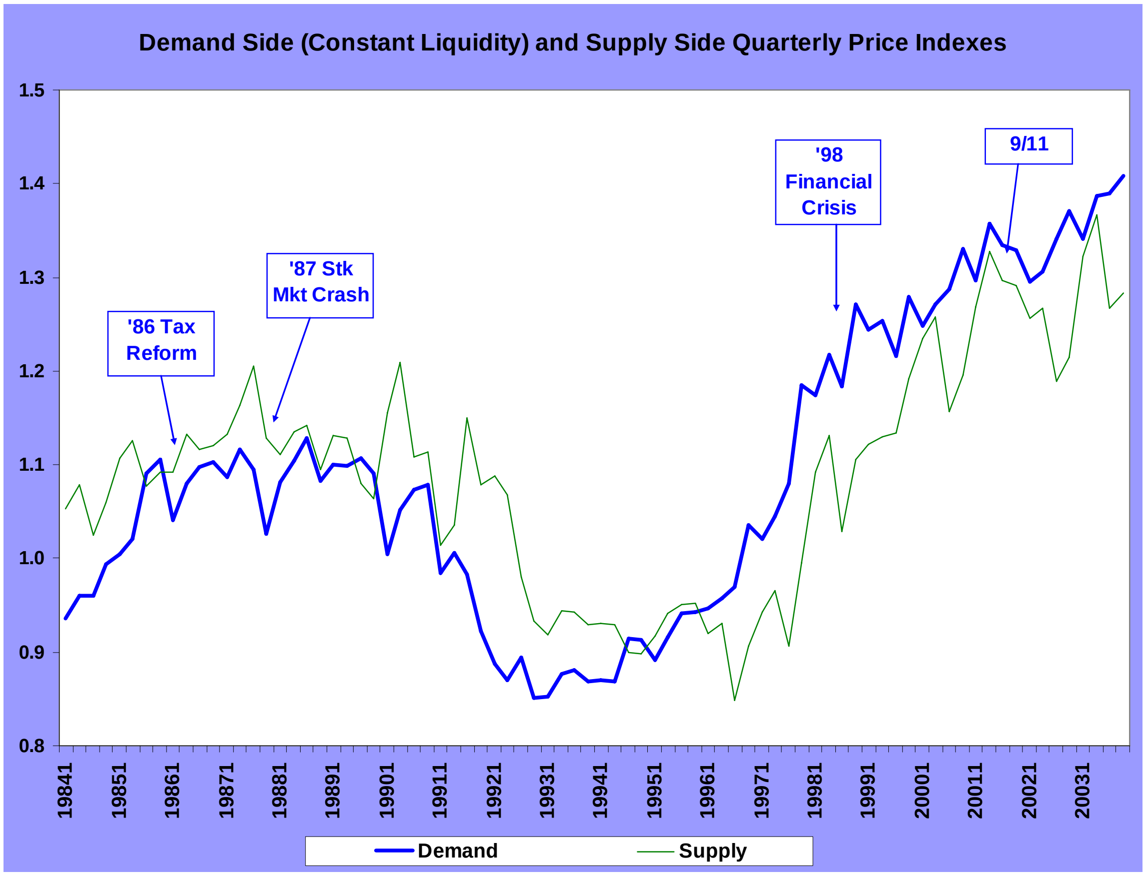

Demand Side (Constant Liquidity) and Supply Side Quarterly Price Indexes
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
'86 Tax Reform
'87 Stk Mkt Crash
'98 Financial Crisis
9/11
19841
19851
19861
19871
19881
19891
19901
19911
19921
19931
19941
19951
19961
19971
19981
19991
20001
20011
20021
20031
Demand
Supply

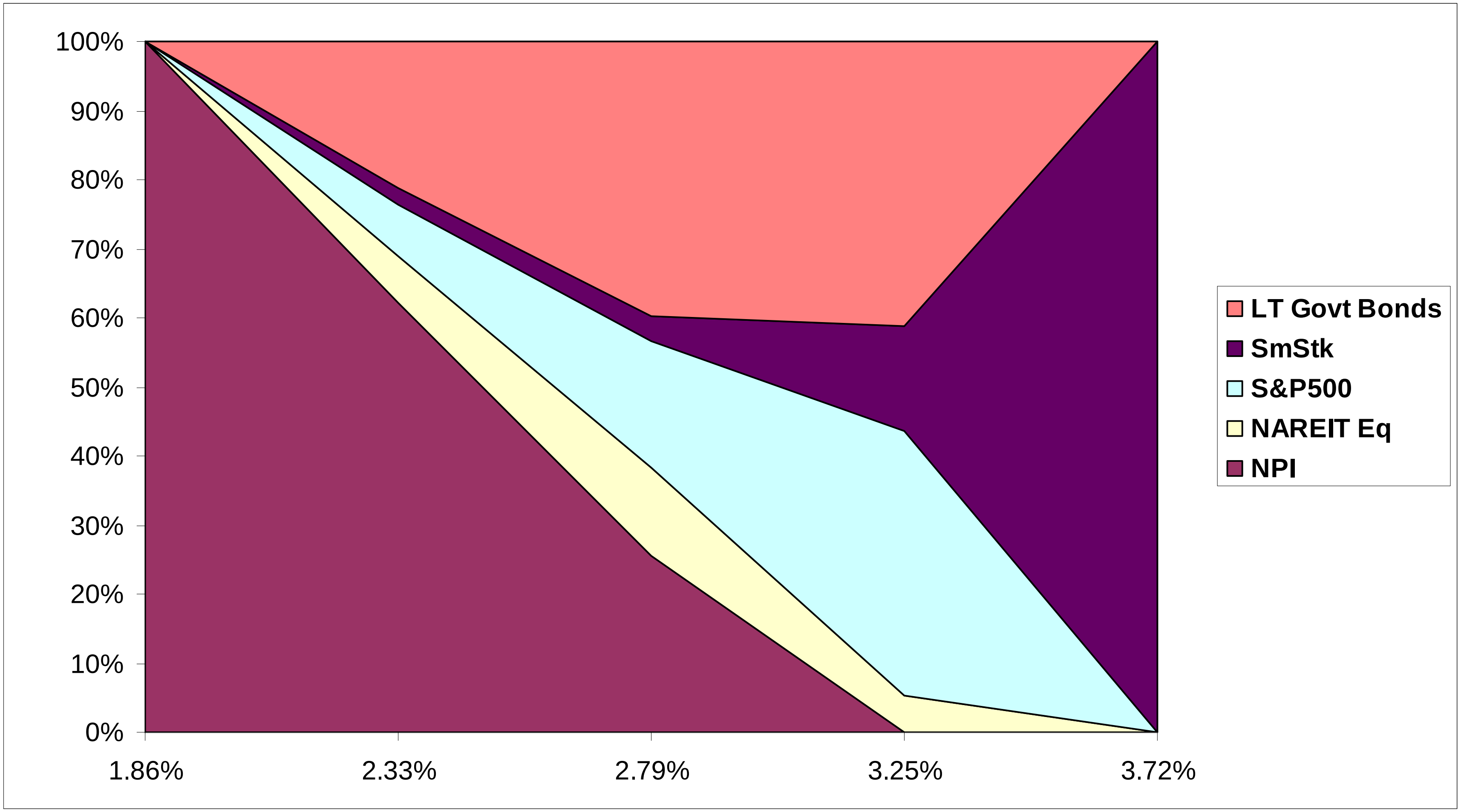
Exhibit 14:

Quarterly Total Return Statistics 1984Q2-2003Q4:

	NPI	Var.Liq. Trans.	NAREIT Eq	S&P500	SmStk	LT Govt Bonds	T-Bills	Inflation
Mean	1.86%	1.97%	3.12%	3.52%	3.72%	2.86%	1.28%	0.75%
Std.Dev.	1.45%	3.24%	6.68%	8.34%	11.54%	5.32%	0.52%	0.54%
1st-Order AutoCorr	79.06%	-12.85%	5.15%	-5.18%	-19.87%	3.17%	95.52%	-7.29%
Sharpe Ratio	0.40	0.21	0.28	0.27	0.21	0.30		

Correlations:	NPI	Var.Liq. Trans.	NAREIT Eq	S&P500	SmStk	LT Govt Bonds	T-Bills	Inflation
NPI	100.00%	46.13%	-2.82%	3.90%	-2.79%	-4.39%	7.95%	-12.06%
Var.Liq.Trans.		100.00%	2.75%	26.51%	15.15%	-7.31%	2.09%	-8.62%
NAREIT Eq			100.00%	49.75%	62.03%	19.58%	-7.11%	-12.68%
S&P500				100.00%	78.05%	4.39%	9.47%	-18.23%
SmStk					100.00%	-9.32%	-16.25%	-10.08%
LT Govt Bonds						100.00%	24.26%	-20.89%
T-Bills							100.00%	32.47%
Inflation								100.00%

Exhibit 15: Optimal Portfolio Shares, with private real estate based on NPI:



(Quarterly target return on horizontal axis.)

Exhibit 16: Optimal Portfolio Shares, with private real estate based on the variable-liquidity transactions based index:

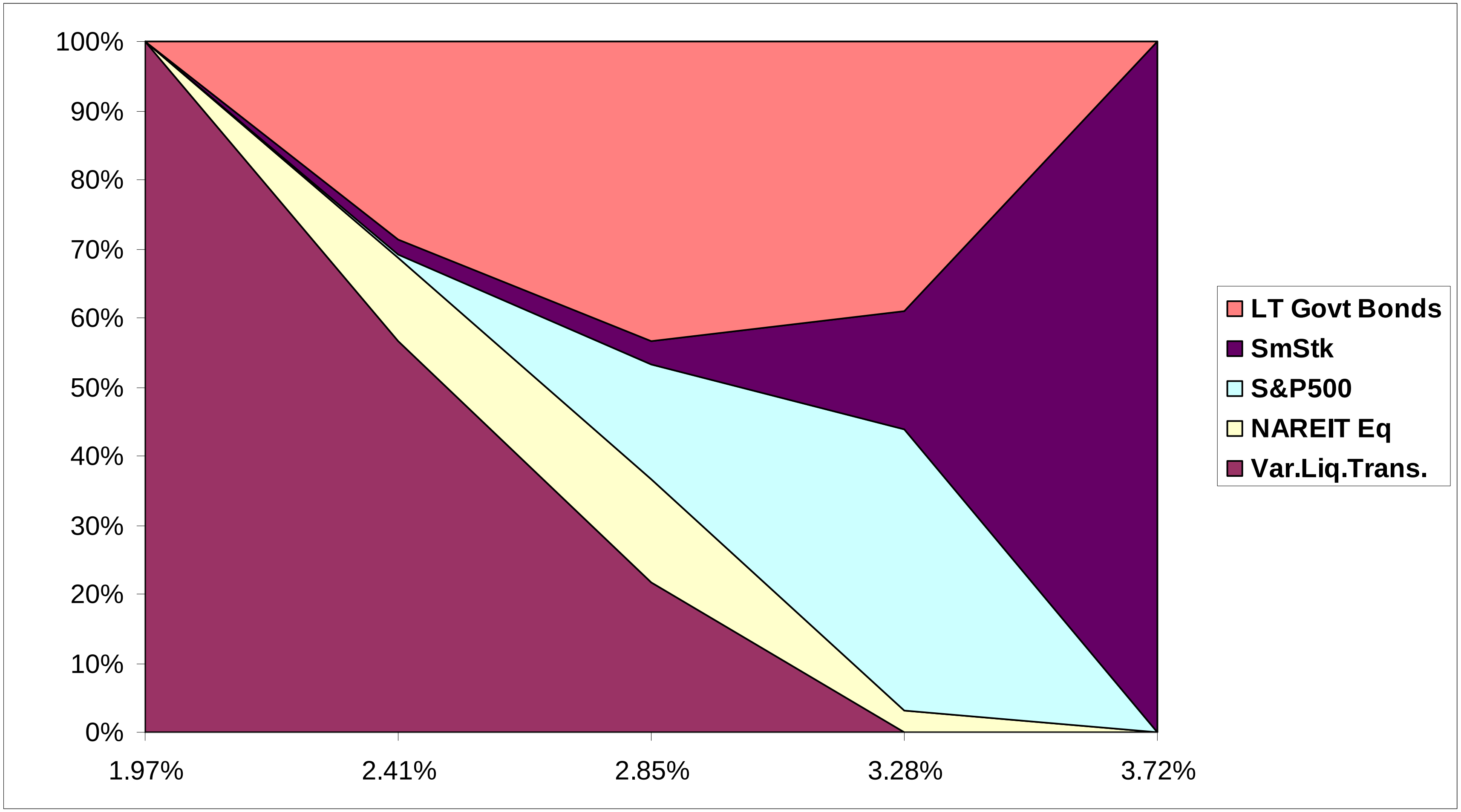


Exhibit 17: Sharpe-Maximizing Optimal Portfolio Shares Under Two Different Private Real Estate Scenarios:

Sharpe-Maximizing Portfolios:		
NPI	**75.26%**	**NA**
Var.Liq.Trans.	**NA**	**40.36%**
NAREIT Eq	**4.58%**	**13.50%**
S&P500	**3.36%**	**8.00%**
SmStk	**2.15%**	**2.60%**
LT Govt Bonds	**14.64%**	**35.54%**

Appendix
Estimation Results for Quarterly All-Property Model (Eqns. 9 & 10)

Appendix: Results of selection-corrected ridge-adjusted hedonic price model, Equation (10)

Dependent Variable: Log (sale price per square foot)

	Coef.	Std. Err.	t
1984.2	-0.0136	0.0484	-0.28
1984.3	-0.0723	0.0485	-1.49
1984.4	-0.0294	0.051	-0.58
1985.1	-0.0046	0.0497	-0.09
1985.2	-0.0113	0.0475	-0.24
1985.3	-0.0253	0.0474	-0.53
1985.4	-0.0247	0.0486	-0.51
1986.1	-0.0839	0.0489	-1.72
1986.2	-0.0411	0.0478	-0.86
1986.3	-0.0508	0.047	-1.08
1986.4	-0.0475	0.0475	-1
1987.1	-0.0542	0.0475	-1.14
1987.2	-0.0127	0.0467	-0.27
1987.3	-0.0101	0.0467	-0.22
1987.4	-0.1155	0.0498	-2.32
1988.1	-0.1038	0.047	-2.21
1988.2	-0.0652	0.047	-1.39
1988.3	-0.0654	0.0464	-1.41
1988.4	-0.1365	0.0461	-2.96
1989.1	-0.1186	0.0466	-2.54
1989.2	-0.1053	0.0466	-2.26
1989.3	-0.1164	0.0483	-2.41
1989.4	-0.1703	0.0461	-3.69
1990.1	-0.1731	0.0472	-3.67
1990.2	-0.0885	0.0524	-1.69
1990.3	-0.1228	0.0474	-2.59
1990.4	-0.1234	0.047	-2.62
1991.1	-0.1876	0.0472	-3.97
1991.2	-0.1905	0.047	-4.05
1991.3	-0.0943	0.0484	-1.95
1991.4	-0.1249	0.0507	-2.47
1992.1	-0.0711	0.0538	-1.32
1992.2	-0.0905	0.0495	-1.83
1992.3	-0.0858	0.0444	-1.93
1992.4	-0.1185	0.0482	-2.46
1993.1	-0.0932	0.0456	-2.05
1993.2	-0.034	0.0447	-0.76
1993.3	-0.0147	0.0448	-0.33
1993.4	-0.0209	0.0465	-0.45
1994.1	-0.0276	0.045	-0.61

Appendix: Results of selection-corrected ridge-adjusted hedonic price model, Equation (10)

Dependent Variable: Log (sale price per square foot)

	Coef.	**Std. Err.**	**t**
1994.2	-0.0338	0.0442	-0.76
1994.3	-0.0177	0.0452	-0.39
1994.4	-0.0313	0.0448	-0.7
1995.1	-0.0462	0.0446	-1.03
1995.2	-0.0186	0.0445	-0.42
1995.3	-0.0026	0.0447	-0.06
1995.4	-0.014	0.0443	-0.32
1996.1	-0.0422	0.0448	-0.94
1996.2	-0.0337	0.0465	-0.72
1996.3	-0.0486	0.0536	-0.91
1996.4	-0.0108	0.049	-0.22
1997.1	-0.0221	0.0477	-0.46
1997.2	-0.0073	0.0469	-0.16
1997.3	0.0074	0.0587	0.13
1997.4	0.0527	0.0508	1.04
1998.1	0.0604	0.047	1.28
1998.2	0.0691	0.0473	1.46
1998.3	-0.0026	0.0552	-0.05
1998.4	0.0221	0.0485	0.46
1999.1	0.0057	0.0514	0.11
1999.2	-0.0158	0.0475	-0.33
1999.3	-0.0379	0.0497	-0.76
1999.4	-0.0159	0.0449	-0.36
2000.1	-0.0319	0.0443	-0.72
2000.2	-0.0252	0.0446	-0.56
2000.3	-0.0439	0.0521	-0.84
2000.4	-0.0585	0.0461	-1.27
2001.1	-0.0663	0.0444	-1.49
2001.2	-0.0097	0.0452	-0.21
2001.3	-0.0376	0.0462	-0.81
2001.4	-0.0621	0.0437	-1.42
2002.1	-0.0782	0.045	-1.74
2002.2	-0.0783	0.0434	-1.8
2002.3	-0.0548	0.0503	-1.09
2002.4	-0.0202	0.0523	-0.39
2003.1	-0.0358	0.0437	-0.82
2003.2	-0.0089	0.0432	-0.21
2003.3	-0.0472	0.0459	-1.03
2003.4	-0.0365	0.047	-0.77
en_div	-0.0285	0.0136	-2.09
me_div	-0.0077	0.014	-0.55
se_div	-0.0035	0.0131	-0.26
sw_div	-0.0052	0.0135	-0.38
wn_div	-0.034	0.0161	-2.11
wp_div	0.0355	0.0125	2.85

Appendix: Results of selection-corrected ridge-adjusted hedonic price model, Equation (10)

Dependent Variable: Log (sale price per square foot)

	Coef.	**Std. Err.**	**t**
ne_div	-0.0021	0.0152	-0.14
regionalma~m	0.0767	0.0319	2.41
retailmall~m	-0.0606	0.0274	-2.21
retailsing~m	0.0439	0.0233	1.88
offcbd_dum	0.0203	0.0153	1.33
offsub_dum	-0.0037	0.009	-0.41
warehouse_~m	-0.0109	0.0096	-1.14
indrd_dum	-0.0177	0.0122	-1.45
indflex_dum	-0.0413	0.0251	-1.64
jointven	0.0063	0.0136	0.46
LaggedLogA~F	1.0141	0.0114	89.28
constant	0.1229	0.1242	0.99
InvMills	-0.0561	0.0607	-0.92
	R2 =	MSE =	
N=3628	0.9981	0.181	

Appendix: Results of probit model of property sale probability (Eqn.(9)

Dependent variable: sale dummy

	Coef.	Std. Err.	z
sqft	-2.45E-07	2.93E-08	-8.34
jointven	0.033	0.032	1.02
1984.2	0.035	0.127	0.28
1984.3	0.455	0.102	4.45
1984.4	-0.136	0.144	-0.95
1985.1	-0.052	0.136	-0.38
1985.2	0.191	0.117	1.63
1985.3	0.303	0.109	2.78
1985.4	0.435	0.103	4.21
1986.1	0.018	0.13	0.14
1986.2	0.389	0.107	3.65
1986.3	0.286	0.111	2.58
1986.4	0.295	0.11	2.67
1987.1	0.159	0.119	1.34
1987.2	0.288	0.108	2.68
1987.3	0.232	0.11	2.11
1987.4	-0.087	0.134	-0.65
1988.1	0.136	0.116	1.17
1988.2	0.382	0.102	3.75
1988.3	0.334	0.103	3.26
1988.4	0.271	0.106	2.56
1989.1	0.137	0.113	1.21
1989.2	0.385	0.1	3.85
1989.3	0.499	0.096	5.21
1989.4	0.302	0.104	2.9
1990.1	0.081	0.117	0.69
1990.2	-0.177	0.139	-1.27
1990.3	0.422	0.097	4.33
1990.4	0.07	0.116	0.6
1991.1	0.469	0.096	4.87
1991.2	0.039	0.115	0.34
1991.3	-0.02	0.12	-0.17
1991.4	-0.162	0.131	-1.24
1992.1	-0.304	0.143	-2.12
1992.2	-0.136	0.121	-1.13
1992.3	0.27	0.096	2.81
1992.4	-0.096	0.117	-0.82
1993.1	0.09	0.107	0.84
1993.2	0.189	0.102	1.86
1993.3	0.323	0.096	3.37
1993.4	-0.007	0.111	-0.07
1994.1	0.115	0.105	1.1

Appendix: Results of probit model of property sale probability (Eqn.(9)

Dependent variable: sale dummy

	Coef.	Std. Err.	z
1994.2	0.201	0.099	2.02
1994.3	0.43	0.091	4.71
1994.4	0.342	0.095	3.6
1995.1	0.292	0.097	3
1995.2	0.238	0.1	2.38
1995.3	0.329	0.096	3.42
1995.4	0.296	0.098	3.02
1996.1	0.326	0.097	3.38
1996.2	0.511	0.09	5.65
1996.3	0.776	0.084	9.27
1996.4	0.632	0.086	7.31
1997.1	0.572	0.088	6.49
1997.2	0.539	0.089	6.08
1997.3	0.933	0.082	11.42
1997.4	0.699	0.087	8.05
1998.1	0.512	0.093	5.48
1998.2	0.559	0.09	6.22
1998.3	0.834	0.084	9.96
1998.4	0.599	0.089	6.71
1999.1	0.686	0.087	7.9
1999.2	0.551	0.09	6.1
1999.3	0.667	0.087	7.67
1999.4	0.382	0.095	4.01
2000.1	0.342	0.095	3.59
2000.2	0.379	0.094	4.05
2000.3	0.758	0.084	9.02
2000.4	0.491	0.09	5.47
2001.1	0.338	0.095	3.58
2001.2	0.444	0.091	4.89
2001.3	0.518	0.087	5.94
2001.4	0.298	0.094	3.17
2002.1	0.464	0.089	5.24
2002.2	0.361	0.091	3.98
2002.3	0.613	0.096	6.41
2002.4	0.706	0.093	7.56
2003.1	0.354	0.09	3.91
2003.2	0.386	0.089	4.36
2003.3	0.554	0.083	6.64
2003.4	0.613	0.083	7.41
LaggedLogA~F	-0.201	0.012	-16.83
constant	-1.412	0.081	-17.42

N = 121,353　　Pseudo R2 = 0.04　　LR chi2(82) = 1295.6