Stability-Adjusted Portfolios

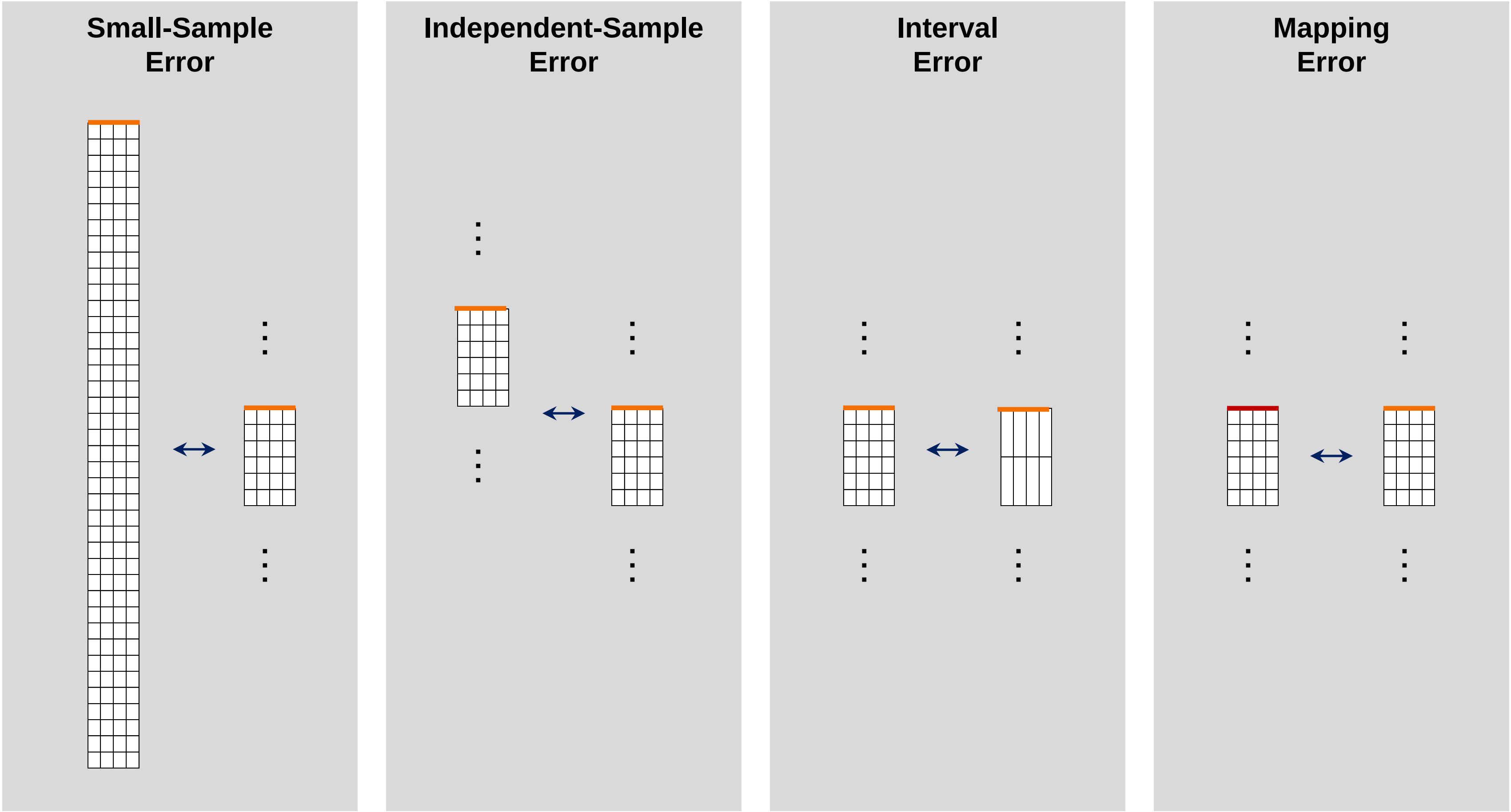
Mark Kritzman
David Turkington
Megan Czasonis

Overview

1. Sources of estimation error
2. Stability-adjusted portfolios
3. Implications for factor investing

Sources of Estimation Error

Sources of estimation error



Vary: **Small samples**

Hold constant:
- Forecasting sample
- Factor mapping
- Measurement interval

Vary: **Forecasting sample**

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- Sample size
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Vary: **Factor mapping**

Hold constant:
- Sample size
- Forecasting sample
- Measurement interval

Notes: Figure is for illustrative purposes only.

Small-sample error

The realization of parameters from a small sample will likely differ from the parameter values of a large sample from which it is selected.

We call this small-sample error.

$$SSE(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\rho_{AB,m,j}\sqrt{\sigma_{A,m,j}\sigma_{B,m,j}} - \rho_{AB,m}\sqrt{\sigma_{A,m}\sigma_{B,m}}}{\sqrt{\sigma_{A,m}\sigma_{B,m}}}\right)^2}$$

When A and B are the same asset, this formula will measure the error in the standard deviation of that asset.

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m, j indicates monthly estimates from a 36-month testing subsample

m alone indicates monthly estimates from the full sample

When A and B are the same asset, this formula will measure the error in the standard deviation of that asset.

Independent-sample error

The realization of parameters from a future sample will likely differ from the parameter values of an independent historical sample.

We call this independent-sample error.

$$ISE(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\rho_{AB,m,j}\sqrt{\sigma_{A,m,j}\sigma_{B,m,j}} - \rho_{AB,\hat{m},j}\sqrt{\sigma_{A,\hat{m},j}\sigma_{B,\hat{m},j}}}{\sqrt{\sigma_{A,m}\sigma_{B,m}}}\right)^2 - SSE(A,B)^2}$$

Notes: In rare instances, small sampler error exceeds the mean squared error between two independent samples. In these cases, we record small sample error as the mean squared error between the two samples, and record independent sample error as zero.

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m, j indicates monthly estimates from a 36-month testing subsample

$\hat{m}$, *j* indicates monthly estimates from a 36-month independent subsample immediately preceding the testing subsample

Notes: In rare instances, small sampler error exceeds the mean squared error between two independent samples. In these cases, we record small sample error as the mean squared error between the two samples, and record independent sample error as zero.

Mapping error

Mapping error can be isolated by comparing the covariance of the best-fit factor-mimicking portfolio to the covariance of a factor-mimicking portfolio estimated from an independent sample, holding the evaluation period constant.

$$ME(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\rho_{AB,m,j}\sqrt{\sigma_{A,m,j}\sigma_{B,m,j}} - \rho_{\hat{A}\hat{B},m,j}\sqrt{\sigma_{\hat{A},m,j}\sigma_{\hat{B},m,j}}}{\sqrt{\sigma_{A,m}\sigma_{B,m}}}\right)^2}$$

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$$ME(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\rho_{AB,m,j}\sqrt{\sigma_{A,m,j}\sigma_{B,m,j}} - \rho_{\hat{A}\hat{B},m,j}\sqrt{\sigma_{\hat{A},m,j}\sigma_{\hat{B},m,j}}}{\sqrt{\sigma_{A,m}\sigma_{B,m}}}\right)^2}$$

A, *B* indicate factor mappings derived from the testing subsample

$\hat{A}$, $\hat{B}$ indicate factor mappings derived from the independent subsample

Interval error

Parameters estimated from monthly or higher-frequency returns often differ from those estimated from lower-frequency returns, within the same sample.

We call this interval error.

Interval error

<u>January 1990 – December 2013</u>

Emerging markets stock return	9.3%
U.S. stock return	9.5%
Correlation of monthly returns	69%

Notes: Data spans Jan 1990 – Dec 2013.

Interval error

<u>January 1990 – December 2013</u>

Emerging markets stock return	9.3%
U.S. stock return	9.5%
Correlation of monthly returns	69%

<u>January 2005 – December 2007</u>

Emerging markets stocks – U.S. stocks	121%

Notes: Data spans Jan 1990 – Dec 2013.

Interval error

<u>January 1990 – December 2013</u>

Emerging markets stock return	9.3%
U.S. stock return	9.5%
Correlation of monthly returns	69%

<u>January 2005 – December 2007</u>

Emerging markets stocks – U.S. stocks	121%

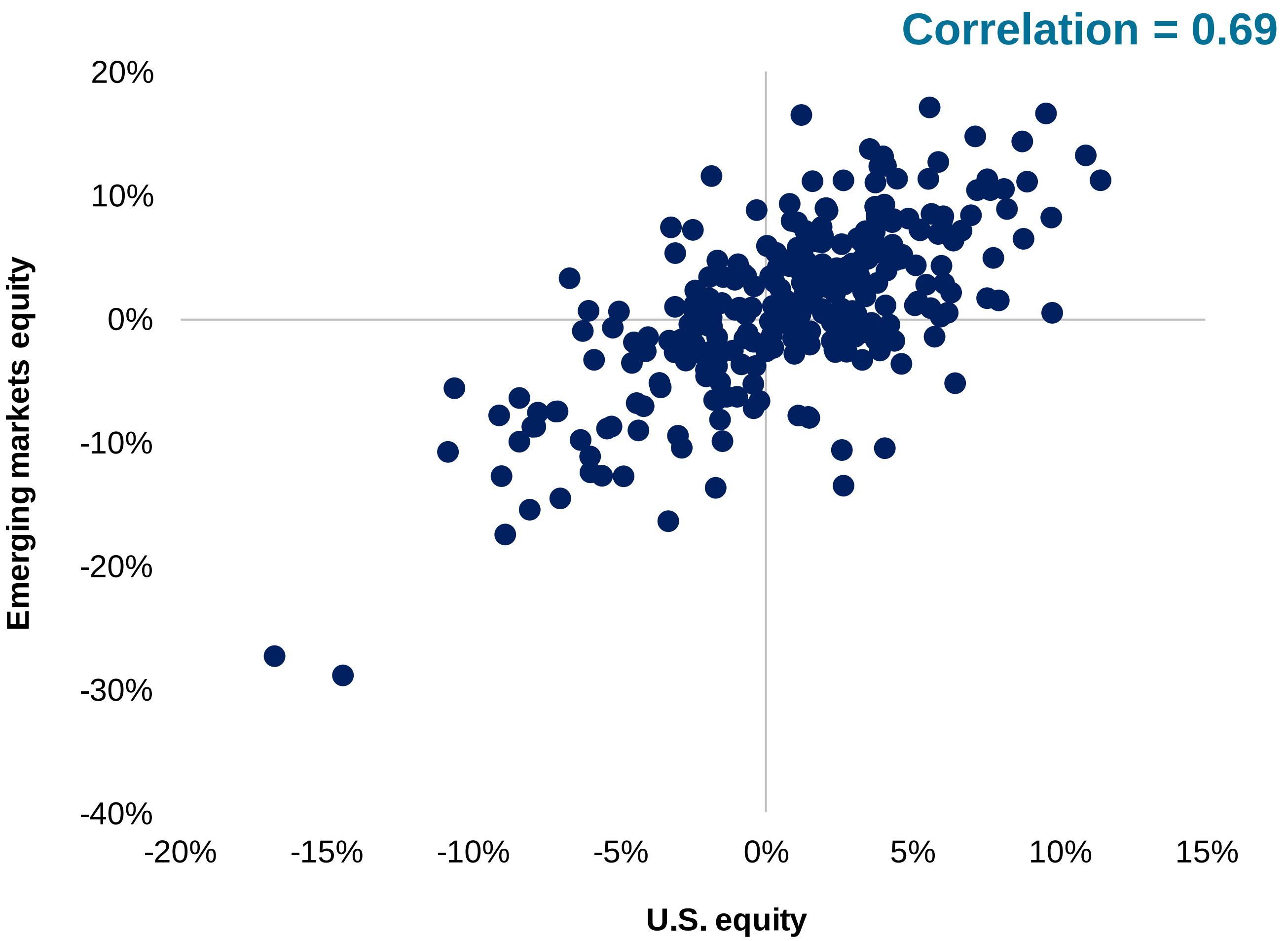
<u>January 2011 – December 2013</u>

Emerging markets stocks – U.S. stocks	-62%

Notes: Data spans Jan 1990 – Dec 2013.

Interval error

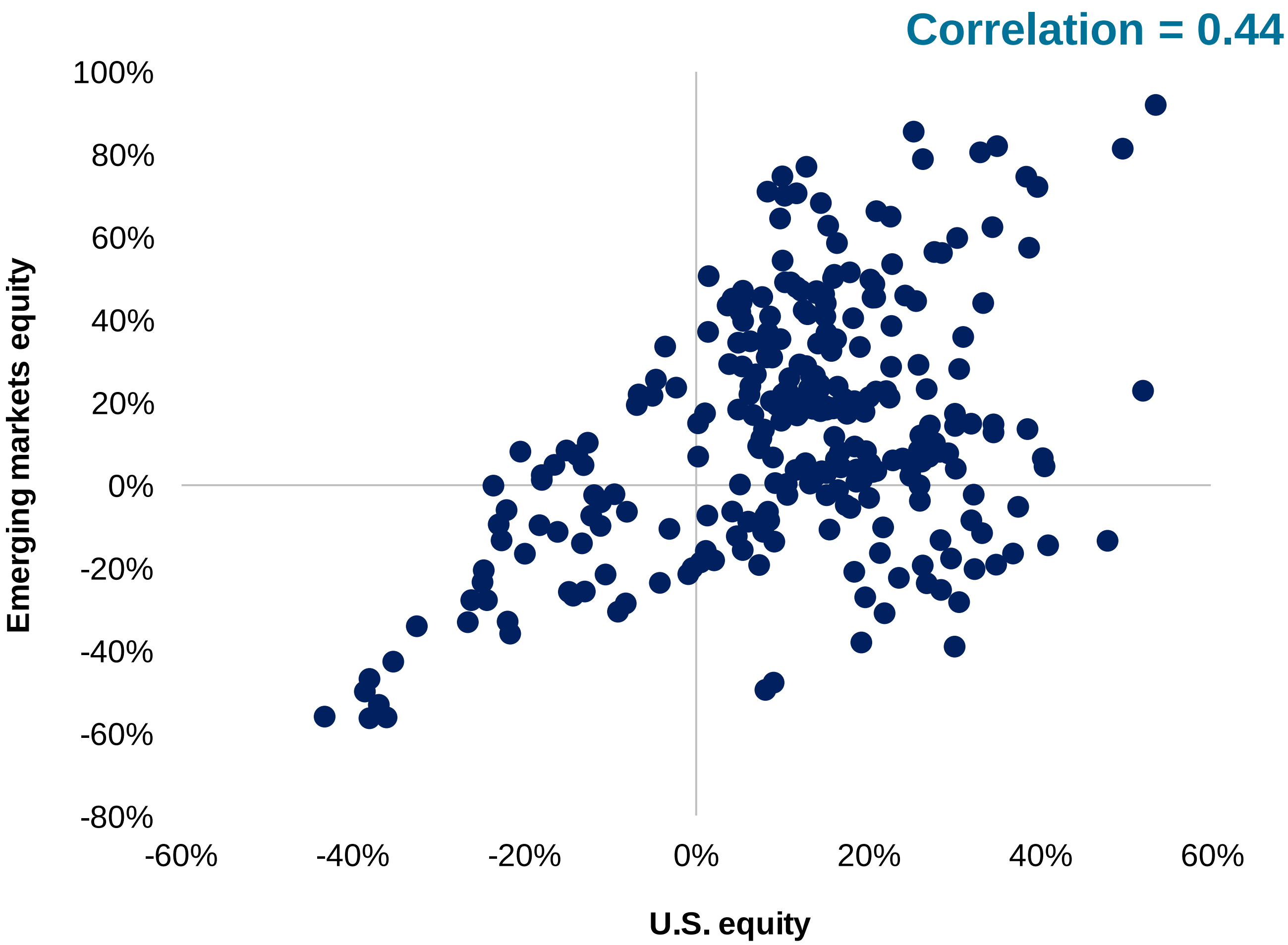
U.S. and emerging markets stocks: monthly returns



Notes: Data spans Jan 1990 – Dec 2013.

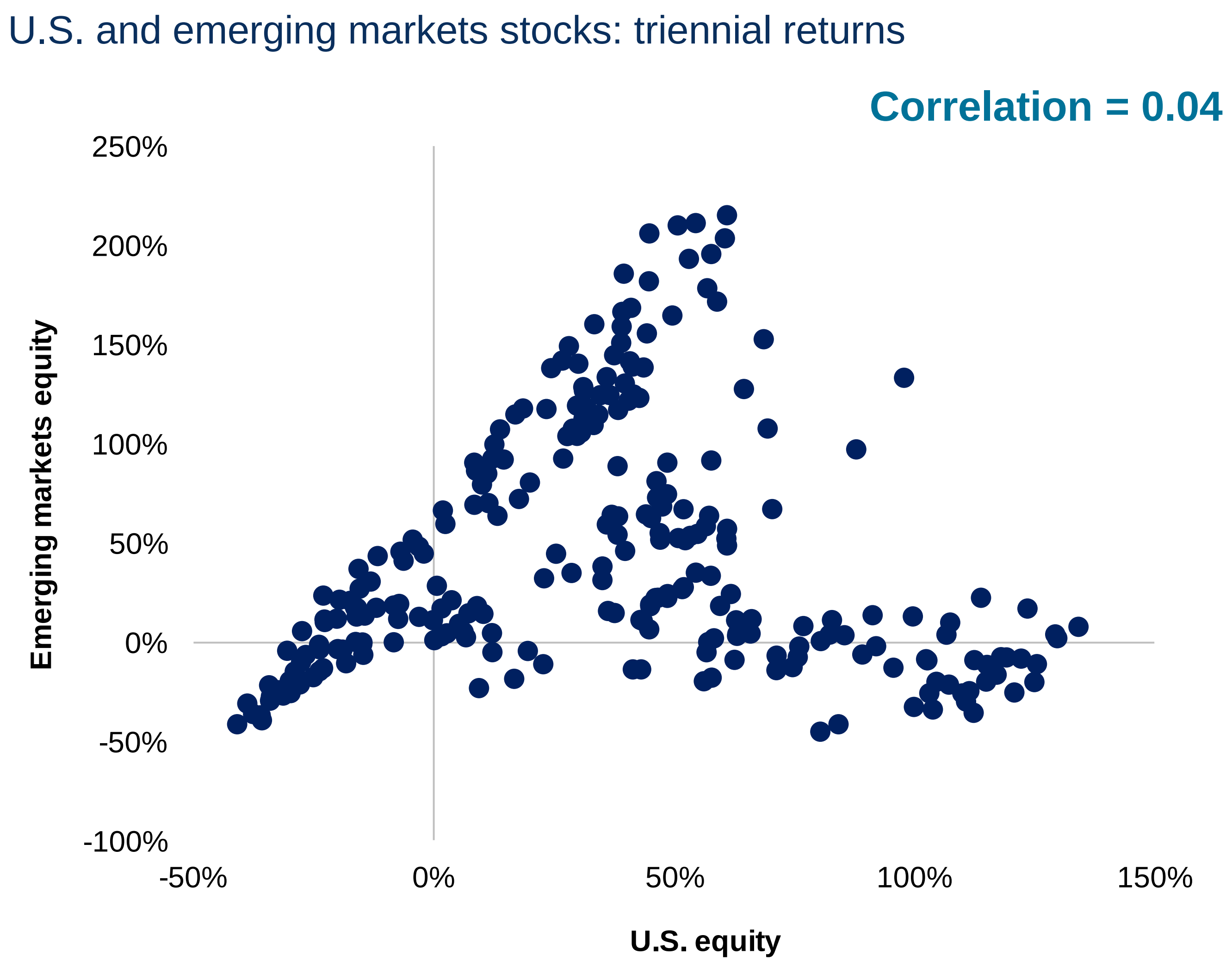
Interval error

U.S. and emerging markets stocks: annual returns



Notes: Data spans Jan 1990 – Dec 2013.

Interval error



Notes: Data spans Jan 1990 – Dec 2013.

Interval error: Long-horizon and short-horizon volatility

The volatility of the cumulative continuous returns of x over q periods is given by:

$$\sigma(x_t + \cdots + x_{t+q-1}) = \sigma_x \sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t, x_{t+k}}}$$

Interval error: Long-horizon and short-horizon volatility

The volatility of the cumulative continuous returns of x over q periods is given by:

$$\sigma(x_t + \cdots + x_{t+q-1}) = \sigma_x \sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t, x_{t+k}}}$$

This term reflects annualization in the absence of lagged effects

Interval error: Long-horizon and short-horizon volatility

The volatility of the cumulative continuous returns of x over q periods is given by:

$$\sigma(x_t + \cdots + x_{t+q-1}) = \sigma_x \sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t, x_{t+k}}}$$

This term captures the impact of auto-correlation

Interval error: Long-horizon and short-horizon correlation

The correlation between the cumulative returns of x and the cumulative returns of y over q periods is given by:

$$\rho(x_t + \cdots + x_{t+q-1}, y_t + \cdots + y_{t+q-1}) =$$

$$\frac{q\rho_{x_t,y_t} + \sum_{k=1}^{q-1}(q-k)(\rho_{x_{t+k},y_t} + \rho_{x_t,y_{t+k}})}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t,x_{t+k}}}\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{y_t,y_{t+k}}}}$$

Interval error: Long-horizon and short-horizon correlation

The correlation between the cumulative returns of x and the cumulative returns of y over q periods is given by:

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$$\frac{q\rho_{x_t, y_t} + \sum_{k=1}^{q-1}(q-k)(\rho_{x_{t+k}, y_t} + \rho_{x_t, y_{t+k}})}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t, x_{t+k}}}\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{y_t, y_{t+k}}}}$$

This term captures the lagged cross-correlation between x and y

Interval error: Long-horizon and short-horizon correlation

The correlation between the cumulative returns of x and the cumulative returns of y over q periods is given by:

$$\rho(x_t + \cdots + x_{t+q-1}, y_t + \cdots + y_{t+q-1}) =$$

$$\frac{q\rho_{x_t,y_t} + \sum_{k=1}^{q-1}(q-k)(\rho_{x_{t+k},y_t} + \rho_{x_t,y_{t+k}})}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t,x_{t+k}}}\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{y_t,y_{t+k}}}}$$

This term captures the auto-correlation of x

This term captures the auto-correlation of y

Interval error

Interval error can be isolated by comparing a covariance matrix estimated from low-frequency returns to a covariance matrix estimated from high-frequency returns.

$$IE(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\rho_{AB,ann,j}\sqrt{\sigma_{A,ann,j}\sigma_{B,ann,j}/12} - \rho_{AB,m,j}\sqrt{\sigma_{A,m,j}\sigma_{B,m,j}}}{\sqrt{\sigma_{A,m}\sigma_{B,m}}}\right)^2}$$

Interval error

Interval error can be isolated by comparing a covariance matrix estimated from low-frequency returns to a covariance matrix estimated from high-frequency returns.

$$IE(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\rho_{AB,ann,j}\sqrt{\sigma_{A,ann,j}\sigma_{B,ann,j}/12} - \rho_{AB,m,j}\sqrt{\sigma_{A,m,j}\sigma_{B,m,j}}}{\sqrt{\sigma_{A,m}\sigma_{B,m}}}\right)^2}$$

ann, j indicates implied 12-month estimates from a 36-month testing subsample

m, j indicates monthly estimates from the same 36-month testing sample

Stability-Adjusted Portfolios

Stability-adjusted optimization, Bayesian shrinkage, and resampling

- Stability-adjusted optimization is a portfolio formation process in which the relative stability of covariances is explicitly incorporated into the optimization process.

- Bayesian shrinkage is a process for mitigating estimation error by blending individual estimates with a cross-sectional mean or some other prior belief.

- Resampling is a process by which optimal portfolio weights are generated many times from a distribution of inputs and then averaged to determine the final portfolio.

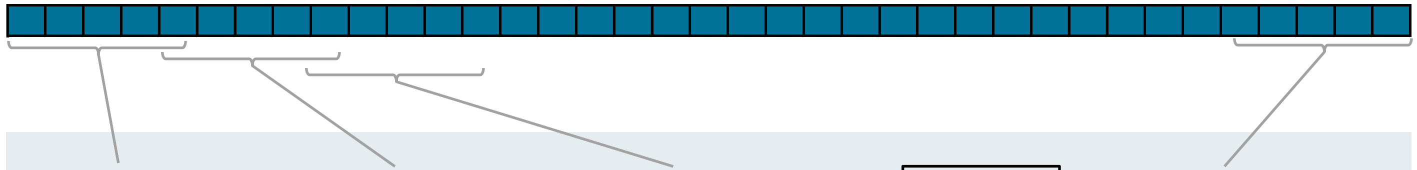
- Both Bayesian shrinkage and resampling have the effect of desensitizing the portfolio composition to input errors and thus rendering the efficient portfolios more similar to each other.

- Stability-adjusted optimization, by contrast, does not attempt to reduce sensitivity to errors. Rather, it attempts to distinguish more stable covariances from less stable covariances and to treat this information as an additional source of risk. Thus the portfolio composition will be more sensitive to some errors and less sensitive to others.

- Moreover, stability-adjusted portfolios in some cases may be more self-similar and in other cases less self-similar.

Constructing a stability-adjusted return distribution

Begin with a long sample of asset returns



Estimate small sample covariance matrices

Σ_{s1} Σ_{s2} Σ_{s3} … Σ_{med} … Σ_{sn}

Compute error matrix for each small sample versus its complementary sample

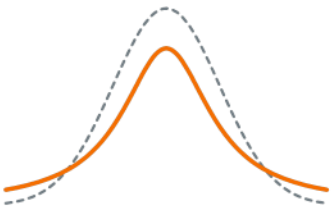
$\Sigma_{e1} = \Sigma_{s1} - \Sigma_{c1}$ $\Sigma_{e2} = \Sigma_{s2} - \Sigma_{c2}$ $\Sigma_{e3} = \Sigma_{s3} - \Sigma_{c3}$ … $\Sigma_{en} = \Sigma_{sn} - \Sigma_{cn}$

Add each error matrix to the median covariance matrix

$\Sigma_{1} = \Sigma_{e1} + \Sigma_{med}$ $\Sigma_{2} = \Sigma_{e2} + \Sigma_{med}$ $\Sigma_{3} = \Sigma_{e3} + \Sigma_{med}$ … $\Sigma_{n} = \Sigma_{en} + \Sigma_{med}$

Draw sample returns from normal distributions

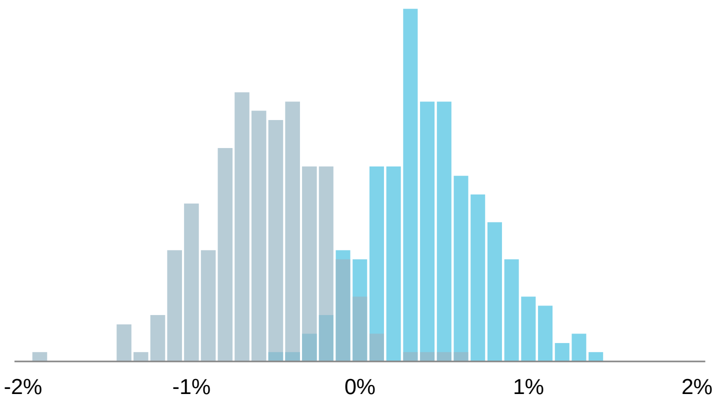
Combine to form a composite non-normal distribution



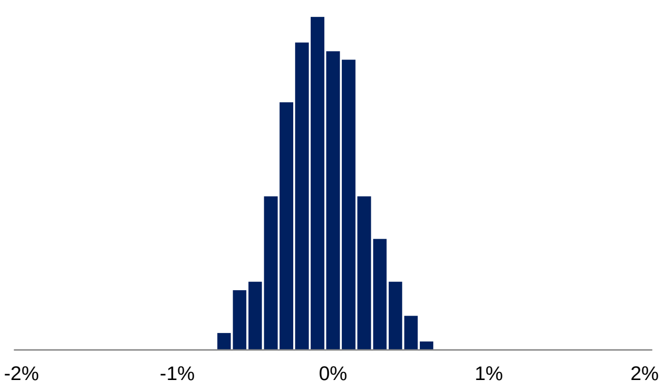
Notes: Figure is for illustrative purposes only.

Constructing a stability-adjusted return distribution

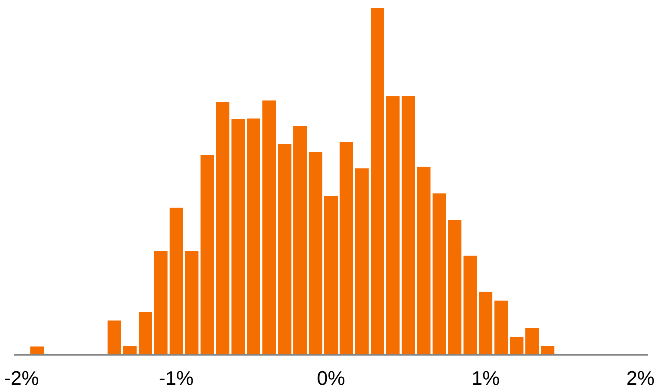
Two distributions



Distribution of the average for days 1, 2, 3, …



Combined distribution for all days



Notes: Figure is for illustrative purposes only. Results are simulated. Each chart shows the frequency of returns for specified return intervals.

Constructing portfolios from stability-adjusted return samples

1. Mean-variance optimization assumes either that returns are elliptically distributed or that investors have preferences that can be well approximated by mean and variance.

2. We have already shown that stability-adjusted return distributions are neither normal nor elliptical.

3. Moreover, we wish to consider investor preferences that change discontinuously at certain thresholds. These preferences cannot be well described by mean and variance.

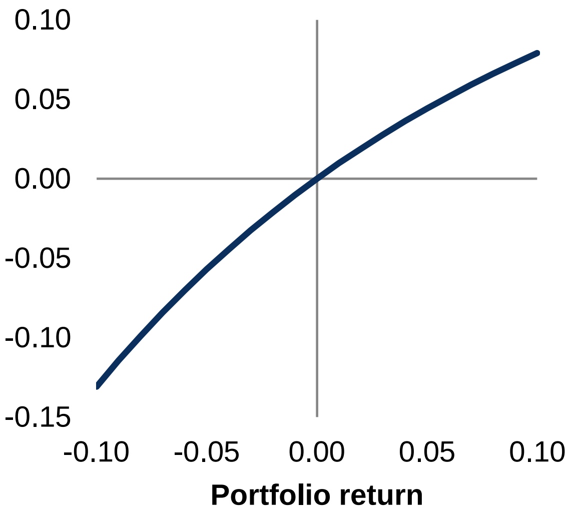
4. Therefore, rather than apply mean-variance optimization, we deploy a portfolio construction process called full-scale optimization.*

* The term, full-scale optimization, was coined by Paul A. Samuelson in correspondence with Mark Kritzman

Constructing portfolios from stability-adjusted return samples

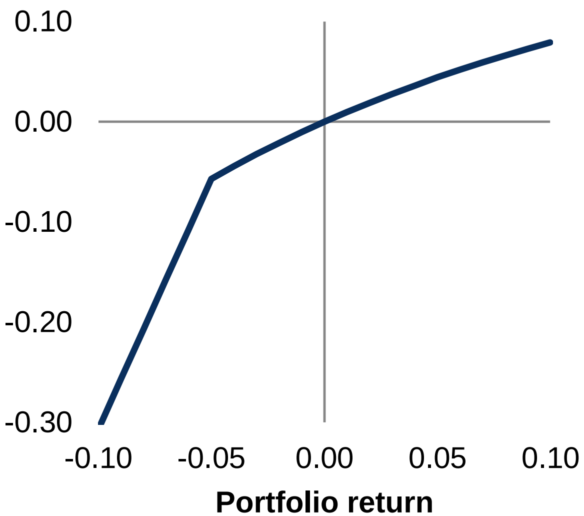
Power Utility

$$U(r) = \frac{1}{1-\theta}\left((1+r)^{1-\theta} - 1\right)$$



Kinked Utility

$$U(r) = \begin{cases} \frac{1}{1-\theta}\left((1+r)^{1-\theta} - 1\right), & if\ r \geq k \\ \frac{1}{1-\theta}\left((1+k)^{1-\theta} - 1\right) - \omega(k-r), & if\ r < k \end{cases}$$



Notes: Images for illustrative purposes only

Constructing portfolios from stability-adjusted return samples

Full-scale optimization: Illustrative example

	Stock Return	Bond Return	Stock Weight	Bond Weight	Utility Calculation	Utility
1993	10.1%	16.2%	**100%**	**0%**	Ln [(1 + 10.1%) x 100% + (1 + 16.2%) x 0%] =	0.0962
1994	1.3%	-7.1%	**100%**	**0%**	Ln [(1 + 1.3%) x 100% + (1 – 7.1%) x 0%] =	0.0129
1995	37.5%	30.0%	**100%**	**0%**	Ln [(1 + 39.5%) x 100% + (1 + 30.0%) x 0%] =	0.3185
1996	22.9%	0.1%	**100%**	**0%**	Ln [(1 + 22.9%) x 100% + (1 + 0.1%) x 0%] =	0.2062
1997	33.3%	14.5%	**100%**	**0%**	Ln [(1 + 33.3%) x 100% + (1 + 14.5%) x 0%] =	0.2874
1998	28.6%	11.8%	**100%**	**0%**	Ln [(1 + 28.6%) x 100% + (1 + 11.8%) x 0%] =	0.2515
1999	20.9%	-7.6%	**100%**	**0%**	Ln [(1 + 20.9%) x 100% + (1 – 7.6%) x 0%] =	0.1898
2000	-9.1%	16.1%	**100%**	**0%**	Ln [(1 – 9.1%) x 100% + (1 + 16.2%) x 0%] =	-0.0954
2001	-11.9%	7.3%	**100%**	**0%**	Ln [(1 – 11.9%) x 100% + (1 + 7.3%) x 0%] =	-0.1267
2002	-22.1%	14.8%	**100%**	**0%**	Ln [(1 – 22.1%) x 100% + (1 + 14.8%) x 0%] =	-0.2497
					Average utility =	**0.0891**

Notes: Data spans Jan 1993 – Dec 2002.

Constructing portfolios from stability-adjusted return samples

Full-scale optimization: Illustrative example

	Stock Return	Bond Return	Stock Weight	Bond Weight	Utility Calculation	Utility
1993	10.1%	16.2%	**50%**	**50%**	Ln [(1 + 10.1%) x 50% + (1 + 16.2%) x 50%] =	0.1235
1994	1.3%	-7.1%	**50%**	**50%**	Ln [(1 + 1.3%) x 50% + (1 – 7.1%) x 50%] =	-0.0294
1995	37.5%	30.0%	**50%**	**50%**	Ln [(1 + 39.5%) x 50% + (1 + 30.0%) x 50%] =	0.2908
1996	22.9%	0.1%	**50%**	**50%**	Ln [(1 + 22.9%) x 50% + (1 + 0.1%) x 50%] =	0.1089
1997	33.3%	14.5%	**50%**	**50%**	Ln [(1 + 33.3%) x 50% + (1 + 14.5%) x 50%] =	0.2143
1998	28.6%	11.8%	**50%**	**50%**	Ln [(1 + 28.6%) x 50% + (1 + 11.8%) x 50%] =	0.1840
1999	20.9%	-7.6%	**50%**	**50%**	Ln [(1 + 20.9%) x 50% + (1 – 7.6%) x 50%] =	0.0644
2000	-9.1%	16.1%	**50%**	**50%**	Ln [(1 – 9.1%) x 50% + (1 + 16.2%) x 50%] =	0.0344
2001	-11.9%	7.3%	**50%**	**50%**	Ln [(1 – 11.9%) x 50% + (1 + 7.3%) x 50%] =	-0.0233
2002	-22.1%	14.8%	**50%**	**50%**	Ln [(1 – 22.1%) x 50% + (1 + 14.8%) x 50%] =	-0.0372
					Average utility =	**0.0930**

Notes: Data spans Jan 1993 – Dec 2002.

Constructing portfolios from stability-adjusted return samples

Full-scale optimization: Illustrative example

	Stock Return	Bond Return	Stock Weight	Bond Weight	Utility Calculation	Utility
1993	10.1%	16.2%	**55%**	**45%**	Ln [(1 + 10.1%) x 55% + (1 + 16.2%) x 45%] =	0.1208
1994	1.3%	-7.1%	**55%**	**45%**	Ln [(1 + 1.3%) x 55% + (1 – 7.1%) x 45%] =	-0.0251
1995	37.5%	30.0%	**55%**	**45%**	Ln [(1 + 39.5%) x 55% + (1 + 30.0%) x 45%] =	0.2936
1996	22.9%	0.1%	**55%**	**45%**	Ln [(1 + 22.9%) x 55% + (1 + 0.1%) x 45%] =	0.1190
1997	33.3%	14.5%	**55%**	**45%**	Ln [(1 + 33.3%) x 55% + (1 + 14.5%) x 45%] =	0.2219
1998	28.6%	11.8%	**55%**	**45%**	Ln [(1 + 28.6%) x 55% + (1 + 11.8%) x 45%] =	0.1910
1999	20.9%	-7.6%	**55%**	**45%**	Ln [(1 + 20.9%) x 55% + (1 – 7.6%) x 45%] =	0.0777
2000	-9.1%	16.1%	**55%**	**45%**	Ln [(1 – 9.1%) x 55% + (1 + 16.2%) x 45%] =	0.0222
2001	-11.9%	7.3%	**55%**	**45%**	Ln [(1 – 11.9%) x 55% + (1 + 7.3%) x 45%] =	-0.0331
2002	-22.1%	14.8%	**55%**	**45%**	Ln [(1 – 22.1%) x 55% + (1 + 14.8%) x 45%] =	-0.0565
					Average utility =	**0.0931**

Notes: Data spans Jan 1993 – Dec 2002.

Stability-adjusted index replication

We seek to minimize the tracking error of a basket of stocks relative to the S&P 500.

- We randomly select two securities from each GICS sector, for a total of 20 stocks.
- We use stability-adjusted weekly returns estimated over the period January 2006 through January 2016. We set the sub-sample windows to equal one year, and evaluate results over horizons of one quarter.
- We assume that each security has a known expected relative return equal to 0%, that their weights sum to 100%, and that the weight of the S&P 500 is fixed at -100%.
- We allow the allocation to each security to be 0%, 10% or 20%.

Stability-adjusted index replication

Two randomly selected stocks from each sector	January 6, 2006 - January 8, 2016
Stock Name	**Sector**
CARNIVAL	CONSUMER DISCRETIONARY
EXPEDIA	CONSUMER DISCRETIONARY
CHURCH & DWIGHT CO.	CONSUMER STAPLES
BROWN-FORMAN 'B'	CONSUMER STAPLES
MURPHY OIL	ENERGY
ONEOK	ENERGY
MOODY'S	FINANCIALS
AMERICAN EXPRESS	FINANCIALS
UNITEDHEALTH GROUP	HEALTH CARE
DENTSPLY INTL.	HEALTH CARE
HUNT JB TRANSPORT SVS.	INDUSTRIALS
EQUIFAX	INDUSTRIALS
APPLIED MATS.	INFORMATION TECHNOLOGY
COGNIZANT TECH.SLTN.'A'	INFORMATION TECHNOLOGY
INTL.FLAVORS & FRAG.	MATERIALS
VULCAN MATERIALS	MATERIALS
CENTURYLINK	TELECOMMUNICATION SERVICES
AT&T	TELECOMMUNICATION SERVICES
EDISON INTL.	UTILITIES
NEXTERA ENERGY	UTILITIES

Relative stability: Individual stocks

- These stability measures are presented as inter-quartile ranges, reflecting small-sample error, independent-sample error, and interval error.

- The inter-quartile ranges for standard deviations are standardized by dividing them by the standard deviation of the stock returns.

		Standard Deviations	Correlations																		
			a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t
a	S&P 500	0.79																				
b	CARNIVAL	0.74	0.73																			
c	EXPEDIA	0.67	0.66	0.56																		
d	CHURCH & DWIGHT CO.	0.47	0.57	0.65	0.77																	
e	BROWN-FORMAN 'B'	0.44	0.62	0.54	0.71	0.48																
f	MURPHY OIL	0.80	0.55	0.90	0.92	0.49	0.79															
g	ONEOK	0.62	0.65	0.59	0.57	0.63	0.49	0.47														
h	MOODY'S	0.73	0.49	0.84	0.63	0.75	0.57	0.72	0.60													
i	AMERICAN EXPRESS	0.79	0.29	0.47	0.69	0.49	0.44	0.72	0.68	0.48												
j	UNITEDHEALTH GROUP	0.81	0.56	0.51	0.63	0.47	0.48	0.69	0.65	0.78	0.58											
k	DENTSPLY INTL.	0.66	0.53	0.69	0.84	0.54	0.53	0.45	0.69	0.92	0.51	0.72										
l	HUNT JB TRANSPORT SVS.	0.80	0.53	0.65	0.68	0.45	0.71	0.74	0.72	0.85	0.42	0.84	0.61									
m	EQUIFAX	0.78	0.46	0.80	0.65	0.42	0.61	0.86	0.70	0.45	0.56	0.72	0.68	0.47								
n	APPLIED MATS.	0.47	0.40	0.90	0.88	0.76	0.46	0.89	0.76	0.70	0.50	0.51	0.78	0.52	0.66							
o	COGNIZANT TECH.SLTN.'A'	0.61	0.56	0.34	0.85	0.63	0.68	0.41	0.70	0.84	0.71	0.83	0.46	0.51	0.63	0.72						
p	INTL.FLAVORS & FRAG.	0.34	0.34	0.91	0.73	0.56	0.74	0.61	0.63	0.68	0.49	0.60	0.77	0.68	0.53	0.29	0.57					
q	VULCAN MATERIALS	0.60	0.55	0.52	0.83	0.44	0.70	0.69	0.48	0.75	0.54	0.84	0.76	0.42	0.44	0.78	0.35	0.44				
r	CENTURYLINK	0.47	0.49	0.73	0.65	0.57	0.54	0.80	0.62	0.60	0.51	0.64	0.65	0.78	0.59	0.52	0.68	0.39	0.53			
s	AT&T	0.36	0.50	0.61	0.65	0.61	0.36	0.67	0.53	0.68	0.51	0.59	0.49	0.48	0.64	0.50	0.72	0.42	0.87	0.55		
t	EDISON INTL.	0.38	0.70	0.85	0.74	0.43	0.49	1.05	0.82	1.07	0.48	0.53	0.53	0.52	0.72	0.60	0.71	0.37	0.63	0.72	0.40	
u	NEXTERA ENERGY	0.85	0.66	0.57	0.66	0.40	0.78	0.98	0.74	0.87	0.49	0.61	0.89	0.70	0.72	0.76	0.85	0.46	0.70	0.55	0.58	0.41

Notes: Data spans Jan 2006 – Jan 2016.

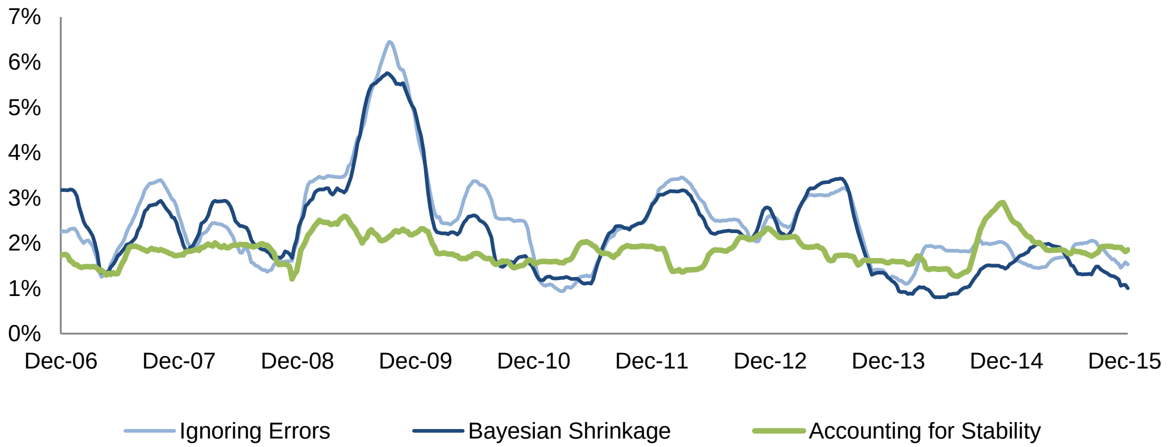
Index replication: Portfolio weights and performance

Random Stock Universe	Power Utility			Kinked Utility		
	Ignoring Errors	Bayesian Shrinkage	Accounting for Stability	Ignoring Errors	Bayesian Shrinkage	Accounting for Stability
CARNIVAL	0%	10%	0%	10%	10%	0%
EXPEDIA	0%	0%	0%	0%	0%	0%
CHURCH & DWIGHT CO.	0%	0%	0%	0%	10%	0%
BROWN-FORMAN 'B'	0%	0%	10%	0%	0%	10%
MURPHY OIL	10%	10%	10%	10%	10%	10%
ONEOK	0%	0%	10%	0%	0%	10%
MOODY'S	10%	10%	0%	10%	10%	0%
AMERICAN EXPRESS	0%	0%	10%	0%	0%	10%
UNITEDHEALTH GROUP	0%	0%	10%	0%	0%	10%
DENTSPLY INTL.	10%	10%	10%	10%	10%	10%
HUNT JB TRANSPORT SVS.	0%	0%	0%	0%	0%	0%
EQUIFAX	10%	10%	10%	10%	0%	10%
APPLIED MATS.	10%	10%	10%	10%	10%	10%
COGNIZANT TECH.SLTN.'A'	0%	0%	0%	0%	0%	0%
INTL.FLAVORS & FRAG.	0%	0%	10%	0%	0%	10%
VULCAN MATERIALS	10%	10%	0%	10%	10%	0%
CENTURYLINK	10%	0%	0%	0%	10%	0%
AT&T	10%	20%	10%	20%	10%	10%
EDISON INTL.	0%	0%	0%	0%	0%	0%
NEXTERA ENERGY	20%	10%	0%	10%	10%	0%
Dispersion of risk	Quarterly tracking error			Quarterly downside tracking error		
90th - 10th percentile	2.0%	2.1%	0.8%	2.4%	2.1%	1.0%
Maximum - minimum	5.5%	5.0%	1.7%	6.4%	4.8%	2.6%

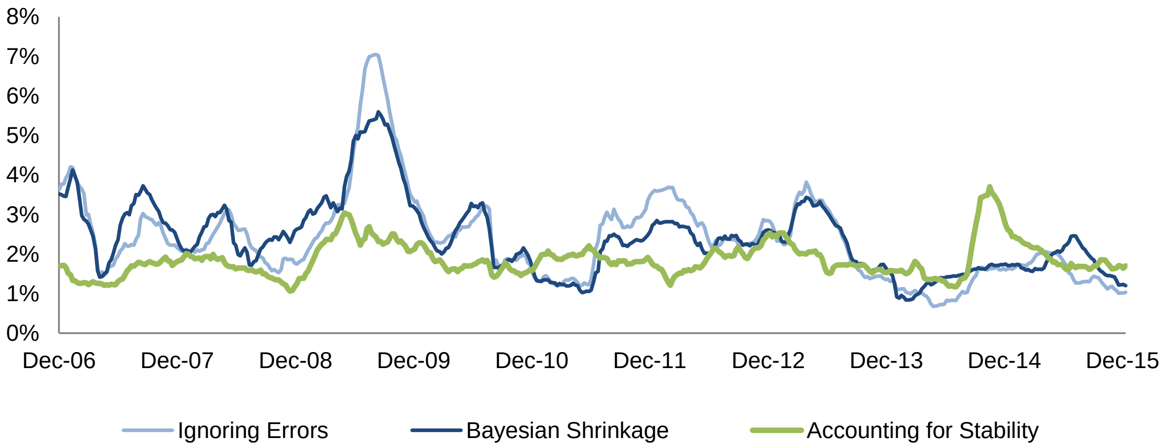
Notes: Data spans Jan 2006 – Jan 2016.

Index replication: Realized tracking error

Quarterly tracking error for rolling 1-year periods, for power utility portfolios



Quarterly downside tracking error for rolling 1-year periods, for kinked utility portfolios



Notes: Data spans Jan 2006 – Jan 2016.

Stability-adjusted currency hedging: Proxy hedging

- Some currencies are expensive to hedge directly – particularly those in emerging markets.

- Proxy hedging using cheaper-to-trade currencies offers an alternative. However, it can be challenging to identify a basket of proxy currencies that will deliver a stable hedge.

- We use stability-adjusted optimization to construct a proxy hedging portfolio. Our objective is to track the currency impact of the MSCI Emerging Markets index from a US dollar base.

- We use stability-adjusted weekly returns estimated over the period January 2006 through January 2016. We set the sub-sample windows to equal one year, and evaluate results over horizons of one quarter. We allow the allocation to each currency to be 0%, 10%, 20%, 30%, 40% or 50%.

Relative stability: Proxy hedging

- These stability measures are presented as inter-quartile ranges, reflecting small-sample error, independent-sample error, and interval error.
- The inter-quartile ranges for standard deviations are standardized by dividing them by the standard deviation of the currency returns.

		Standard Deviations	Correlations								
			a	b	c	d	e	f	g	h	i
a	EM currency basket	0.68									
b	AUD	0.40	0.19								
c	CAD	0.66	0.39	0.30							
d	CHF	0.58	0.51	0.55	0.62						
e	EUR	0.74	0.43	0.55	0.43	0.26					
f	GBP	0.63	0.33	0.45	0.48	0.41	0.22				
g	JPY	0.52	0.69	0.74	0.84	0.63	0.73	0.93			
h	NOK	0.49	0.38	0.42	0.31	0.38	0.27	0.30	0.76		
i	SEK	0.62	0.35	0.26	0.54	0.56	0.61	0.57	0.77	0.56	
j	NZD	0.76	0.33	0.41	0.54	0.28	0.20	0.30	0.70	0.23	0.59

Notes: Data spans Jan 2006 – Jan 2016.

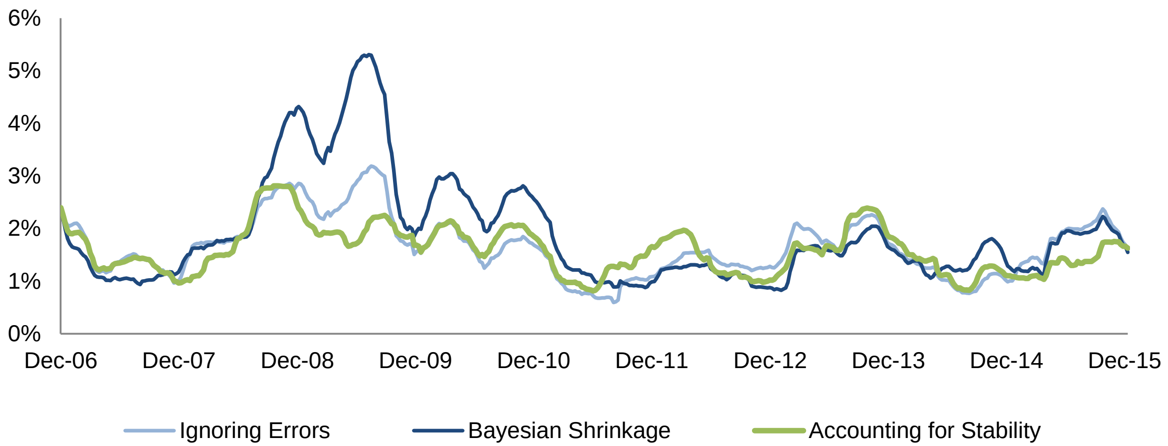
Proxy hedging: Portfolio weights and performance

	Power Utility			Kinked Utility		
	Ignoring Errors	Bayesian Shrinkage	Accounting for Stability	Ignoring Errors	Bayesian Shrinkage	Accounting for Stability
Optimal tracking basket						
AUD	20%	20%	10%	10%	20%	20%
CAD	30%	10%	30%	30%	20%	30%
CHF	0%	0%	20%	0%	0%	20%
EUR	0%	0%	0%	0%	0%	0%
GBP	20%	30%	10%	20%	30%	10%
JPY	10%	0%	10%	10%	10%	10%
NOK	10%	20%	0%	10%	10%	0%
SEK	10%	20%	20%	20%	10%	10%
NZD	0%	0%	0%	0%	0%	0%
Dispersion of risk	Quarterly tracking error			Quarterly downside tracking error		
90th - 10th percentile	1.4%	2.4%	1.2%	1.6%	1.8%	1.4%
Maximum - minimum	2.6%	4.5%	2.0%	3.5%	3.8%	2.4%

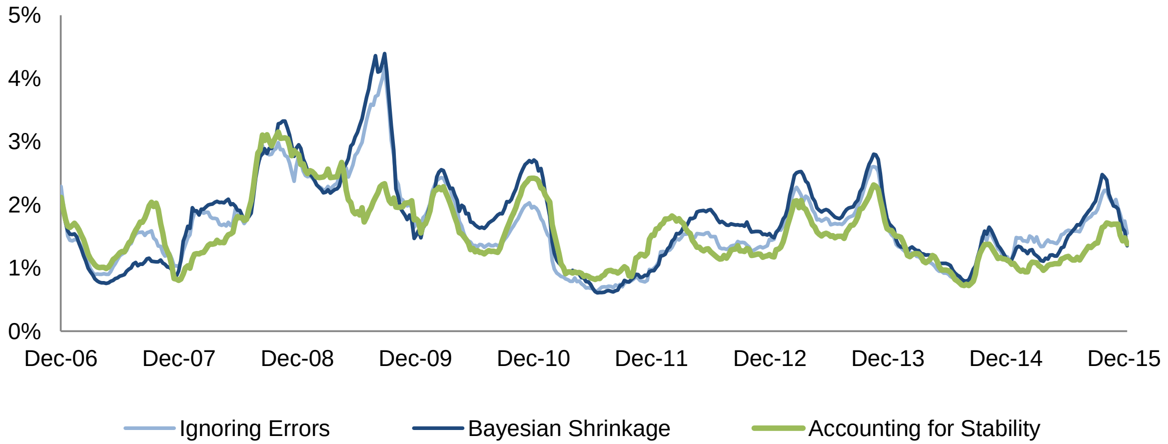
Notes: Data spans Jan 2006 – Jan 2016.

Proxy hedging: Realized tracking error

Quarterly tracking error for rolling 1-year periods, for power utility portfolios



Quarterly downside tracking error for rolling 1-year periods, for kinked utility portfolios



Notes: Data spans Jan 2006 – Jan 2016.

The persistence of relative stability

We collect data for two universes:

- 288 individual stocks from the S&P 500 (those with 20+ years of available data).
- Pooled collection of 24 currencies, 10 US equity sectors, 22 country equities, 21 commodities, 8 US treasury maturities, and 5 hedge fund styles.

For each universe, we:

- Compute the **historical instability score** for each asset's volatility using monthly data from February 1995 to December 2010. These scores represent the risk surprises that occurred over all overlapping 5-year periods in the past.
- Record the **historical spread** in instability between the 20% least stable and 20% most stable assets.
- Compute the **realized error** for each asset's volatility in the most recent 5-year period (January 2011 to December 2015), as compared to prior full sample volatility.
- Record the **realized spread** in instability for the assets that were historically the 20% least stable and 20% most stable.
- Compute the **correlation** between predicted instability and realized instability.

The persistence of relative stability

	Correlation of predictions with future errors	Correlation p-value	Spread between top-bottom 20%: **Predicted**	Spread between top-bottom 20%: **Future**
Individual stocks (288)	**0.37**	**0.00**	**0.17**	**0.18**
Pooled asset categories (90)	**0.38**	**0.00**	**0.17**	**0.19**

Data spans Feb 1995 – Dec 2015.

Implications for factor investing

Implications for factors

- Recently there has been growing interest in using factors rather than assets as the building blocks for forming portfolios.
- Both assets and factors are subject to small-sample error, independent-sample error, and interval error.
- Factors, however, are vulnerable to an additional source of error: mapping error.
- Thus, factors tend to have less stable covariances than assets.

Composite instability score

The four sources of estimation error are independent from one another, which means we can sum the variances of each error and then take the square root of this sum to compute a composite instability score.

$$Composite\ Instability\ Score\ (CIS) = \sqrt{SSE^2 + ISE^2 + ME^2 + IE^2}$$

Asset classes, fundamental factors and principal components

Asset Classes		
Equity	U.S. Large Cap	S&P 500
	U.S. Small Cap	Russell 2000
Fixed Income	U.S. Government Bonds	Barclays U.S. Aggregate Government
	U.S. Corporate Bonds	Barclays U.S. Aggregate Corporate
Alternatives	Commodities	S&P GSCI Commodities
	Hedge Funds	HFRI Fund of Funds Composite
Fundamental Factors		
Macro	Inflation	U.S. Consumer Price Index, Seasonally Adjusted
	Growth	One year ahead U.S. GDP growth forecast
Fixed Income	Term Premium	10-year minus 2-year U.S. treasury yield
	Credit Premium	Baa U.S. corporate yield minus 10-year U.S. treasury yield
Equity	Small Cap Premium	Fama-French Small-minus-Big factor
	Value Premium	Fama-French High-minus-Low factor

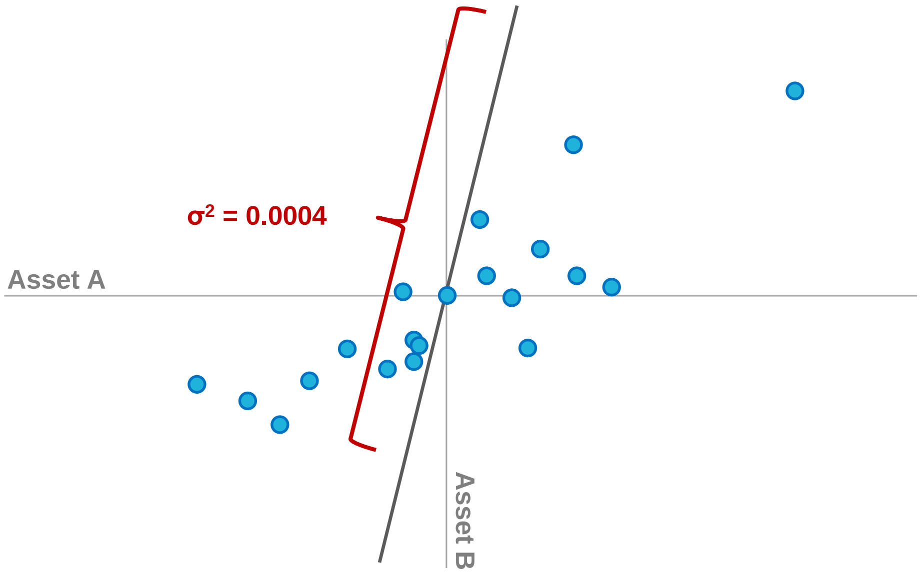
Notes: Our analysis uses monthly data over the period Jan 1990 – Dec 2015. We proxy U.S. large cap, U.S. small cap, U.S. government bonds, U.S. corporate bonds, commodities, and hedge funds with the following indices: S&P 500, Russell 2000, Barclays U.S. Agg Government, Barclays U.S. Agg. Corporate, S&P GSCI Commodity, and HFRI Fund of Funds Composite, respectively. We proxy inflation, growth, term premium, credit premium, small premium, and value premium with the following: U.S. CPI, U.S. GDP growth forecast, 10-2 yr U.S. Treasury, Baa Corp-10 yr U.S. Treasury, Fama-French SMB factor, and Fama-French HML factor, respectively.
To analyze fundamental factors, we build six factor-mimicking portfolios by regressing each factor on the six asset classes.
To analyze principal components, we build six portfolios using principal components analysis (each portfolio corresponds to an eigenvector).

Industries, attributes and principal components

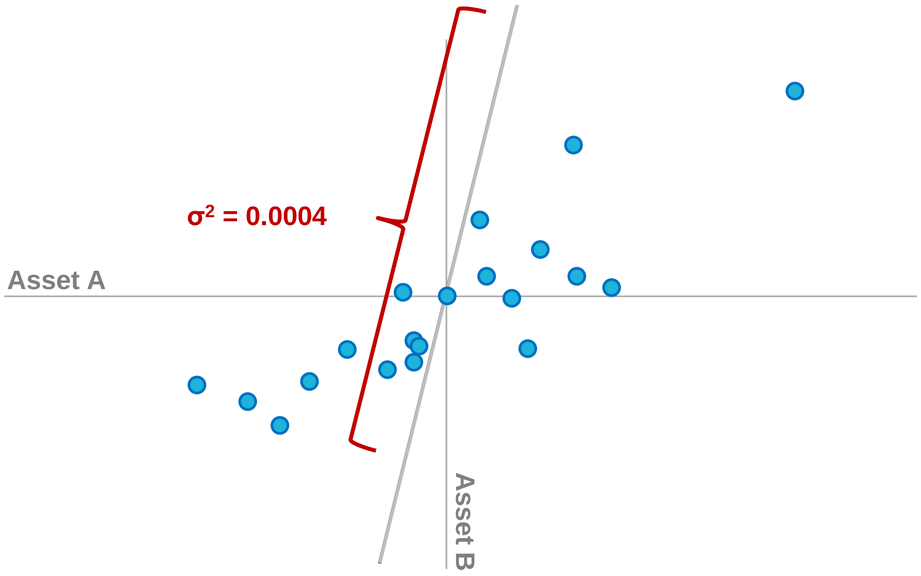
Grouping	Methodology
Industries	10 portfolios formed on GICS level I
Size	10 portfolios formed on capitalization
Value	10 portfolios formed on book-to-market
Momentum	10 portfolios formed on trailing 1y return
Principal Components	Top 10 principal components

Notes: Our analysis uses monthly returns over the period Jan 1989 through Dec 2015. We narrow the universe of 400 stocks (based on the constituents in the MSCI U.S. Index as of Dec 2015) to the 288 stocks that have full price, market cap, and book-to-market history over this sample.
To form industry portfolios, we use GICS classifications. The Global Industry Classification Standard (GICS) was developed by and is the exclusive property of MSCI Inc. and Standard & Poor's. GICS is a service mark of MSCI and S&P and has been licensed for use by State Street.
To form attribute portfolios, we use average market cap, book-to-market ratios, and 1-year returns, respectively.
To form principal components portfolios, we run a PCA on the 288 stocks and form portfolios corresponding to the top 10 eigenvectors.

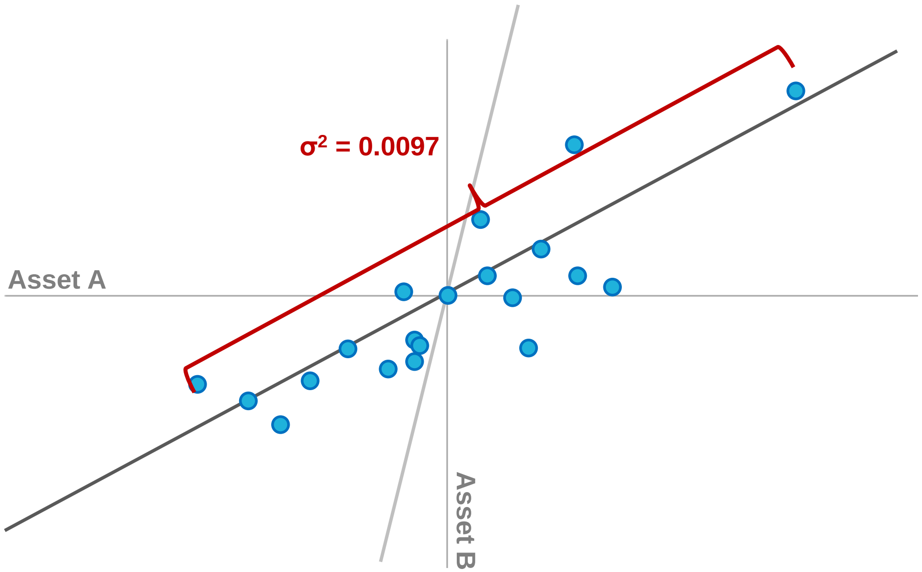
Principal components



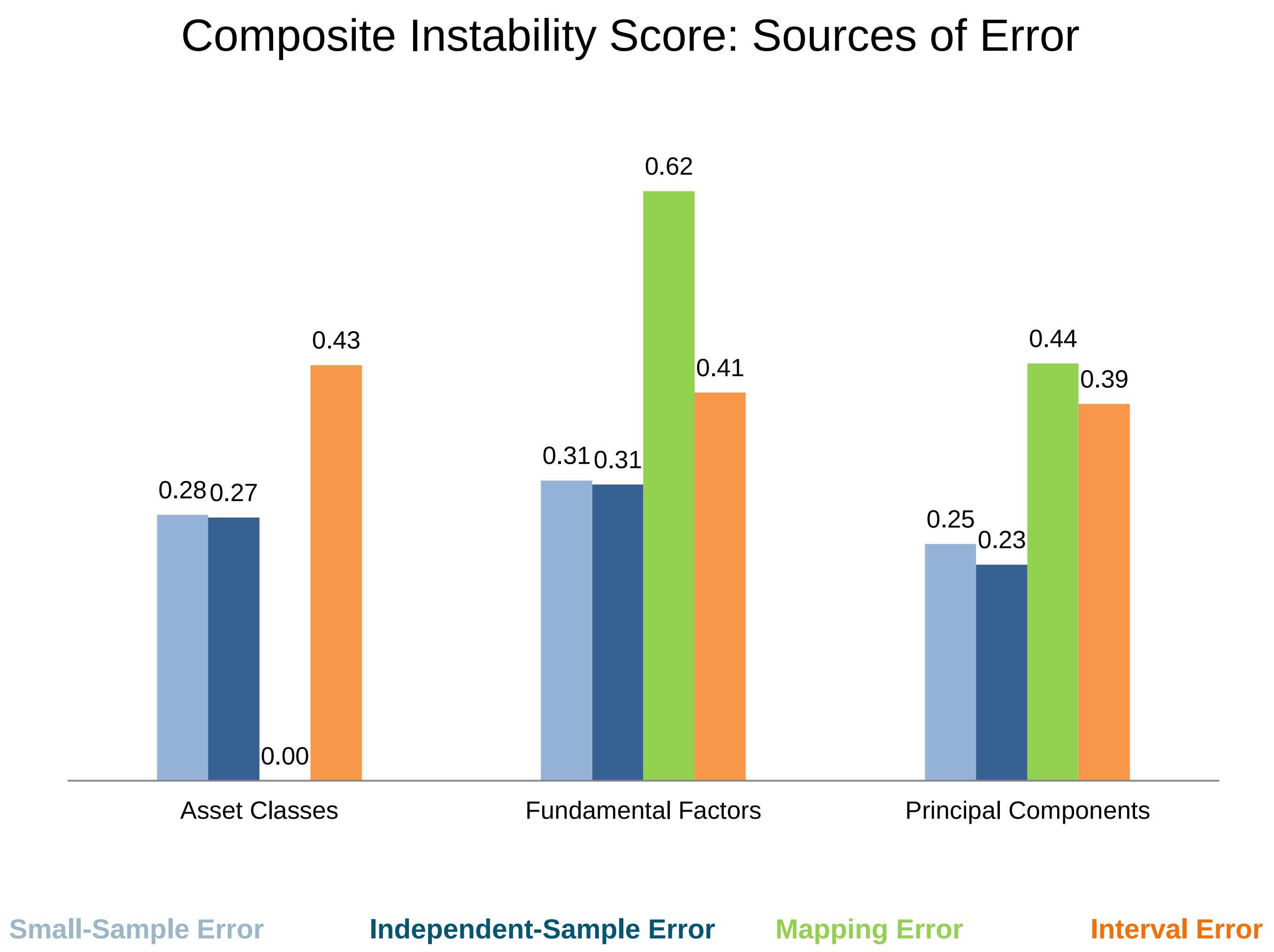
Principal components



Principal components

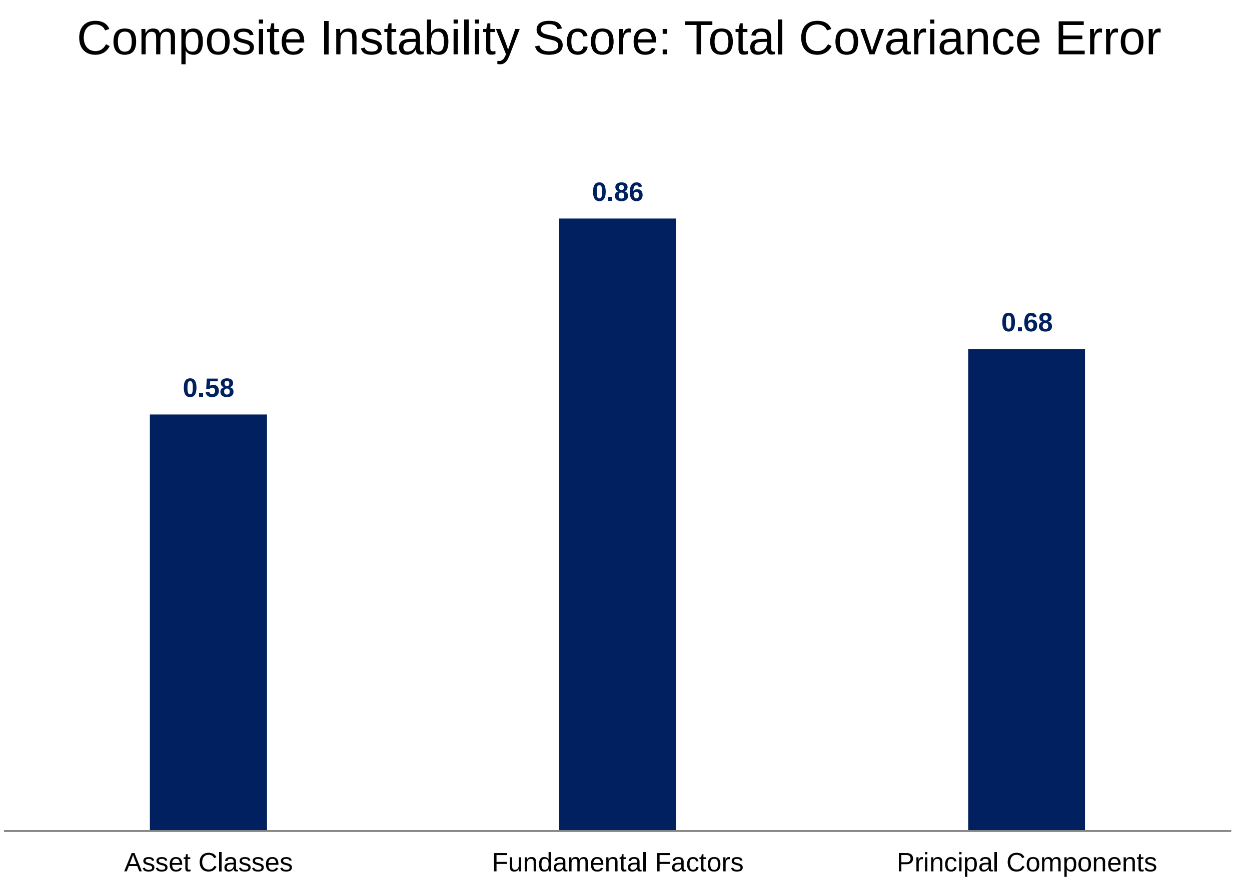


Asset classes, fundamental factors, and principal components



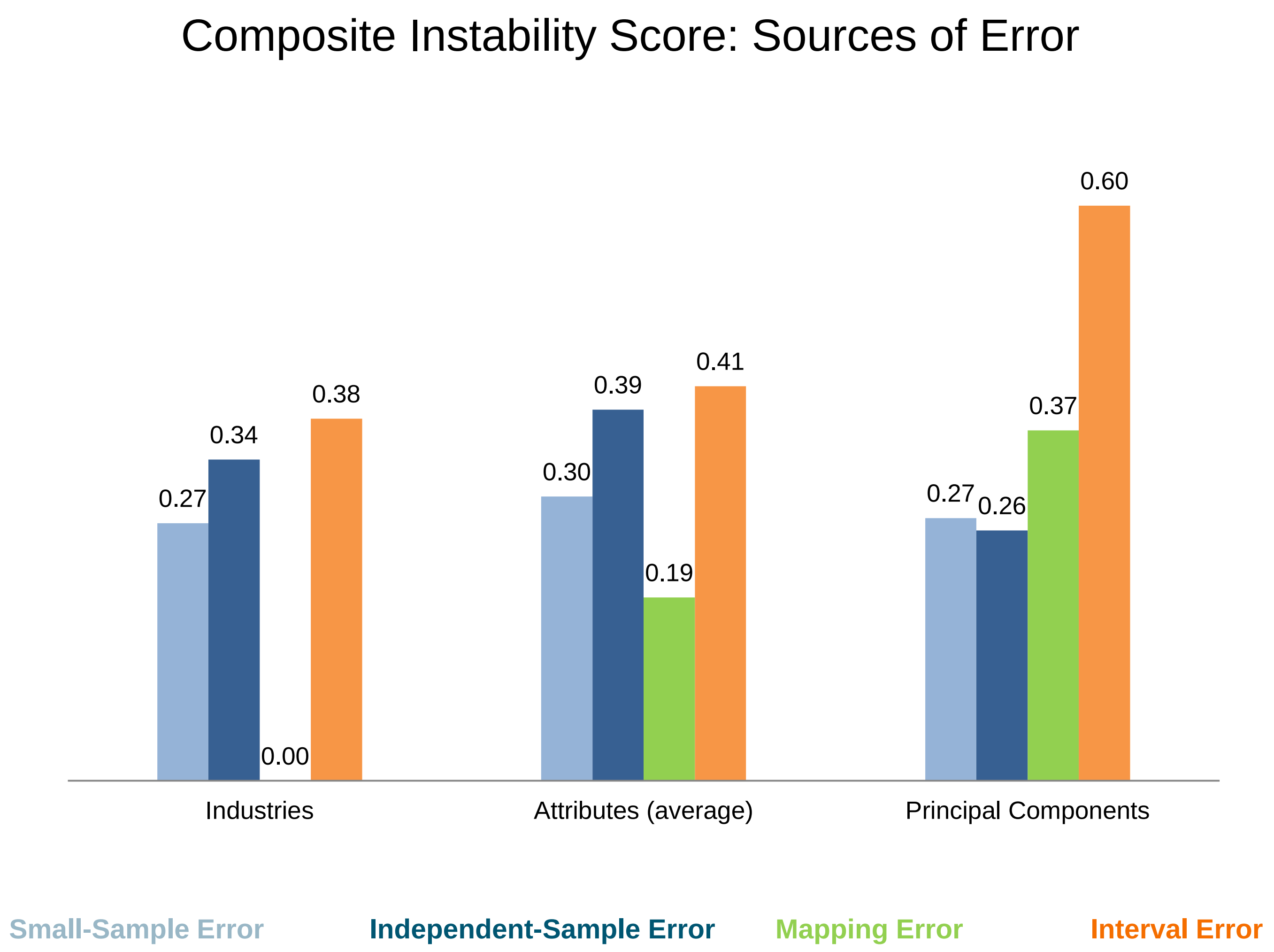
Notes: Data spans Jan 1990 – Dec 2015.

Asset classes, fundamental factors, and principal components



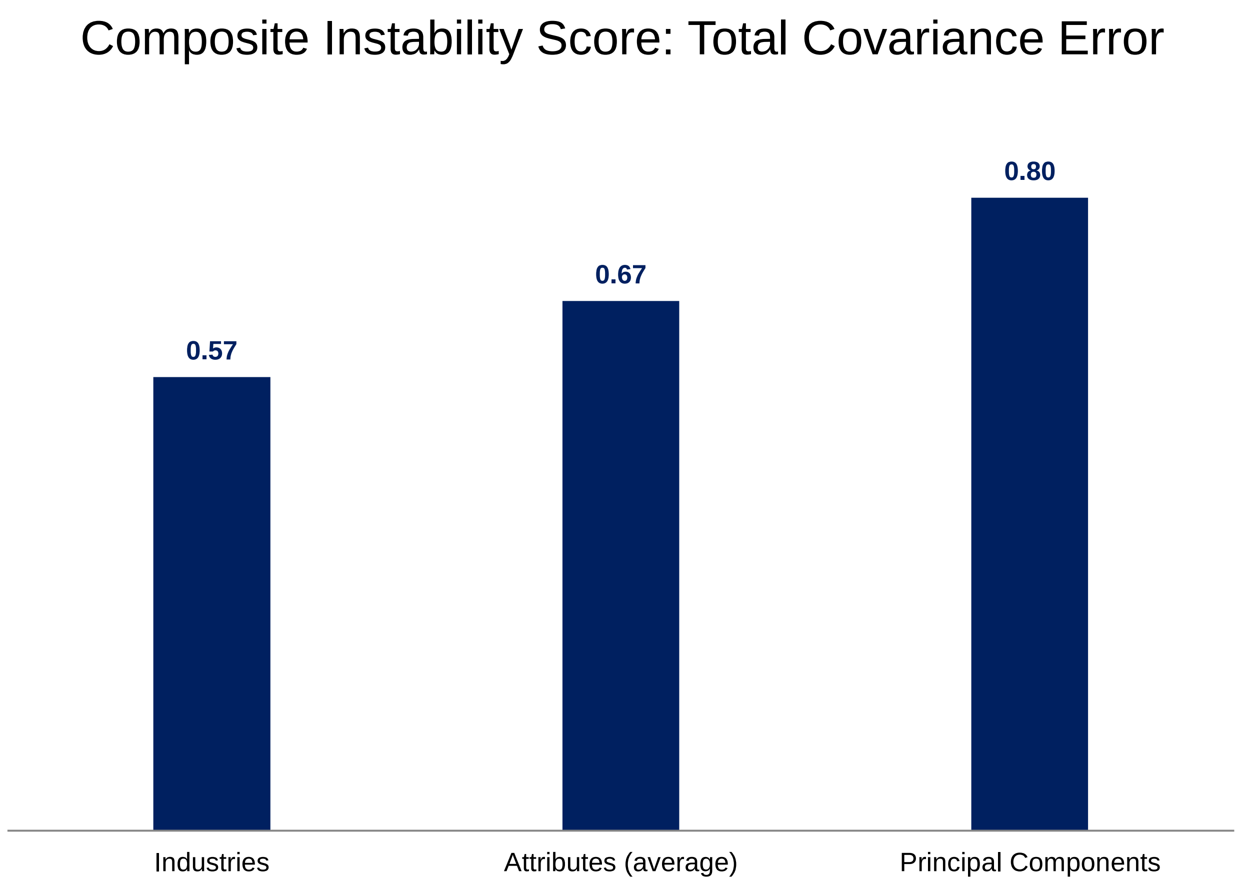
Notes: Data spans Jan 1990 – Dec 2015.

Industries, attributes, and principal components (10 groups)



Notes: Data spans Jan 1989 – Dec 2015.

Industries, attributes, and principal components (10 groups)



Notes: Data spans Jan 1989 – Dec 2015.

Summary

- We describe the major sources of estimation error and show how to quantify these errors in comparable units that are mutually independent.
- We then describe a process for forming portfolios that treats the relative stability of covariances as a distinct component of risk.
- We show that stability-adjusted optimization, on balance, delivers better results than optimization that ignores errors and optimization that applies Bayesian shrinkage.
- We also show that asset classes serve as better building blocks than factors for forming portfolios when we account for the relative stability of covariances.

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