Uncertain Covariance Models

RISK FORECASTS THAT KNOW HOW ACCURATE THEY ARE AND WHERE

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The Math of Uncertain Covariance

Straightforward … and not for a presentation

Shah, A. (2015). Uncertain Covariance Models

http://ssrn.com/abstract=2616109

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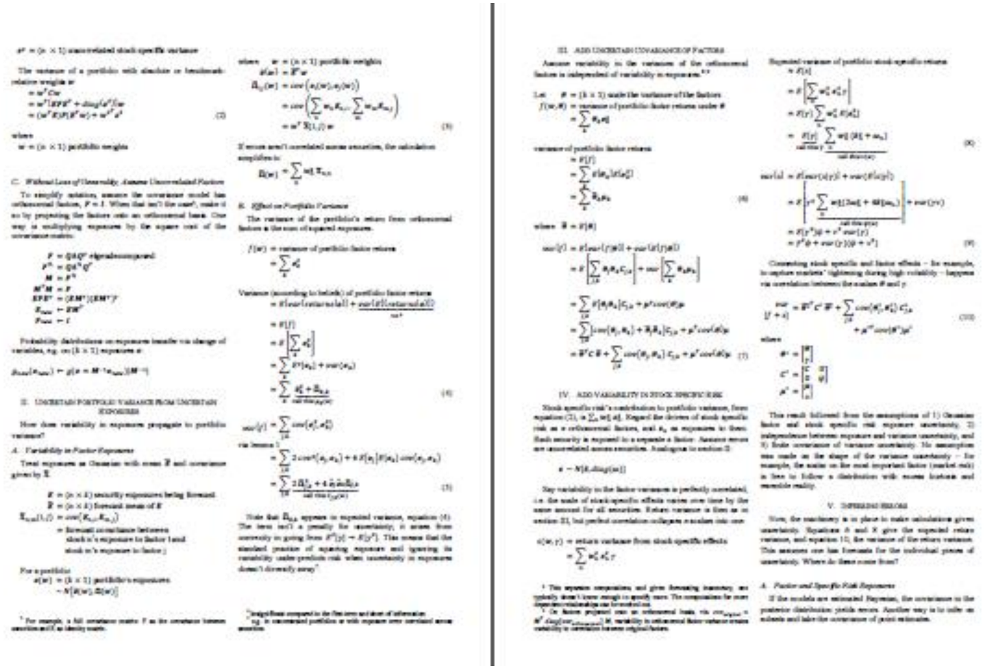


Background: Why Care About Covariance?

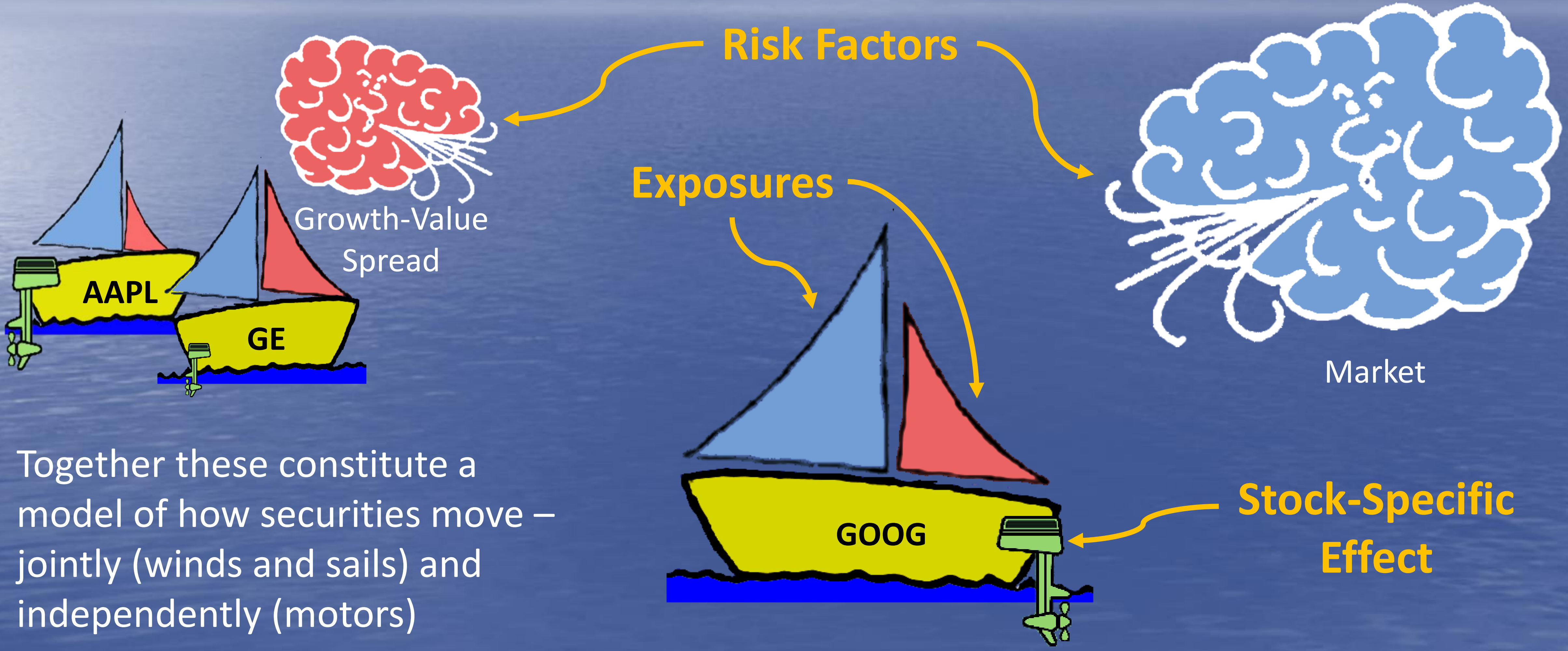
Notions of co-movement are needed to make decisions throughout the investment process

1. Estimating capital at risk and portfolio volatility
2. Hedging
3. Constructing and rebalancing portfolios through optimization
4. Algorithmic trading
5. Evaluating performance
6. Making sense of asset allocation

Background: Why Care About Uncertainty?

1. Nothing is exactly known. Everything is a forecast
2. However, one can estimate the accuracy of individual numbers
   - Suppose two stocks have the same expected covariance against other securities
   - The first has been well-predicted in the past, the other not
   - Which is a safer hedge?
3. It's imprudent to make decisions without considering accuracy
   - A fool, omitting accuracy from his objective, curses optimizers for taking numbers at face value
4. **Relying on wrong numbers can cost you your shirt**
   - under leverage
   - when you can be fired and have assets assigned to another manager

A Factor Covariance Model in Pictures



Together these constitute a model of how securities move – jointly (winds and sails) and independently (motors)

The Math of a Factor Covariance Model

1. Say a stock's return is partly a function of pervasive factors, e.g. the return of the market and oil

$$r_{GOOG} = h_{GOOG}(f_{mkt},\ f_{oil}) + \text{stuff assumed to be independent of the factors and other securities}$$

2. Imagine linearly approximating this function

$$r_{GOOG} \approx \frac{\partial h_{GOOG}}{\partial f_{mkt}} f_{mkt} + \frac{\partial h_{GOOG}}{\partial f_{oil}} f_{oil} + \text{constant} + \text{stuff}$$

3. Model a stock's variance as the variance of the approximation

$$var(r_{GOOG}) \approx \underbrace{\begin{bmatrix} \frac{\partial h_{GOOG}}{\partial f_{mkt}} & \frac{\partial h_{GOOG}}{\partial f_{oil}} \end{bmatrix}}_{\substack{\text{exposures} \\ \text{sails}}} \underbrace{\begin{bmatrix} var(f_{mkt}) & cov(f_{mkt}, f_{oil}) \\ cov(f_{mkt}, f_{oil}) & var(f_{oil}) \end{bmatrix}}_{\substack{\text{factor covariance} \\ \text{how wind blows}}} \underbrace{\begin{bmatrix} \frac{\partial h_{GOOG}}{\partial f_{mkt}} \\ \frac{\partial h_{GOOG}}{\partial f_{oil}} \end{bmatrix}}_{\substack{\text{exposures} \\ \text{sails}}} + \underbrace{var(\text{stuff})}_{\substack{\text{stock specific variance} \\ \text{size of motor}}}$$

The Math of a Factor Covariance Model

4. Covariance between stocks is modeled as the covariance of their approximations

$$cov(r_{GOOG}, r_{GE}) \approx \underbrace{\begin{bmatrix} \frac{\partial h_{GOOG}}{\partial f_{mkt}} & \frac{\partial h_{GOOG}}{\partial f_{oil}} \end{bmatrix}}_{\substack{\text{exposures} \\ \text{sails } (GOOG)}} \underbrace{\begin{bmatrix} var(f_{mkt}) & cov(f_{mkt}, f_{oil}) \\ cov(f_{mkt}, f_{oil}) & var(f_{oil}) \end{bmatrix}}_{\substack{\text{factor covariance} \\ \text{how wind blows}}} \underbrace{\begin{bmatrix} \frac{\partial h_{GE}}{\partial f_{mkt}} \\ \frac{\partial h_{GE}}{\partial f_{oil}} \end{bmatrix}}_{\substack{\text{exposures} \\ \text{sails } (GE)}}$$

Note: no motors here – they are assumed independent across stocks

5. Parts aren't known but inferred, typically by regression, or by more sophisticated tools

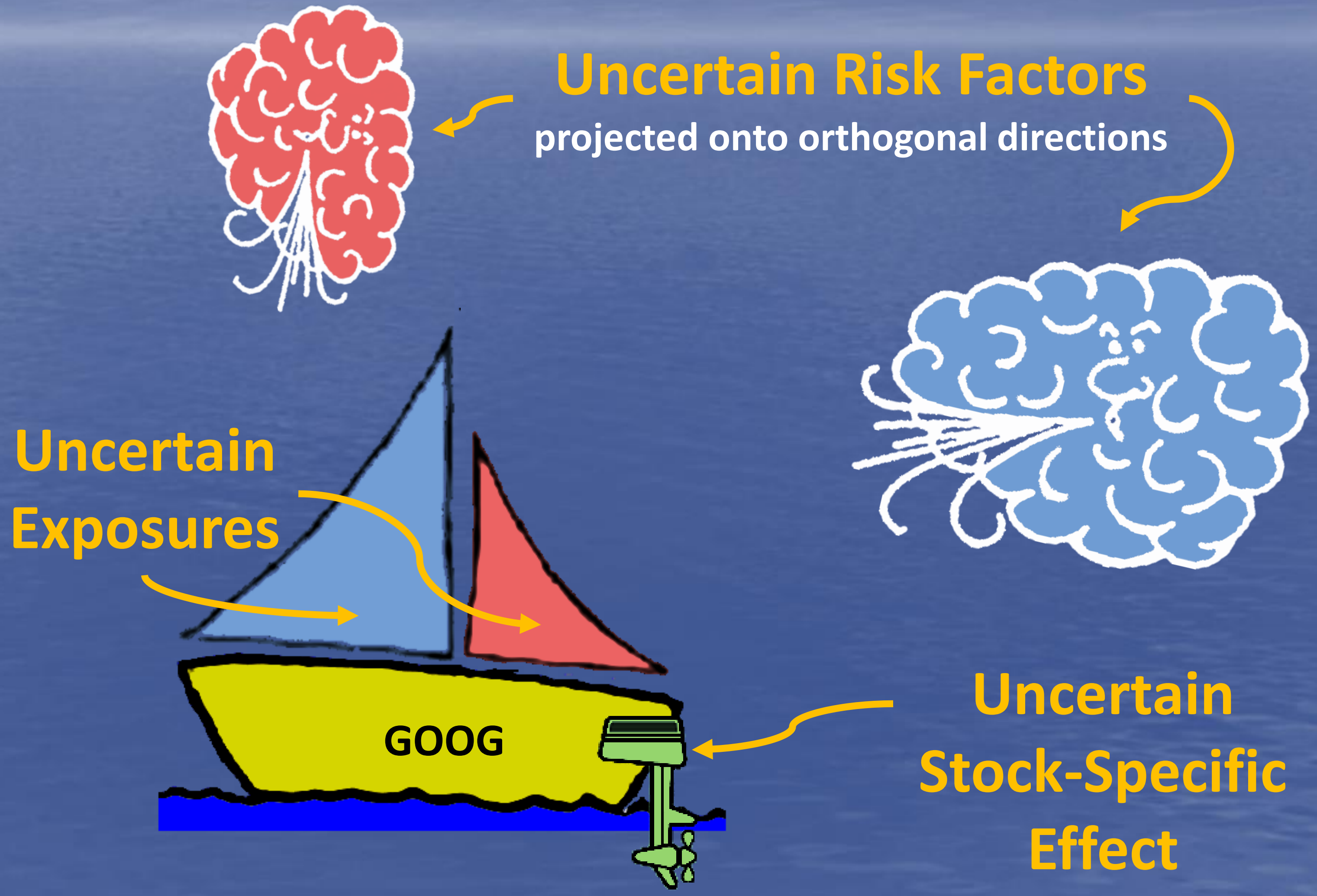
6. Best is to forecast values over one's horizon

An Uncertain Factor Covariance Model

Welcome to reality!
Nothing is known with certainty.
But some forecasts are believed more accurate than others

A portfolio's risk has – according to beliefs – an expected value and a variance

Good decisions come from considering uncertainty explicitly.
Ignoring doesn't make it go away



Uncertain Exposures

Beliefs about future exposure to the factors are communicated as Gaussian

- $\boldsymbol{e}_{GOOG} \sim N\left[\hat{\boldsymbol{e}}_{GOOG}, \widehat{\boldsymbol{\Omega}}_{GOOG}\right]$
- Exposures can be correlated across securities
- Estimates of mean and covariance come from the method to forecast exposures and historical accuracy

So, a portfolio's exposures are also Gaussian

- This fact is used in the math to work out variance (from uncertainty) of portfolio return variance

Sidebar: $E[\beta^2] > (E[\beta])^2$
Ignoring Uncertainty Underestimates Risk

A CAPM flavored bare bones example to illustrate the idea:

- Stock's return = market return × β
- Variance of stock's return = market var × β^2

Say β isn't known exactly

- E[variance of stock's return] $= \text{market var} \times E[\beta^2]$

$$= \text{market var} \times \left(E^2[\beta] + \text{var}[\beta] \right)$$

$$> \text{market var} \times E^2[\beta]$$

"uncertainty correction"

Ignoring uncertainty underestimates risk

- Note: this has nothing to do with aversion to uncertainty

Uncertain Factor Variances

Beliefs about the future factor variances are communicated as their mean and covariance

- Forecasts are the mean and covariance – according to uncertainty – of return variances
- Not Gaussian since variances ≥ 0

How the heck do you generate these?

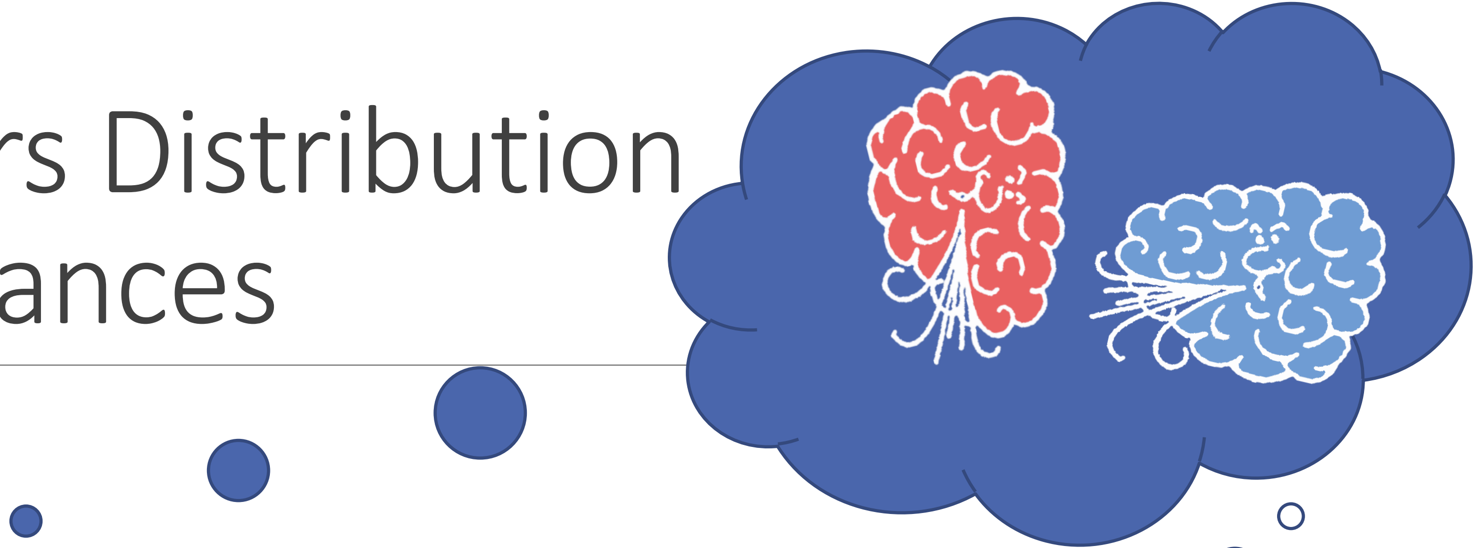
Shah, A. (2014). Short-Term Risk and Adapting Covariance Models to Current Market Conditions

- http://ssrn.com/abstract=2501071

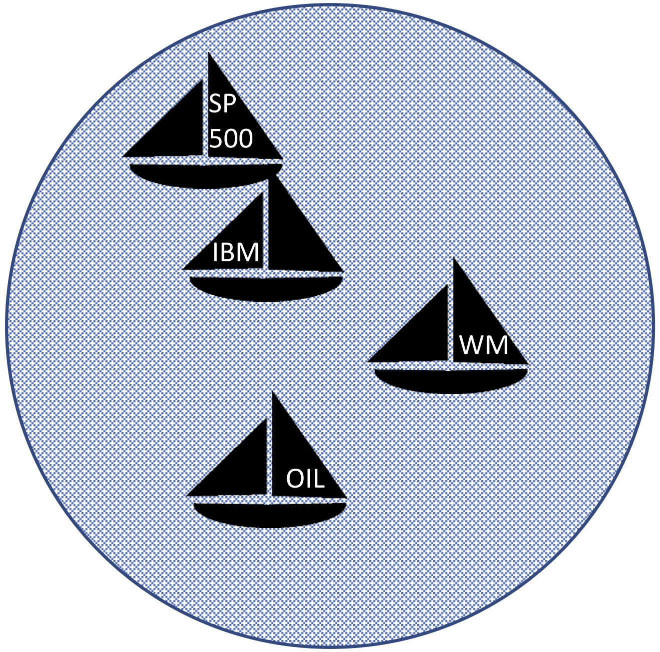
1. Forecast whatever you can, e.g. from VIX and cross-sectional returns, the volatility of S&P 500 daily returns over the next 3 months will be 25% ± 5% annualized
2. The states of quantities measured by the risk model imply a configuration of factor variances

Since this inferred distribution of factor variances arises from predictions, it is a forecast

Adapting Infers Distribution Of Factor Variances

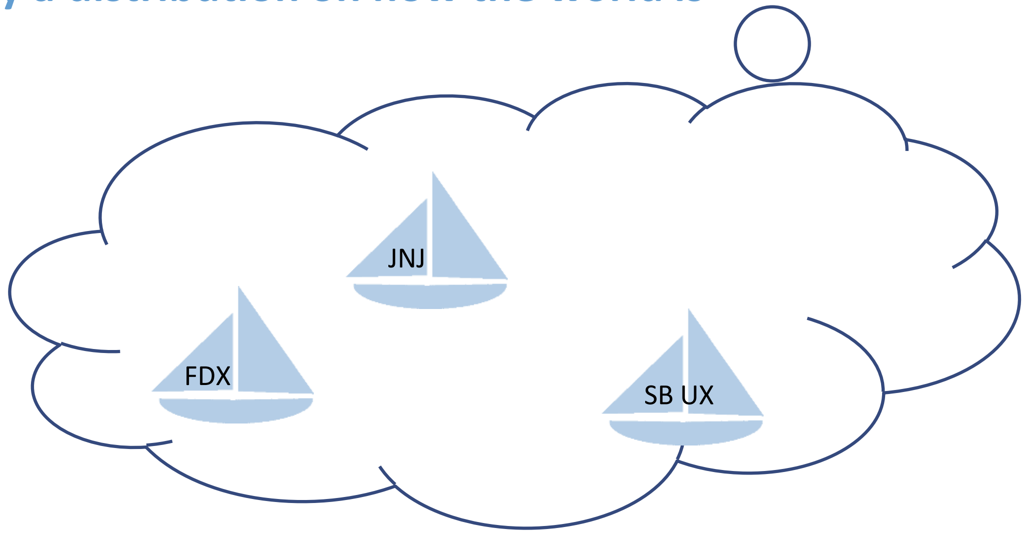


2. Imply a distribution on how the world is



1. Noisy variance forecasts via all manner of Information sources

Implied vol, intraday price movement, news and other big data, …



3. An aside: this extends to the behavior of other securities

All risk forecasts are improved

Uncertain Stock-Specific Effects

Stock-specific effects (motors) are best regarded as both exposures (sails) and factors (wind)

1. A motor is like wind that affects just 1 stock
2. Its size is estimated with error like a sail
3. The average size of motors across securities varies over time
   - e.g. stock-specific effects can shrink as market volatility rises
   - Since it might depend on the variance of other factors, average size gets treated like one

Thus, uncertainty from stock-specific effects is captured using both types of uncertainty in the preceding slides.

All Set with Machinery: Uncertain Portfolio Variance

Math then yields for a portfolio
- expected variance
- standard deviation of variance

according to beliefs (estimates) about uncertainty in the pieces
- exposures
- factor variances
- stock-specific effects

Expectation and standard deviation are with respect to beliefs
- Not reality, but one's best assessment of it
- "Given my beliefs, the portfolio's tracking variance is E ± sd"
- What a person (or computer) needs to make good decisions from the information at hand

Applications

Portfolio Optimization: Uncertain Utility

Conventional

$\max_{\mathbf{w}} U(\mathbf{w}) = r(\mathbf{w}) - \lambda \times v(\mathbf{w})$ where $r(\mathbf{w})$ = mean return, $v(\mathbf{w})$ = variance of return

Uncertain

$\max_{\mathbf{w}} O(\mathbf{w}) = E[U(\mathbf{w})] - \gamma \times \text{stdev}[U(\mathbf{w})]$

$\text{var}[U(\mathbf{w})] = \text{var}[r(\mathbf{w})] + \lambda^2 \text{var}[v(\mathbf{w})] - 2\lambda\, \text{stdev}[r(\mathbf{w})]\, \text{stdev}[v(\mathbf{w})] \times \rho_{\mathbf{w}}$

$\rho_{\mathbf{w}} = \text{cor}[r(\mathbf{w}), v(\mathbf{w})]$ for portfolio **w**, the correlation of uncertainty in mean and in variance

$r(\mathbf{w}) \sim N[\mathbf{w}^T\boldsymbol{\mu}, \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}]$ assume mean returns have Gaussian error

All pieces are known except $\rho_{\mathbf{w}}$ which, absent beliefs, can be set to 0

Portfolio Optimization: Maximize Risk Adjusted Return

Risk-adjusted return ≡ portfolio alpha - ½ portfolio tracking variance

Say alpha is known exactly

Randomly pick 10 securities – ½ are eq wt benchmark, ½ are optimized into fully invested portfolio

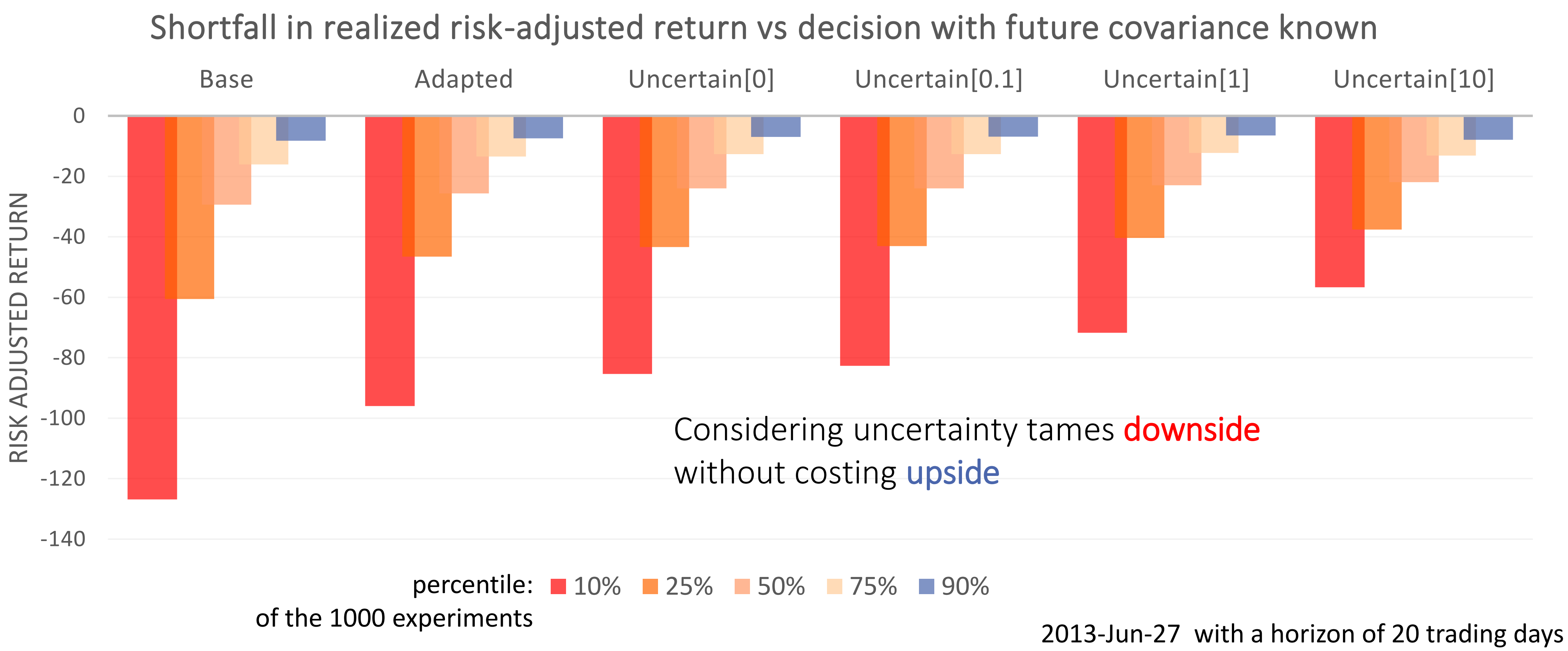
Optimize under the following covariance models

Base	conventional Bayesian 15 factor PCA model
Adapted	base model adapted to forecasts of future market conditions
Uncertain[γ]	adapted model with uncertainty information
	maximize alpha - ½ [variance + γ stdev(variance)]
	note: Uncertain[0] has uncertainty correction, but no penalty on uncertainty

Measure the shortfall in realized risk-adjusted return vs. the ideal (w/future covariance known)

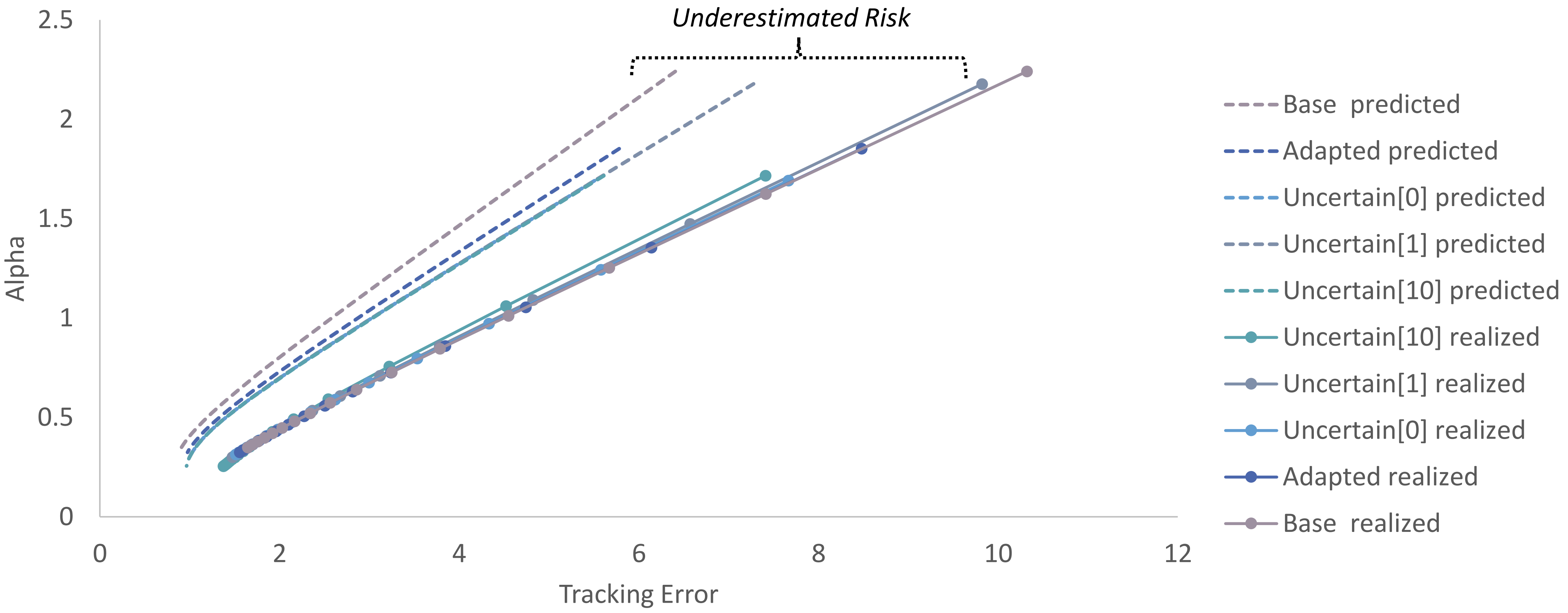
Repeat the experiment 1000 times

Portfolio Optimization: Maximize Risk Adjusted Return (cont)



Predicted vs Realized Efficient Frontier

Perfectly known alphas
Random 5 stock portfolio, 5 stock benchmark

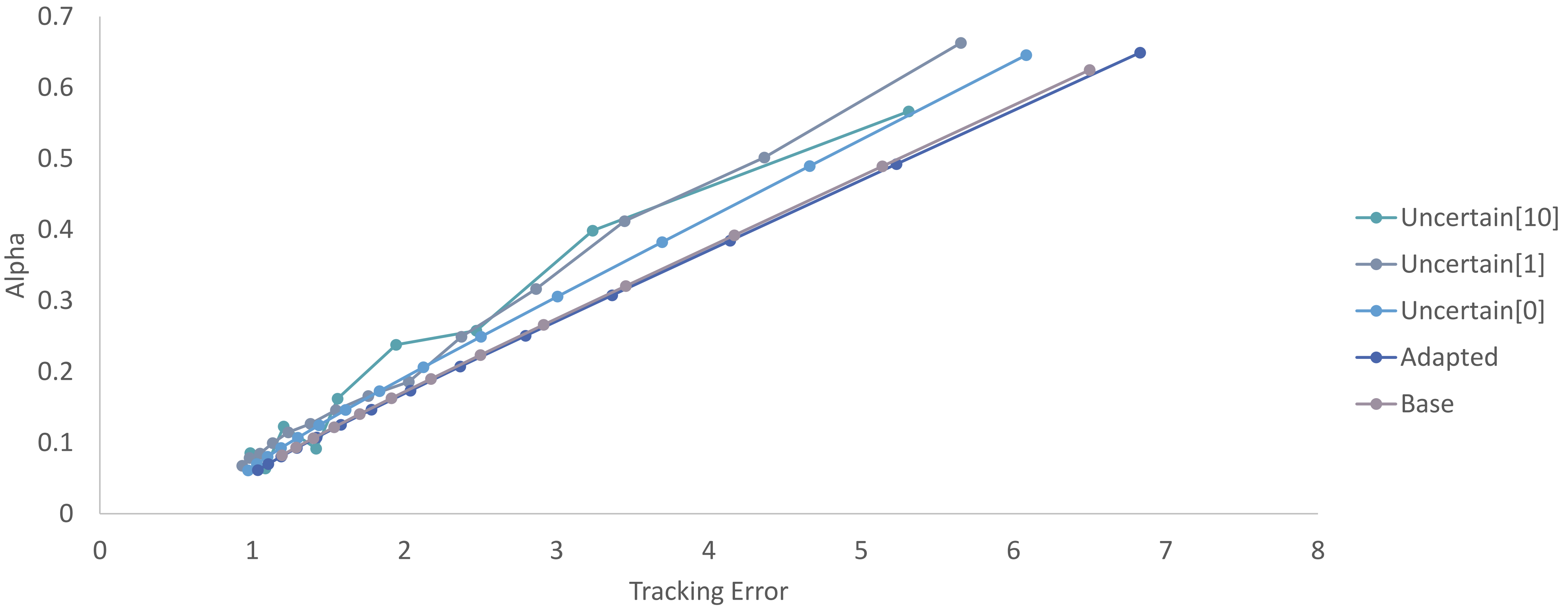


Realized Efficient Frontier, Imperfect Alphas

cor(Forecast, Realized Alpha) = .07
Random 5 stock portfolio, 5 stock benchmark

Realized Efficient Frontier, Imperfect Alphas

cor(Forecast, Realized Alpha) = .07
Random 15 stock portfolio, 15 stock benchmark



Pairs Trading

Improved via knowledge of uncertainty

- Feel bullish (or bearish) about one or several similar securities
- Have candidates you feel the opposite about
- Choose – from the two sets – the long-short pair with lowest uncertainty penalized risk
- If you have explicit alpha forecasts, instead maximize uncertain utility
- Better risk control = safer leverage and more room to pursue alpha

A toy example, hedging with 3 securities

- AAPL is the reference security
- Every 11 trading days from 2012 through 2013, find the best 3 hedges (ignoring stock-specific effects) from a universe of tech stocks and equal weight-them
- Calculate the subsequent 10 day volatility of daily returns of long AAPL, short the equal weighted

Model	Avg 1 Day TE
Base	1.34
Uncertain[0]	1.13
Uncertain[0.5]	1.14
Uncertain[1]	1.13
Uncertain[100]	1.15

Investment Grade Modeling LLC
Uncertain Covariance Models

Run nightly in the cloud. Bloomberg BBGID or ticker

Built uniquely for each use

- Risk factors arise from one's universe (e.g. only healthcare + tech, all US equities + commodity indices)
- .. and horizon (e.g. 1 day, 6 months) and return frequency (daily or weekly)
- Exposures (sails) are forecasts of the average over the horizon
- Factors volatilities (winds) and specific risks (motors) are adapted to forecasts of future volatility over the horizon made from broad set of information
- Though I believe risk crowding is baloney: zero chance of crowding from others using an identical model

"Augmented" PCA

- PCA + (as necessary) factors to cover important not-stock-return-pervasive effects, e.g. VIX and certain commodities

Java library does uncertainty calculations

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