The Triumph of Mediocrity: A Case Study of “Naïve Beta”

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Definition

What do they mean?

» “Naïve”
  » showing unaffected simplicity; a lack of judgment, or information

» “Smart”
  » showing intelligence or good judgment

» “Mediocrity”
  » of ordinary or moderate quality; neither good nor bad; barely adequate

» “Why all the quotation marks”?
  » Few things are what they seem in investment industry.
Definition

What do they really mean?

» “Securities”
  » Risky investments

» “High yield bonds”
  » Junk bonds

» “Private equity”
  » Leveraged buyout, accounting arbitrage

» “60/40 balanced funds”
  » Portfolios with non-diversified equity risk

» “Hedge funds”
  » Unhedged investments for regressive wealth distribution

» “Smart beta”
  » ???
Definition

“Smart beta”

- Quantitative Equity Portfolio Management
  - Co-authors Ron Hua, Eric Sorensen
- Second edition?
  - New chapters on “smart beta”
- “Smart beta”
  - Factor-based
  - Diversification-based
CHAPTER 13

FACTOR-BASED “SMART BETA”

13.1 Please refer to chapter 5 on quantitative equity factors

13.2 Discard risk model

13.3 Use equal-weighting or capitalization-weighting method

13.4 Call it smart beta, scientific beta, advanced beta, exotic beta, or indexing

13.5 Make no reference to active quantitative equity
Introduction

Naïve beta

» What are naïve betas?

» Why do they outperformed the S&P 500 index?

» Not all naïve betas are created equal
Naïve Beta

Everybody is mediocre in someway

» Dimension of equality and corresponding naïve beta
  » Equal weight: Equal weight (EQ)

  » Equal expected return: Minimum variance (MV)

  » Equal risk-adjusted return: Maximum diversification (MD)

  » Equal risk contribution: Risk parity (RP)
Same expected return, then mean-variance optimal portfolio is min variance portfolio

\[ \mathbf{i} = (1, \ldots, 1)' \]

\[
\min \mathbf{w}'\Sigma \mathbf{w}, \text{subject to } w_1 + w_2 + \cdots + w_N = 1.
\]

\[ \mathbf{w}' \cdot \mathbf{i} = 1 \]

\[ \mathbf{w}_{\text{MV}} = \frac{1}{\lambda_{\text{MV}}} \Sigma^{-1} \mathbf{i} = \frac{1}{(\mathbf{i}'\Sigma^{-1}\mathbf{i})} \Sigma^{-1} \mathbf{i}. \]
Naïve Beta
Max diversification

» Same risk-adjusted return, then mean-variance optimal portfolio is maximum diversification portfolio

\[ \mu_i = k \sigma_i, \quad i = 1, \ldots, N \]
\[ \mu = k \sigma \]

\[ W_{MD} = \frac{1}{\lambda_{MD}} \Sigma^{-1} \sigma = \frac{1}{\left(i' \Sigma^{-1} \sigma\right)} \Sigma^{-1} \sigma \]
Naïve Beta

Risk parity

» Same risk contribution leads to risk parity portfolio
  » Risk contribution = weight x marginal contribution

\[
RC = w \otimes (\Sigma w)
\]

\[
w_{RP} \otimes (\Sigma w_{RP}) = \lambda_{RP} i.
\]
A Case Study

Naïve beta versus the S&P 500 index

» Data
  » From Jan 1990 to Nov 2014
  » Monthly return for the S&P 500 index
  » Monthly return for the 10 S&P 500 index sectors
  » Monthly sector weights

» Backtest for EQ/MV/MD/RP
  » Long-only, fully invested
  » In sample
  » Out-of-sample from Jan 1992 to Nov 2014
    » Update covariance matrix (half-life 5 years)
A Case Study
Naïve beta versus the S&P 500 index

» Return statistics
  » High risk sectors: FIN/TEC
  » Low risk sectors: CSS/HLT/UTL

<table>
<thead>
<tr>
<th>Sector</th>
<th>Return</th>
<th>Volatility</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Staples (CSS)</td>
<td>11.40%</td>
<td>13.34%</td>
<td>0.58</td>
</tr>
<tr>
<td>Consumer Discretionary (CSD)</td>
<td>10.32%</td>
<td>17.86%</td>
<td>0.37</td>
</tr>
<tr>
<td>Energy (ENE)</td>
<td>10.67%</td>
<td>18.29%</td>
<td>0.38</td>
</tr>
<tr>
<td>Financials (FIN)</td>
<td>8.62%</td>
<td>21.98%</td>
<td>0.23</td>
</tr>
<tr>
<td>Health Care (HLT)</td>
<td>12.24%</td>
<td>15.68%</td>
<td>0.54</td>
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<tr>
<td>Industrials (IND)</td>
<td>10.10%</td>
<td>17.41%</td>
<td>0.37</td>
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<tr>
<td>Information Technology (TEC)</td>
<td>11.06%</td>
<td>25.39%</td>
<td>0.29</td>
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<td>Materials (MAT)</td>
<td>8.20%</td>
<td>19.92%</td>
<td>0.23</td>
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<td>Telecommunication Services (TEL)</td>
<td>5.89%</td>
<td>19.26%</td>
<td>0.12</td>
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<tr>
<td>Utilities (UTL)</td>
<td>8.12%</td>
<td>15.05%</td>
<td>0.30</td>
</tr>
</tbody>
</table>
A Case Study
Naïve beta versus the S&P 500 index

» Return statistics
  » CAPM was wrong

\[ y = -2.517x + 0.8046 \]
\[ R^2 = 0.393 \]
A Case Study

Naïve beta versus the S&P 500 index

» Correlation matrix
» Cyclical sectors tend to have high correlations with each other

<table>
<thead>
<tr>
<th></th>
<th>CSS</th>
<th>CSD</th>
<th>ENE</th>
<th>FIN</th>
<th>HLT</th>
<th>IND</th>
<th>TEC</th>
<th>MAT</th>
<th>TEL</th>
<th>UTL</th>
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<tr>
<td>CSS</td>
<td>1.00</td>
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<td>0.71</td>
<td>0.59</td>
<td>0.29</td>
<td>0.49</td>
<td>0.40</td>
<td>0.44</td>
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<tr>
<td>CSD</td>
<td>0.56</td>
<td>1.00</td>
<td>0.44</td>
<td>0.78</td>
<td>0.51</td>
<td>0.85</td>
<td>0.71</td>
<td>0.74</td>
<td>0.53</td>
<td>0.28</td>
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<tr>
<td>ENE</td>
<td>0.37</td>
<td>0.44</td>
<td>1.00</td>
<td>0.48</td>
<td>0.36</td>
<td>0.58</td>
<td>0.37</td>
<td>0.64</td>
<td>0.32</td>
<td>0.50</td>
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<tr>
<td>FIN</td>
<td>0.61</td>
<td>0.78</td>
<td>0.48</td>
<td>1.00</td>
<td>0.59</td>
<td>0.81</td>
<td>0.52</td>
<td>0.69</td>
<td>0.45</td>
<td>0.39</td>
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<td>HLT</td>
<td>0.71</td>
<td>0.51</td>
<td>0.36</td>
<td>0.59</td>
<td>1.00</td>
<td>0.55</td>
<td>0.38</td>
<td>0.44</td>
<td>0.41</td>
<td>0.40</td>
</tr>
<tr>
<td>IND</td>
<td>0.59</td>
<td>0.85</td>
<td>0.58</td>
<td>0.81</td>
<td>0.55</td>
<td>1.00</td>
<td>0.66</td>
<td>0.83</td>
<td>0.50</td>
<td>0.40</td>
</tr>
<tr>
<td>TEC</td>
<td>0.29</td>
<td>0.71</td>
<td>0.37</td>
<td>0.52</td>
<td>0.38</td>
<td>0.66</td>
<td>1.00</td>
<td>0.54</td>
<td>0.49</td>
<td>0.16</td>
</tr>
<tr>
<td>MAT</td>
<td>0.49</td>
<td>0.74</td>
<td>0.64</td>
<td>0.69</td>
<td>0.44</td>
<td>0.83</td>
<td>0.54</td>
<td>1.00</td>
<td>0.39</td>
<td>0.33</td>
</tr>
<tr>
<td>TEL</td>
<td>0.40</td>
<td>0.53</td>
<td>0.32</td>
<td>0.45</td>
<td>0.41</td>
<td>0.50</td>
<td>0.49</td>
<td>0.39</td>
<td>1.00</td>
<td>0.35</td>
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<td>UTL</td>
<td>0.44</td>
<td>0.28</td>
<td>0.50</td>
<td>0.39</td>
<td>0.40</td>
<td>0.40</td>
<td>0.16</td>
<td>0.33</td>
<td>0.35</td>
<td>1.00</td>
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<tr>
<td>Avg</td>
<td>0.55</td>
<td>0.64</td>
<td>0.50</td>
<td>0.63</td>
<td>0.53</td>
<td>0.68</td>
<td>0.51</td>
<td>0.61</td>
<td>0.49</td>
<td>0.42</td>
</tr>
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A Case Study

Naïve beta versus the S&P 500 index

» In sample results sector weights

<table>
<thead>
<tr>
<th>Sector</th>
<th>MV</th>
<th>MD</th>
<th>RP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Staples (CSS)</td>
<td>42.6%</td>
<td>16.9%</td>
<td>13.4%</td>
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<tr>
<td>Consumer Discretionary (CSD)</td>
<td>1.8%</td>
<td>0.0%</td>
<td>8.7%</td>
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<tr>
<td>Energy (ENE)</td>
<td>9.2%</td>
<td>13.3%</td>
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<tr>
<td>Financials (FIN)</td>
<td>0.0%</td>
<td>0.0%</td>
<td>7.1%</td>
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<tr>
<td>Health Care (HLT)</td>
<td>6.2%</td>
<td>10.4%</td>
<td>11.6%</td>
</tr>
<tr>
<td>Industrials (IND)</td>
<td>0.0%</td>
<td>0.0%</td>
<td>8.4%</td>
</tr>
<tr>
<td>Information Technology (TEC)</td>
<td>3.8%</td>
<td>16.6%</td>
<td>7.5%</td>
</tr>
<tr>
<td>Materials (MAT)</td>
<td>0.0%</td>
<td>3.2%</td>
<td>8.2%</td>
</tr>
<tr>
<td>Telecommunication Services (TEL)</td>
<td>7.7%</td>
<td>12.9%</td>
<td>10.2%</td>
</tr>
<tr>
<td>Utilities (UTL)</td>
<td>28.9%</td>
<td>26.6%</td>
<td>14.5%</td>
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</tbody>
</table>
A Case Study

Naïve beta versus the S&P 500 index

» In sample results – performance
  » Turnover 30-35% two-way
A Case Study
Naïve beta versus the S&P 500 index

» Out-of-sample results – MV sector weights
  » Dominated by UTL and CSS
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Naïve beta versus the S&P 500 index

» Out-of-sample results – MD sector weights
  » UTL/TEL/TEC/HLH/ENE/CSS
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Naïve beta versus the S&P 500 index

» Out-of-sample results – RP sector weights
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Naïve beta versus the S&P 500 index

» Out-of-sample results – performance
  » Turnover – MV 77% MD 64% RP 32%
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Naïve beta versus the S&P 500 index

» Out-of-sample results – the S&P index sector weights
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Naïve beta versus the S&P 500 index

» Four naïve betas outperformed the index in sample
  » In order of Sharpe MV/MD/RP/EQ/Index

» Four naïve betas still outperformed the index out-of-sample
  » In order of Sharpe RP/EQ/MD/MV/Index

» Sector perspective
  » The index is dominated by cyclical sectors
  » MV is concentrated in low-vol sectors: CSS/UTL
  » MD is concentrated in defensive sectors plus TEC/ENE
  » RP is balanced with tilts to low-vol sectors
A Case Study
What’s wrong with the S&P 500 index?

» Nothing is wrong
  » Rooted in Nobel-prize winning theory – efficient market hypothesis, pretty “smart”
  » Most of active managers don’t beat the index

» Something is wrong
  » Why naïve betas beat the index?
  » Why the index is loaded with cyclical sectors?
  » High volatility, tail risks
  » Not truly diversified
  » Is it really passive?
A Case Study

What’s wrong with the S&P 500 index?

» Cumulative sector weight change net of drift
A Case Study
What’s wrong with the S&P 500 index?

Cumulative # of name changes in the sectors
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What’s wrong with the S&P 500 index?

“Value added” of sector shifts by the S&P 500 index
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Comparison of three risk-based naïve betas

» MV/MD/RP all use risk models
  » MV/MD use optimization
  » RP uses risk budgeting – no optimization

» MV is concentrated in low vol sectors
» MD is concentrated in defensive sectors with a couple of cyclical sectors
» RP is balanced in sectors with a tilt to low vol sectors

» None of the theoretical solutions is easy to solve
A Case Study

Comparison of three risk-based naïve betas

» Solutions recap

\[ \mathbf{w}_{MV} \propto \Sigma^{-1} \mathbf{i} \]

\[ \mathbf{w}_{MD} \propto \Sigma^{-1} \mathbf{\sigma} \]

\[ \mathbf{w}_{RP} \propto \left( \Sigma \mathbf{w}_{RP} \right) \propto \mathbf{i} \]
A Case Study
Comparison of three risk-based naïve betas

» Decomposition of covariance matrix into correlation matrix and volatilities

\[ \Sigma = \text{diag}(\sigma) \cdot C \cdot \text{diag}(\sigma). \]

\[ \Sigma^{-1} = \text{diag}(\sigma^{-1}) \cdot C^{-1} \cdot \text{diag}(\sigma^{-1}). \]
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Comparison of three risk-based naïve betas

» Risk-modified weights = weight x volatility
  » MV weights inversely proportional to variance
  » MD and RP weights inversely proportional to volatility

\[ W_i = \sigma_i w_i, i = 1, \ldots, N, W = \sigma \otimes w. \]

\[ W_{MV} \propto C^{-1} \sigma^{-1}. \]

\[ W_{MD} \propto C^{-1} i. \]

\[ W_{RP} \otimes (C W_{RP}) \propto i. \]
A Case Study
Comparison of three risk-based naïve betas

» Modeling two groups of securities
   » Homogeneous within each group; heterogeneous across the groups

\[
C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix},
\]

\[
C_{11} = \begin{pmatrix} 1 & \rho_1 & \cdots & \rho_1 \\ \rho_1 & 1 & \cdots & \rho_1 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_1 & \rho_1 & \cdots & 1 \end{pmatrix}_{N_1 \times N_1}, \quad C_{22} = \begin{pmatrix} 1 & \rho_2 & \cdots & \rho_2 \\ \rho_2 & 1 & \cdots & \rho_2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_2 & \rho_2 & \cdots & 1 \end{pmatrix}_{N_2 \times N_2},
\]

\[
C_{12} = C'_{21} = \rho_{12} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \end{pmatrix}_{N_1 \times N_2}.
\]
A Case Study

Comparison of three risk-based naïve betas

» Modeling two groups of securities
  » Risk-modified weights are the same within each group

\[ W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}, \quad W_1 = \begin{pmatrix} W_1 \\ \vdots \\ W_1 \end{pmatrix}_{N_1 \times 1}, \quad W_2 = \begin{pmatrix} W_2 \\ \vdots \\ W_2 \end{pmatrix}_{N_2 \times 1} \]

» We are interested in the ratio of the weights for MV/MD/RP portfolios

\[ \begin{pmatrix} \frac{W_1}{W_2} \end{pmatrix} = ??? \]
A Case Study

Comparison of three risk-based naïve betas

» Ratio of risk-modified weights for the two groups

\[
\begin{align*}
\left(\frac{W_1}{W_2}\right)_{MV} & = \frac{1+(N_2-1)\rho_2 - (\sigma_1 / \sigma_2)N_2\rho_{12}}{(\sigma_1 / \sigma_2)\left[1+(N_1-1)\rho_1\right] - N_1\rho_{12}}. \\
\left(\frac{W_1}{W_2}\right)_{MD} & = \frac{1+(N_2-1)\rho_2 - N_2\rho_{12}}{1+(N_1-1)\rho_1 - N_1\rho_{12}}. \\
\left(\frac{W_1}{W_2}\right)_{RP} & = \frac{(N_1-N_2)\rho_{12} + \sqrt{\left[(N_1-N_2)\rho_{12}\right]^2 + 4\left[1+(N_1-1)\rho_1\right]\left[1+(N_2-1)\rho_2\right]}}{2\left[1+(N_1-1)\rho_1\right]}.
\end{align*}
\]
A Case Study
Comparison of three risk-based naïve betas

» Application to the S&P 500 sectors
  » Group 1: consumer discretionary, financials, industrials, and materials
  » Group 2: consumer staples, energy, health care, technology, telecom, and utilities
  » Cross correlation

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.78</td>
<td>0.40</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>19.3%</td>
<td>17.8%</td>
</tr>
</tbody>
</table>

$\rho_{12} = 0.51$
A Case Study

Comparison of three risk-based naïve betas

» Application to the S&P 500 sectors
  » Group 1: consumer discretionary, financials, industrials, and materials
  » Group 2: consumer staples, energy, health care, technology, telecom, and utilities
  » According to the theoretical solution, MV/MD should have zero weight in group 1 under long-only constraint
  » The ratio should be 0.81 for RP weights, the actual ratio is 0.805

\[
\frac{W_1}{W_2} \text{MV} < 0, \frac{W_1}{W_2} \text{MD} < 0, \frac{W_1}{W_2} \text{RP} = 0.81.
\]
Triumph of “Naïve Beta”

Conclusion

» EQ/MV/MD/RP can be thought of as “naïve beta”
   » They are naively diversified in some dimension
   » In contrast, the S&P 500 index is “smart”
   » There is a blurred line between being “smart” and “naïve”
   » “Naïve” beta beating the index is “the triumph of mediocrity”

» MV/MD portfolios are highly concentrated and sensitive to risk inputs and risk models. RP portfolio is the most diversified portfolio

» We provide a two-group correlation matrix, as a model the sector portfolios of MV/MD/RP with reasonable accuracy
Triumph of “Naïve Beta”

Some quotes

» “Prediction is very difficult, especially about the future.“

» Niels Bohr

» “The fundamental cause of trouble in the world today is that the stupid are cocksure while the intelligent are full of doubt.”

» Bertrand Russell