Asset Allocation:

Fallacies, Challenges, and Solutions

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Fallacies of Asset Allocation

Asset allocation determines more than 90 percent of performance

Time diversifies risk

Optimized portfolios are hypersensitive to input errors

Factors offer superior diversification and noise reduction

Equally weighted portfolios are superior to optimized portfolios

FALLACIES OF ASSET ALLOCATION

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THE IMPORTANCE OF ASSET ALLOCATION

Consider the following example:

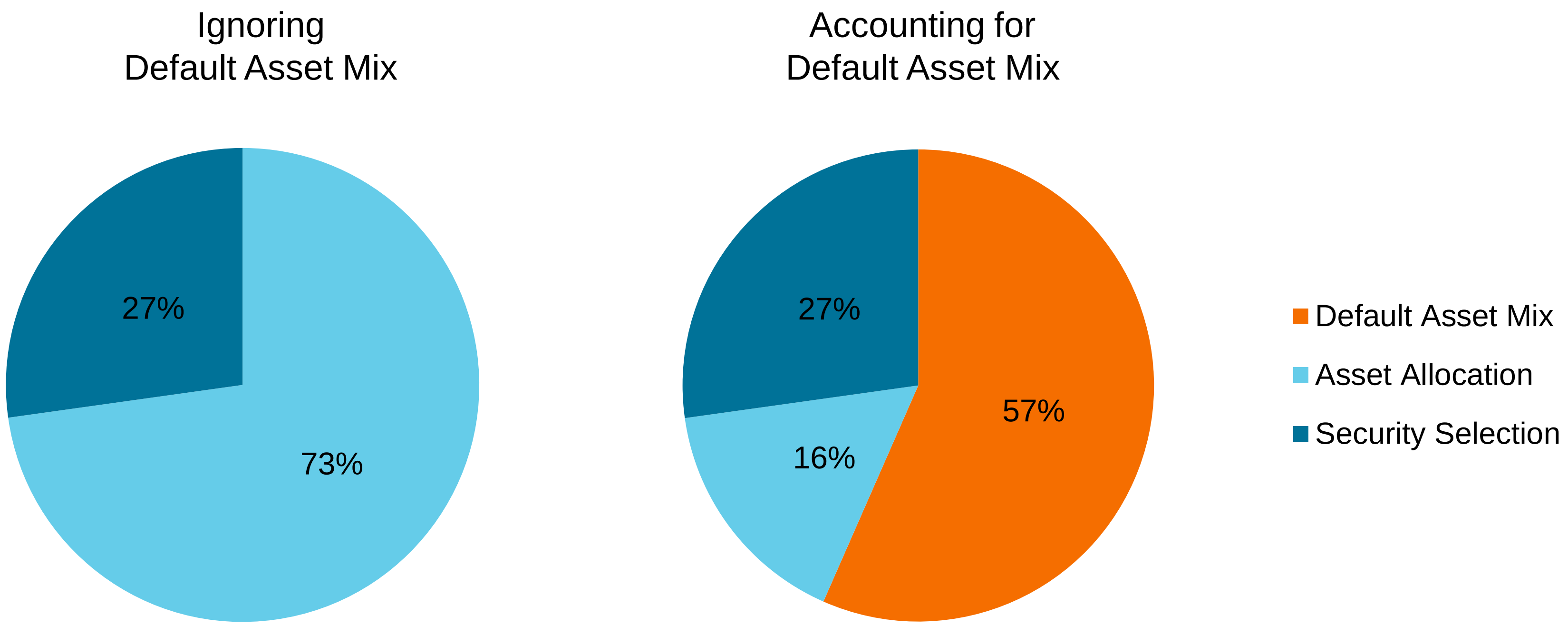
- A portfolio has an asset mix policy of 75% technology stocks and 25% U.S. bonds.
- We compute monthly returns from January 2006 to December 2012.
- What percentage of return variation is explained by the asset mix policy?

BHB Methodology:

- The contribution of the asset mix policy is measured as the percentage of return variation explained by the returns of a portfolio invested 75% in a broad stock market index and 25% in a U.S. bond index.
- This approach completely ignores the notion of a default asset mix, such as a portfolio that invests 60% in U.S. stocks and 40% U.S. bonds. It implicitly assumes that the portfolio would be otherwise uninvested.

THE IMPORTANCE OF ASSET ALLOCATION

Fractional Contribution to Total Variance



Notes: Data spans Jan 2006 to Dec 2012. The hypothetical portfolio consists of 75% S&P 500 information technology sector index plus 25% Barclays US government bond index. The default asset mix consists of 60% S&P 500 composite index and 40% Barclays US government bond index.

TIME DIVERSIFICATION

It is widely assumed that investing over long horizons is less risky than investing over short horizons, because the likelihood of loss is lower over long horizons.

Time, Volatility, and Probability of Loss

Expected continuous return: 10%

Continuous standard deviation: 20%

Investment Horizon	Annualized Continuous Standard Deviation	Probability of Loss (<0%) on Average over Horizon
1 Year	20.0%	30.9%
5 Years	8.9%	13.2%
10 Years	6.3%	5.7%
20 Years	4.5%	1.3%

TIME DIVERSIFICATION

Paul A. Samuelson showed that time does not diversify risk, because though the probability of loss decreases with time, the magnitude of potential losses increases with time.

Expected utility accounts for both the likelihood and magnitude of changes in wealth.

A certainty equivalent is the certain amount that conveys the same expected utility as a risky gamble.

$ln(\$100) = 4.6052$

$50\% \times ln(\$100 \times 1.3333) + 50\% \times ln(\$100 \times 0.75) = 4.6052$

Expected Wealth and Expected Utility

	Initial Wealth	1st Period Distribution	2nd Period Distribution	3rd Period Distribution
				237.04 x .125
			177.78 x .25	
				133.33 x .125
		133.33 x .50		
				133.33 x .125
			100.00 x .25	
				75.00 x .125
	100.00			
				133.33 x .125
			100.00 x .25	
				75.00 x .125
		75.00 x .50		
				75.00 x .125
			56.25 x .25	
				42.19 x .125
Expected wealth	100.00	104.17	108.51	113.03
Expected utility	4.6052	4.6052	4.6052	4.6052

Time diversification

It is also true that the probability of loss within an investment horizon never decreases with time.

$$Pr_W = N\left[\frac{\ln(1+L)-\mu T}{\sigma\sqrt{T}}\right] + N\left[\frac{\ln(1+L)+\mu T}{\sigma\sqrt{T}}\right](1+L)^{\frac{2\mu}{\sigma^2}}$$

Probability of a Within-Horizon Loss

Continuous Expected Return: 10%

Continuous Standard Deviation: 20%

Investment Horizon	Probability of -10%
0.25 Years	22.1%
1 Year	44.1%
5 Years	56.7%
10 Years	58.4%
20 Years	59.0%
100 Years	59.1%

TIME DIVERSIFICATION

Finally, the cost of a protective put option increases with time to expiration. Therefore, because it costs more to insure against losses over longer periods than shorter periods, it follows that risk does not diminish with time.

Risky asset	100
Risk-free rate	3%
Volatility	20%
Strike Price	95

Time to Expiration	Price of Put Option
0.25	1.67
1	4.39
5	8.61
10	9.49

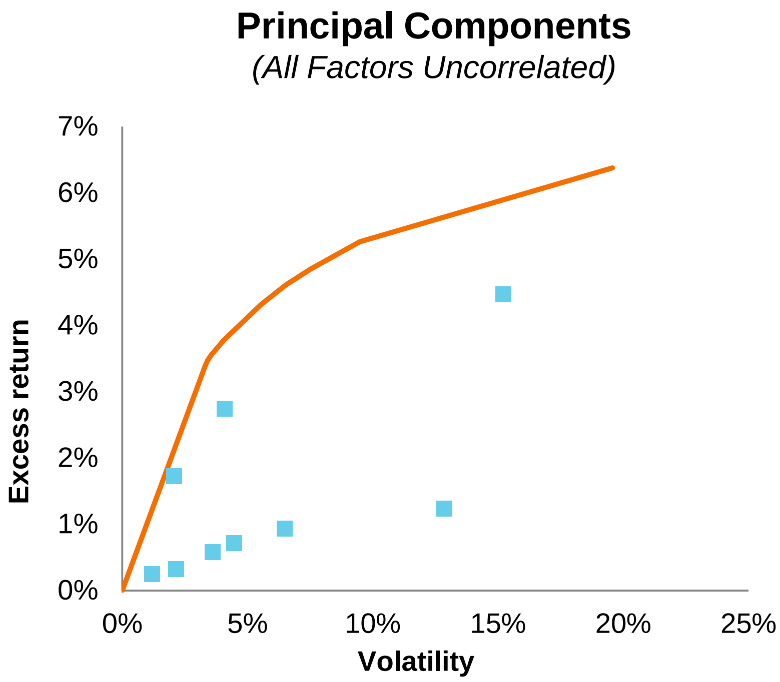
FACTORS

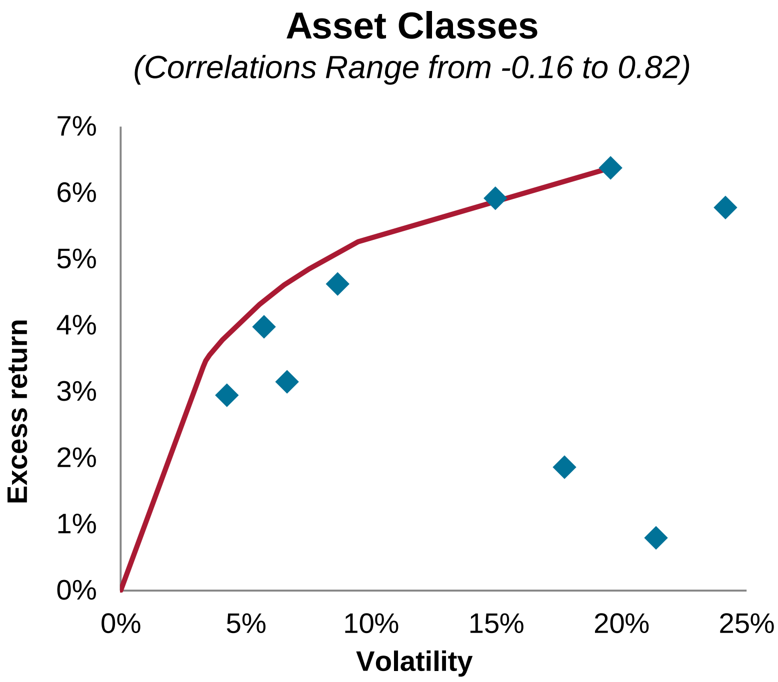
- Some investors believe that factors offer greater potential for diversification than asset classes because they appear less correlated than asset classes.

- Factors appear less correlated only because the portfolio of assets designed to mimic them includes short positions.

- Given the same constraints and the same investible universe, it is mathematically impossible to regroup assets into factors and produce a better efficient frontier.

Factors





Source: A Practitioner's Guide to Asset Allocation, Wiley 2017
Analysis is based on data spanning Jan 1976 through Dec 2015.

FACTORS

- Some investors believe that consolidating a large group of securities into a few factors reduces noise more effectively than consolidating them into a few asset classes.

- Consolidation reduces noise around means but no more so by using factors than by using asset classes.

- Consolidation does not reduce noise around covariances.

Challenges to Asset Allocation

Necessary conditions for optimization

Constraints

Currency risk

Optimal exposure to illiquid assets

Risk measurement

Estimation error

Leverage versus concentration

Rebalancing

Shifting risk regimes

CHALLENGES TO ASSET ALLOCATION

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CONSTRAINTS

Mean-variance optimization:

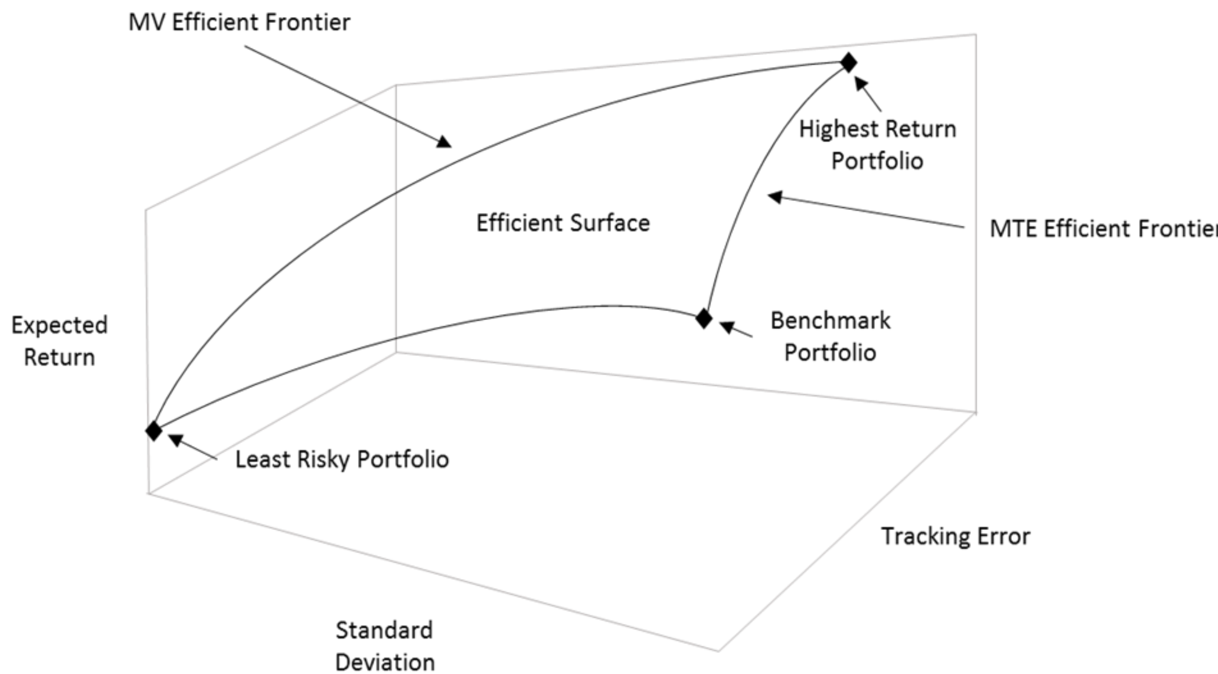
$$E(U) = \mu_p - \lambda_{RA}\sigma_p^2$$

Mean-variance-tracking error optimization:

$$E(U) = \mu_p - \lambda_{RA}\sigma_p^2 - \lambda_{TEA}\xi_p^2$$

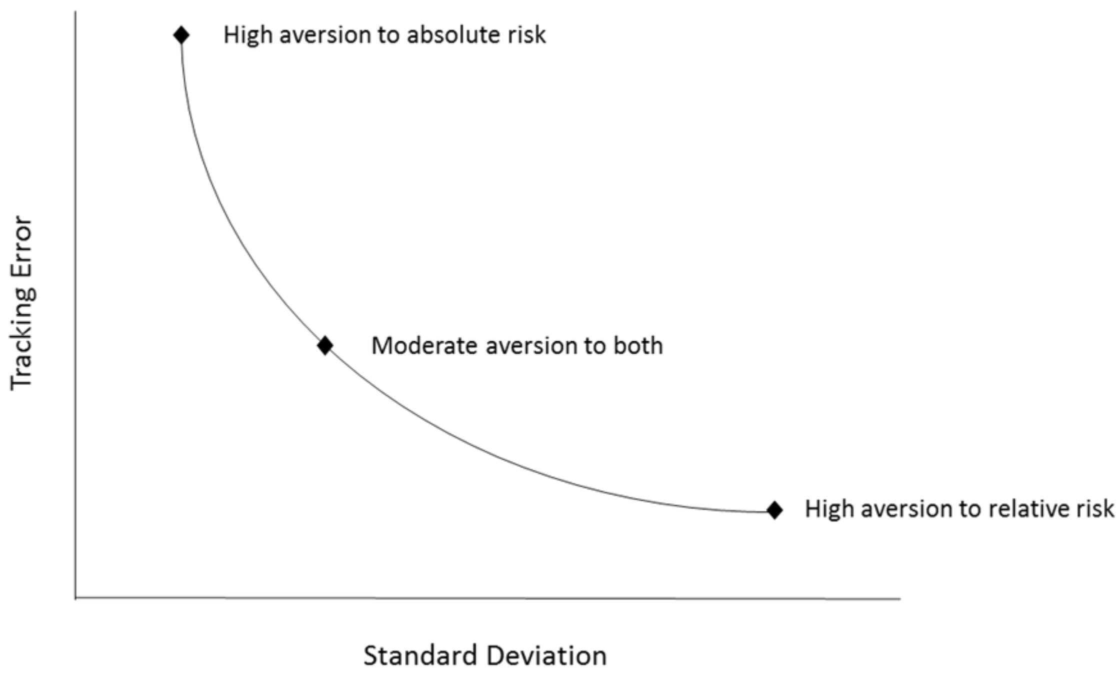
CONSTRAINTS

Efficient Surface



CONSTRAINTS

Iso-Expected Return Curve



Illiquidity

We treat liquidity as a shadow allocation.

If liquidity is deployed to raise expected utility, we attach a shadow asset to tradable assets to capture this incremental benefit.

If liquidity is deployed to prevent a decline in expected utility, we attach a shadow liability to assets that are not tradable.

ILLIQUIDITY

Investors benefit from liquidity in a variety of ways

- Rebalance a portfolio
- Meet capital calls
- Engage in tactical asset allocation
- Seize new opportunities
- Respond to shifts in risk tolerance

Even though these liquidity benefits are driven by different purposes, we can measure all of them in units of expected return and risk.

Required Equity Return

	Public Equity	Private Equity Ignoring Illiquidity	Private Equity Accounting for Illiquidity
Equity standard deviation	18.00%	22.00%	22.00%
Other assets return	5.00%	5.00%	7.00%
Other assets standard deviation	8.00%	8.00%	8.94%
Equity/other assets correlation	0.5000	0.4000	0.3578
Shadow asset return			2.00%
Shadow asset standard deviation			4.00%
Risk aversion	1	1	1
Equity weight	50%	50%	50%
Other assets weight	50%	50%	50%
Required equity return	**7.60%**	**9.20%**	**11.04%**
Marginal utility of equity	0.0364	0.0366	0.0550
Marginal utility of other assets	0.0364	0.0366	0.0550
Difference in marginal utilities	0	0	0

Optimal Allocation to Illiquid Asset

	Public Equity	Private Equity Ignoring Illiquidity	Private Equity Accounting for Illiquidity
Equity standard deviation	18.00%	22.00%	22.00%
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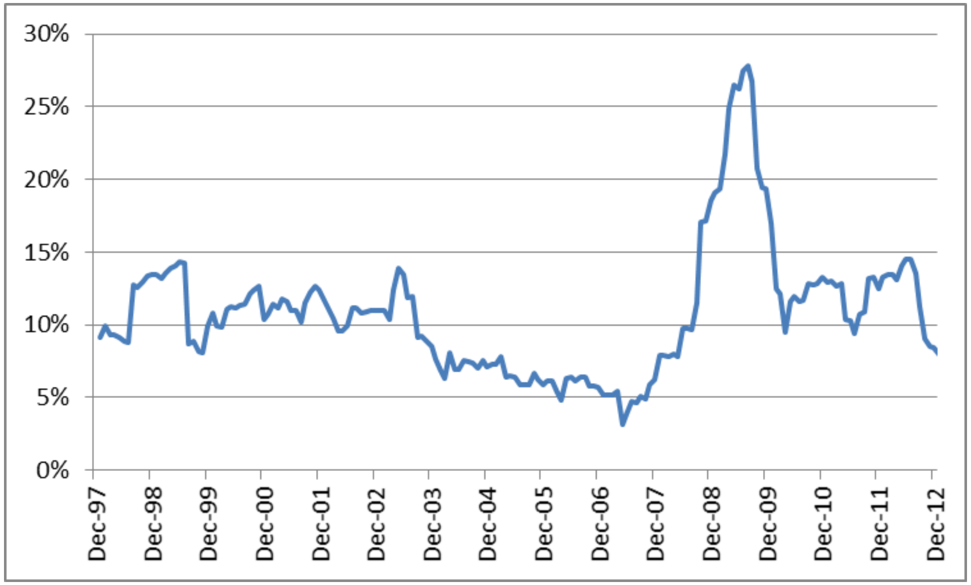
Equity/other assets correlation	0.5000	0.4000	0.3578
Shadow asset return			2.00%
Shadow asset standard deviation			4.00%
Risk aversion	1	1	1
Equity weight	50%	50%	**28%**
Other assets weight	50%	50%	**72%**
Required equity return	7.60%	9.20%	**9.20%**
Marginal utility of equitie	0.0364	0.0366	0.0545
Marginal utility of other assets	0.0364	0.0366	0.0545
Difference in marginal utilities	0	0	0

REGIME SHIFTS

Trailing 12-Month Annualized Portfolio Volatility

January 1998 through February 2013



Regime shifts

$$Turbulence_t = \frac{1}{N}(\boldsymbol{x}_t - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_t - \boldsymbol{\mu})$$

$\boldsymbol{x}_t$ is a vector of monthly returns across asset classes.

$\boldsymbol{\mu}$ is a vector of average returns for each asset class over the full 40-year sample.

$\boldsymbol{\Sigma}^{-1}$ is the inverse of the covariance matrix computed from the 40-year sample.

Regime Shifts

Hidden Markov Model Fit and Conditional Asset Class Performance

Hidden Markov Model Fit: Turbulence	Calm	Moderate	Turbulent
Regime Persistence	92%	75%	67%
Turbulence Average	0.7	1.1	1.7
Turbulence Standard Deviation	0.2	0.3	0.6

Average Annual Asset Return	Calm	Moderate	Turbulent
U.S. Equities	15.0%	13.6%	–27.7%
Foreign Developed Market Equities	15.3%	5.7%	–12.0%
Emerging Market Equities	17.2%	21.7%	–26.0%
Treasury Bonds	5.9%	9.6%	12.3%
U.S. Corporate Bonds	7.5%	10.2%	4.2%
Commodities	7.8%	7.8%	–17.1%
Cash Equivalents	3.9%	5.9%	7.4%

Asset Standard Deviations	Calm	Moderate	Turbulent
U.S. Equities	12.6%	20.2%	19.9%
Foreign Developed Market Equities	14.7%	19.6%	31.0%
Emerging Market Equities	21.3%	30.5%	32.5%
Treasury Bonds	4.2%	6.4%	12.1%
U.S. Corporate Bonds	5.1%	7.8%	16.6%
Commodities	18.1%	21.3%	30.3%
Cash Equivalents	0.8%	1.1%	1.7%

REGIME SHIFTS

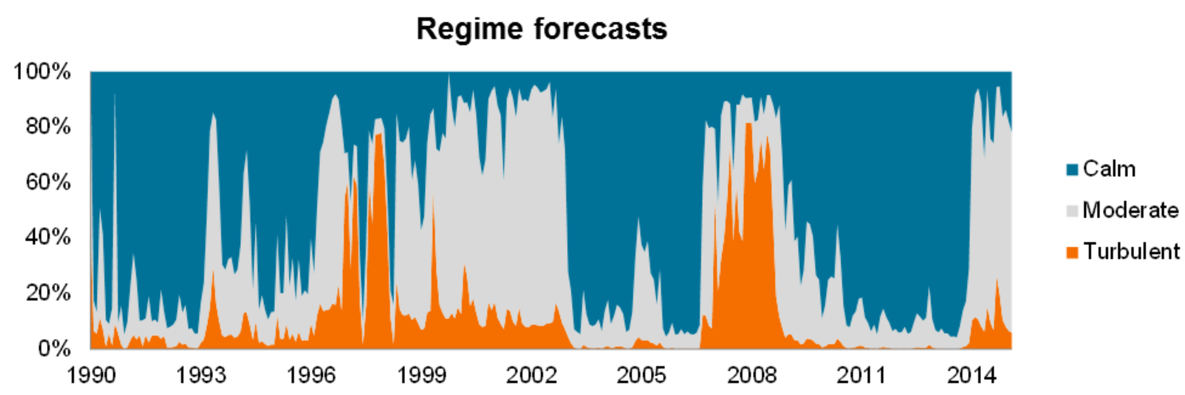
Next Period Probability of Each Regime

$$\begin{bmatrix} P(\varphi_{t+1} = A) \\ P(\varphi_{t+1} = B) \\ P(\varphi_{t+1} = C) \end{bmatrix} =$$

$$\begin{bmatrix} P(\varphi_{t+1} = A|\varphi_t = A) & P(\varphi_{t+1} = A|\varphi_t = B) & P(\varphi_{t+1} = A|\varphi_t = C) \\ P(\varphi_{t+1} = B|\varphi_t = A) & P(\varphi_{t+1} = B|\varphi_t = B) & P(\varphi_{t+1} = B|\varphi_t = C) \\ P(\varphi_{t+1} = C|\varphi_t = A) & P(\varphi_{t+1} = C|\varphi_t = B) & P(\varphi_{t+1} = C|\varphi_t = C) \end{bmatrix} \begin{bmatrix} P(\varphi_t = A) \\ P(\varphi_t = B) \\ P(\varphi_t = C) \end{bmatrix}$$

Regime shifts

Estimated probability of each regime occurring next month, calibrated on prior data at each point in time.



Cumulative returns of static portfolios, and tactical strategy that allocates proportional to regime forecasts.