Outline

Introduction to Hidden Markov Models

Turbulence, inflation, and economic growth regimes

Investable risk premia: out-of-sample performance

Dynamic asset allocation: out-of-sample performance

Summary
Introduction to Hidden Markov Models

A simple example
Introduction to Hidden Markov Models

• Imagine someone is in bed wearing a heart monitor and that we receive this person’s heart rate data at one-minute intervals.

• While the person is sleeping, we observe a low average heart rate with low volatility.

• When the person wakes up, we notice a sudden rise in the average level of the heart rate and its volatility.

• Without seeing the person, we can reasonably conclude which “state” he or she is in. The heart rate data follows a Markov process – at any point in time, a “state” (or regime) generates observations from a specific distribution.
A simple example

- Laverty, Miket, and Kelly (2002) provide a simple illustration of a Markov-Switching process via simulation. The initial probability of being in regime \( i \) is given by:

\[
\Pr(X_1 = i) = p_i
\]

where \( X_1 \) is the first regime in the Markov chain.

- The elements of the transition probability matrix, \( \Gamma \), denote the probability of a transition into regime \( j \) from regime \( i \), as follows:

\[
\Gamma = \begin{bmatrix}
\gamma_{11} & \gamma_{12} \\
\gamma_{21} & \gamma_{22}
\end{bmatrix}
\]

\[
\gamma_{ij} = \Pr(X_t = j \mid X_{t-1} = i)
\]

- Over time, the Markov chain is either in regime 1 or 2. Each regime generates observations \( Y_t \) that are consistent with a given distribution \( \pi_i \).
A simple example

Suppose the following:

- Regime 1 is normally distributed with a mean of 2 and a sigma of 1
- Regime 2 is normally distributed with a mean of 4 and a sigma of 6
- The initial probability of being in Regime 1 is 70%
- Regime shifts are generated by the following transition matrix:

\[
\Gamma = \begin{bmatrix}
    0.95 & 0.05 \\
    0.02 & 0.98
\end{bmatrix}
\]
A simple example
Why bother?

• When dealing with changing distributions, we can expect Markov-Switching models to perform better than simple data partitions based on thresholds.

• In this example, had we simply classified all top-quartile observations as Regime 2, we would have misclassified 40 out of 200 observations.

• A well calibrated Markov-Switching model would have misclassified only 3 observations.

• Arbitrary thresholds give false signals for two reasons:
  – they fail to capture the persistence in regimes, and
  – they fail to capture shifts in volatility.

• Moreover, what appear to be fat tails in the full sample may in fact be an artifact of the attempt to model two distinct regimes with a single distribution.
A few samples of previous research

- Many studies have found that return and risk parameters are not stable through time.

- Clark and de Silva (1998) showed that in a world with more than one economic regime, an expanded opportunity set exists for investors who can take advantage of regime-specific return and risk.

- Ang and Bekaert (2004) proposed a regime-switching model for country allocation based on modeling changes in the systematic risk of each country. They found that using a two-state Markov-Switching model to estimate returns and covariances significantly improved the performance of optimized equity portfolios.

- Guidolin and Timmerman (2006) used a four-state Markov-Switching model to explain the joint returns of stocks and bonds, and found some predictive capacity in using a vector autoregressive forecasting model based on prior returns and dividend yields.
Our approach

• Our approach differs from these previous studies in that we did not rely on a specific asset pricing model nor did we model regimes in returns directly.

• Kritzman and Li (2010) presented a static solution to non-stationarity by designing event-sensitive portfolios.

• We extended the Kritzman and Li (2010) approach by using Markov-Switching models to reallocate dynamically across event-sensitive portfolios.
Turbulence, inflation, and economic growth regimes

In-sample performance
Motivation

• Harvey and Dalquist (2001) suggest that if economic conditions are (1) persistent and (2) strongly linked to asset performance, then a dynamic asset allocation process should add value.

• We employ Maximum Likelihood Estimation to build a simple regime-switching model for the following variables:
  – FX market turbulence [December 1977 through December 2009]
  – Equity market turbulence [December 1975 through December 2009]
  – Inflation (CPI) [February 1947 through December 2009]
  – Gross National Product [April 1947 through December 2009]

• We then measure the conditional performance of a variety of risk premia and asset classes during each regime.
In-sample Markov-Switching results

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th></th>
<th></th>
<th>Regime 2 (&quot;event regime&quot;)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Persistence*</td>
<td>Mu</td>
<td>Sigma</td>
<td>Persistence*</td>
<td>Mu</td>
<td>Sigma</td>
</tr>
<tr>
<td>Equity Turbulence</td>
<td>92%</td>
<td>0.65</td>
<td>0.28</td>
<td>90%</td>
<td>1.89</td>
<td>1.13</td>
</tr>
<tr>
<td>Currency Turbulence</td>
<td>92%</td>
<td>0.88</td>
<td>0.33</td>
<td>68%</td>
<td>2.14</td>
<td>1.22</td>
</tr>
<tr>
<td>Inflation Rate</td>
<td>98%</td>
<td>2.62%</td>
<td>0.70%</td>
<td>95%</td>
<td>6.66%</td>
<td>1.81%</td>
</tr>
<tr>
<td>Economic Growth</td>
<td>90%</td>
<td>1.09%</td>
<td>0.84%</td>
<td>68%</td>
<td>-0.14%</td>
<td>0.96%</td>
</tr>
</tbody>
</table>

*Persistence is defined as the estimated transition probability of staying in the current regime.
In-sample Markov-Switching results (with standard errors)

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2 (&quot;event regime&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Persistence*</td>
<td>Mu</td>
</tr>
<tr>
<td>Equity Turbulence</td>
<td>92%</td>
<td>0.65</td>
</tr>
<tr>
<td>Standard Error</td>
<td>8%</td>
<td>0.00</td>
</tr>
<tr>
<td>Currency Turbulence</td>
<td>92%</td>
<td>0.88</td>
</tr>
<tr>
<td>Standard Error</td>
<td>5%</td>
<td>0.00</td>
</tr>
<tr>
<td>Inflation Rate</td>
<td>98%</td>
<td>2.62%</td>
</tr>
<tr>
<td>Standard Error</td>
<td>5%</td>
<td>0.12%</td>
</tr>
<tr>
<td>Economic Growth</td>
<td>90%</td>
<td>1.09%</td>
</tr>
<tr>
<td>Standard Error</td>
<td>9%</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

*Persistence is defined as the estimated transition probability of staying in the current regime.
Regime persistence

Probability of remaining in event regime

- Equity Turbulence: 60% (Fixed threshold), 90% (Hidden Markov Model)
- Currency Turbulence: 47% (Fixed threshold), 68% (Hidden Markov Model)
- Inflation Rate: 46% (Fixed threshold), 62% (Hidden Markov Model)
- Economic Growth: 46% (Fixed threshold), 68% (Hidden Markov Model)
Probability that the event regime prevails

**Equity Turbulence**
- End of energy crisis
- Recession of early 1980s
- 1987 stock market crash
- Recession of early 1990s
- Dot-com bubble / collapse
- Recent financial crisis

**Currency Turbulence**
- Brief run on USD
- NZD begins to float and USD/GBP speculation
- Plaza Accord
- ERM crisis
- Asian financial crisis
- Russian default
- Sept 11, 2001
- Recent financial crisis
Probability that the event regime prevails

Inflation Rate

- Post-Korean war
- Vietnam war / high Government spending
- Energy crisis and stagflation
- Brief oil price shock
- 2007-2008 oil shock

Economic Growth

- Recession of 1947
- Recession of 1953
- Recession of 1957
- Oil crisis
- Recession of early 1980s
- Recession of early 1990s
- Recession of early 2000s
- Recent financial crisis
Risk premia: in-sample performance*

\[
\frac{(\text{Event Mean} - \text{Non-Event Mean})}{\text{Full Sample Standard Deviation}}
\]

- Global Stocks - Bonds: -2.00
- Equity Mkt Neutral HF - Cash: -1.68
- Emerging - Developed Equity: -1.13
- Small Cap Premium: -0.34
- Equity Momentum: -1.70
- Credit Spread: -2.37
- High Yield Spread: -1.02
- Emerging Market Bond Spread: -1.88
- FX Carry Strategy**: 0.45
- FX Valuation Strategy**: 0.90
- Gold - Cash: 0.78
- TIPS - Nominal Bonds: -1.13
- US Yield Curve (10y-2y): -1.44
- Global Stocks - Bonds: -1.31
- US Cyclical - Non-Cyclical Stocks:

* Time period ends in December 2009 and starts at various points (as early as 1947) depending on data availability.

** Based on Currency Turbulence
Investable risk premia

Out-of-sample performance
Backtest procedure: Investable risk premia

At the beginning of each month in the backtest, we:

1. Calibrate our Markov-Switching model using a growing window of data available up to that point in time.

2. Tilt our risk premia allocation defensively when the model indicates a high probability that an event regime is imminent.

3. Compare the performance of the dynamic risk premia portfolio with the performance of the constant risk premia portfolio.

4. Roll the backtest forward one month and repeat.
## Risk premia tilts

<table>
<thead>
<tr>
<th>Risk Premia</th>
<th>Default Exposure</th>
<th>Turbulence</th>
<th>Recession</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global Stocks – Bonds</td>
<td>10%</td>
<td>-5%</td>
<td>-5%</td>
<td></td>
</tr>
<tr>
<td>Small Cap Premium</td>
<td>10%</td>
<td>-5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity Momentum</td>
<td>10%</td>
<td>-5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity Mk Neutral HF – Cash</td>
<td>10%</td>
<td>-5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emerging – Developed Equity</td>
<td>10%</td>
<td>-5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit Spread</td>
<td>10%</td>
<td>-5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Yield Spread</td>
<td>10%</td>
<td>-5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Yield Curve (10y-2y)</td>
<td>10%</td>
<td></td>
<td>-5%</td>
<td></td>
</tr>
<tr>
<td>Emerging Market Bond Spread</td>
<td>10%</td>
<td>-5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FX Carry Strategy*</td>
<td>10%</td>
<td>-5%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Defensive Trades</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold – Cash</td>
<td>0%</td>
<td></td>
<td>+10%</td>
<td></td>
</tr>
<tr>
<td>TIPS – Nominal Bonds</td>
<td>0%</td>
<td></td>
<td>+10%</td>
<td></td>
</tr>
<tr>
<td>US Non-Cyclical – Cyclical Stocks</td>
<td>0%</td>
<td></td>
<td>+10%</td>
<td></td>
</tr>
<tr>
<td>FX Valuation Strategy</td>
<td>0%</td>
<td></td>
<td>+10%</td>
<td></td>
</tr>
</tbody>
</table>

| Total Notional Exposure                          | 100%             | 55%        | 15%       | 25%       |
Out-of-sample event regime forecasts

**Equity Turbulence**

Mar-78, Mar-80, Mar-82, Mar-84, Mar-86, Mar-88, Mar-90, Mar-92, Mar-94, Mar-96, Mar-98, Mar-00, Mar-02, Mar-04, Mar-06, Mar-08

**Currency Turbulence**

Mar-78, Mar-80, Mar-82, Mar-84, Mar-86, Mar-88, Mar-90, Mar-92, Mar-94, Mar-96, Mar-98, Mar-00, Mar-02, Mar-04, Mar-06, Mar-08
Out-of-sample event regime forecasts

Inflation

Recession
Out-of-sample performance*

<table>
<thead>
<tr>
<th>[Feb 1978 - Dec 2009]</th>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Excess Return</td>
<td>5.99%</td>
<td>6.28%</td>
</tr>
<tr>
<td>Annualized Volatility</td>
<td>8.37%</td>
<td>6.83%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>0.72</td>
<td>0.92</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.56</td>
<td>-1.01</td>
</tr>
<tr>
<td>5% Value-at-Risk</td>
<td>-3.39%</td>
<td>-2.72%</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-41.48%</td>
<td>-32.69%</td>
</tr>
</tbody>
</table>

* Includes transaction costs of 40 basis points. The dynamic strategy turns over approximately 1.5 times per year.
Dynamic asset allocation

Out-of-sample performance
Backtest procedure: Dynamic asset allocation

At the beginning of each month in the backtest, we:

1. Calibrate our Markov-Switching model using a growing window of data available up to that point in time.
2. Tilt our asset allocation defensively when the model indicates a high probability that an event regime is imminent.
3. Compare the performance of the portfolio with dynamic tilts with the performance of the (static) strategic allocation.
4. Roll the backtest forward one month and repeat.

<table>
<thead>
<tr>
<th></th>
<th>Strategic Allocation</th>
<th>Turbulence Tilt</th>
<th>Recession Tilt</th>
<th>Inflation Tilt</th>
<th>Possible Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equity</td>
<td>30%</td>
<td>-5%</td>
<td>-10%</td>
<td></td>
<td>15-30%</td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>30%</td>
<td>-5%</td>
<td></td>
<td></td>
<td>25-30%</td>
</tr>
<tr>
<td>US Government Bonds</td>
<td>20%</td>
<td>+5%</td>
<td>+10%</td>
<td>-5%</td>
<td>15-35%</td>
</tr>
<tr>
<td>US Corporate Bonds</td>
<td>20%</td>
<td>+5%</td>
<td></td>
<td>-5%</td>
<td>15-25%</td>
</tr>
<tr>
<td>Cash</td>
<td>0%</td>
<td></td>
<td>+10%</td>
<td></td>
<td>0-10%</td>
</tr>
</tbody>
</table>
Performance results

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Return</td>
<td>9.45%</td>
<td>9.29%</td>
<td></td>
</tr>
<tr>
<td>Annual 5% Value-at-Risk</td>
<td>-10.44%</td>
<td>-8.34%</td>
<td></td>
</tr>
<tr>
<td>Return-to-VaR</td>
<td>0.90</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>Annualized Volatility</td>
<td>9.88%</td>
<td>8.98%</td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.36</td>
<td>-0.34</td>
<td></td>
</tr>
<tr>
<td>Worst Year</td>
<td>-34.93%</td>
<td>-29.51%</td>
<td></td>
</tr>
</tbody>
</table>

* Includes transaction costs of 40 basis points. Average yearly turnover associated with the dynamic tilts is 34%.
## Drawdown analysis: the five worst drawdown periods

<table>
<thead>
<tr>
<th></th>
<th>Static Allocation</th>
<th>With Dynamic Tilts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Maximum Loss</strong></td>
<td><strong>Length of Drawdown</strong></td>
<td><strong>Maximum Loss</strong></td>
</tr>
<tr>
<td>-35.2%</td>
<td>Ongoing</td>
<td>-30.1%</td>
</tr>
<tr>
<td>-27.7%</td>
<td>34</td>
<td>-25.7%</td>
</tr>
<tr>
<td>-21.6%</td>
<td>45</td>
<td>-17.1%</td>
</tr>
<tr>
<td>-12.8%</td>
<td>14</td>
<td>-12.0%</td>
</tr>
<tr>
<td>-11.9%</td>
<td>14</td>
<td>-10.8%</td>
</tr>
</tbody>
</table>
Summary

• We employ a Markov-Switching process to model economic conditions as opposed to directly modeling asset returns.

• Our results confirm that inflation, economic growth, and market turbulence are persistent and are directly and intuitively linked to asset performance.

• We find that dynamic allocation to investable risk premia based on regime forecasts outperforms constant exposure.

• We find that dynamic asset allocation based on regime forecasts outperforms static asset allocation.
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