Models Behaving Badly

QWAFAFEW

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The Trouble With Finance

Fundamental Theorem of Finance. *Security prices exclude arbitrage if and only if there exists a strictly positive value functional, under the technical restrictions that the space of portfolios and the space of contingent claims are locally convex topological vector spaces and the positive cone of the space of contingent claims is compactly generated, that is, there exists a compact set K of X (not containing the null element of X) such that*

$$C = \{x \in X : x \geq 0\} = \bigcup_{\lambda \geq 0} \lambda K.$$

- The fundamental theorem of arithmetic: Every natural number greater than 1 can be written as a unique product of prime numbers.
- The fundamental theorem of algebra: Every polynomial equation of degree n with complex number coefficients has n complex roots.
- Economists don't understand the difference between theorems and laws.
- Economists have fallen in love with mathematics, rigor and formalism for their own sake, irrespective of their efficacy. It's not their fault that they can't find better models; economics is a social science and people are difficult to theorize about. It's their fault that they don't understand the difference.

Metaphors

- *Sleep is the interest we have to pay on the capital which is called in at death; and the higher the rate of interest and the more regularly it is paid, the further the date of redemption is postponed."*
  - Arthur Schopenhauer
- *Periodicity* of *sleep* and *coupons* is the common element of the metaphor.
- Metaphors are an insight that state that something *is something else.*
- But metaphors also extend our understanding:
  - Analytic continuation of the factorial function.
- Much of our knowledge is a layer of metaphors.
- Metaphors and language build on our physical nature:
  - Elation/depression
  - Light/dark

Models

- Models are elaboration of metaphorical insights.
- Model airplanes: what makes it a model?
  Limited but significant resemblance to properties that are important to user.
- Fashion model: Only the surface is important.
- Weather model: measurable abstractions like pressure, temperature, and known/tested equations; limitations are the omissions.
- Economic model: invisible abstractions like supply, demand, untested equations; limitations are the concepts and equations and the omissions.
- Liquid Drop Model of the Nucleus.
- The Standard Model: unifies previously disconnected theories via a symmetry; predicts the existence of previously unobserved interactions and particles; more than a model.

Why Is A Model A Model?

- A model is not the thing in itself: there is a gap between the model and the system.
  - It is a caricature.
  - It focuses on parts.
  - It is a metaphor with limited applicability.
  - It is a fetish, and therefore attractively dangerous.
- Models transform consciously learned knowledge into unconscious visceral knowledge.
- One must digest previous metaphors to move up the hierarchy to wider ones.
- Models are labor-saving devices, allow you to avoid thinking/working for yourself.
  - Feynman diagrams, like silicon chips, bring computation to the masses.
- Models reduce dimensionality
- Models extrapolate or interpolate
- Sometimes it's time to make unconscious knowledge conscious again.

Why Is A Theory A Theory?

- Models are *analogies* and *relative*. Theories are **the real thing**.
- A theory is the ultimate non-metaphor: אֶהְיֶה אֲשֶׁר אֶהְיֶה *I am that which I am.*
- Theories tell you what something **is**. Models tell you what something is **more or less** like.
- The Dirac equation for the electron: $(-i\partial\!\!\!/ + m)\psi = 0$
- Maxwell's Equations

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \cdot \mathbf{D} = \rho$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

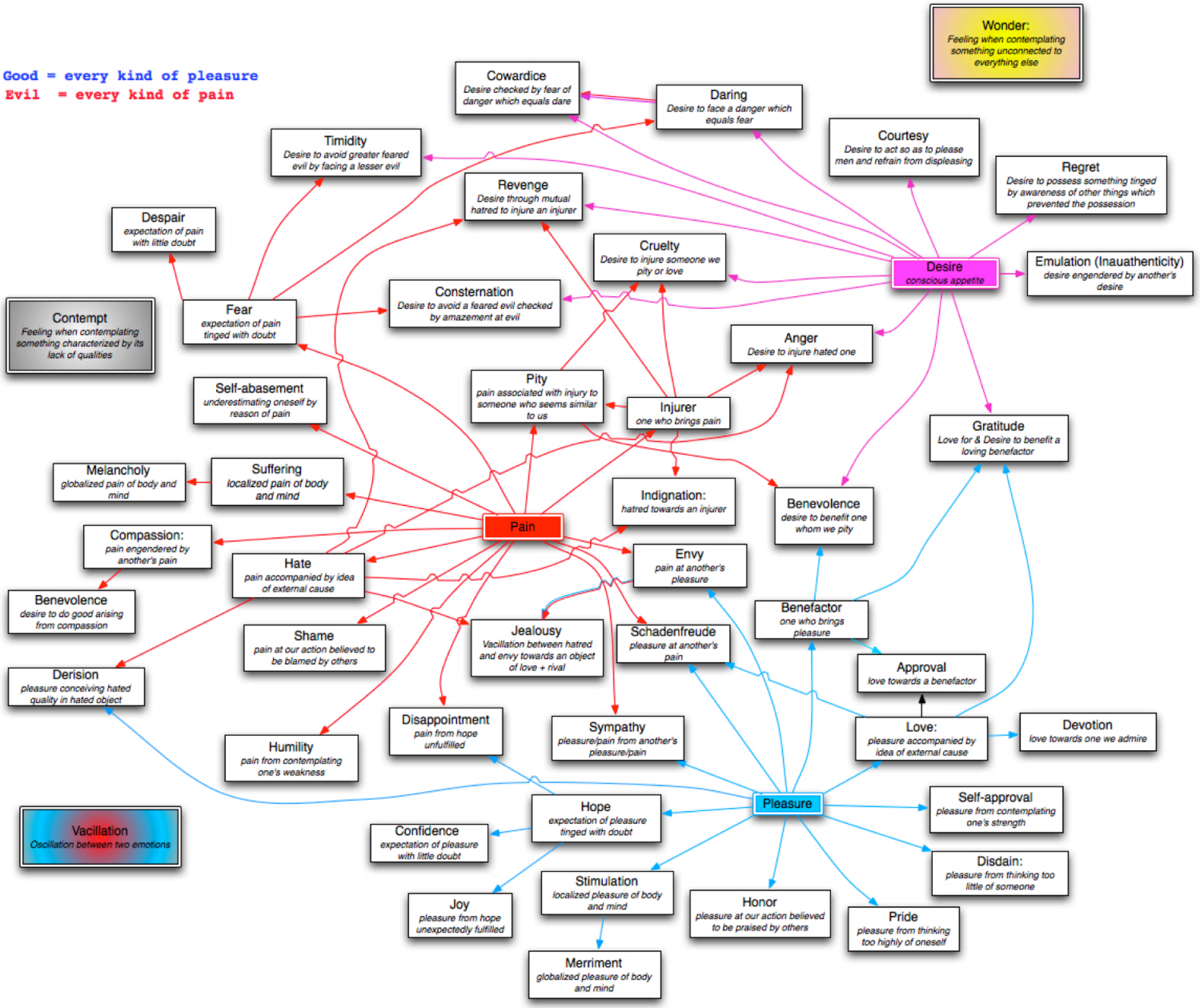
Feynman's Nobel Prize Lecture:
Many different physical ideas can describe the same physical reality. Thus, classical electrodynamics can be described by a field view, or an action at a distance view, etc.
Originally, Maxwell filled space with idler wheels, and Faraday with fields lines, but somehow the Maxwell equations themselves are pristine and independent of the elaboration of words attempting a physical description.

- Why is the Dirac equation a **theory**?
- A theory provides **non-metaphorical** insight, deals in **absolutes**; hence often uses math.
- Theories are deep, models are shallow. Nothing wrong with that.
- A correct theory is almost indistinguishable from the world.

An Example: Spinoza's Theory of Emotions ...

- Spinoza's treats emotions like Euclid treats geometry: emotions are derivatives.
  - Primitives are Desire, Pleasure, Pain.
- *Good* is everything that brings pleasure, and *Evil* is everything that brings pain.
- Love: Pleasure associated with an external object.
- Hate: Pain associated with an external object.
- Envy: Pain at another's Pleasure.
- Hope: Expectation of future Pleasure tinged with doubt.
- Fear: Expectation of future Pain.
- Cruelty: Desire to inflict Pain on a someone Loved.
- Three more primitives:
  - Vacillation, Wonder, Contempt.

A Non-Physics Theory …Spinoza's Theory of Emotions as Derivatives



Intuition

- It takes intuition to discover the nature of the world.
  - Kepler, Newton, Ampere, Maxwell.
- Intuition may sound casual but it takes intimate knowledge of the world acquired by careful observation and painstaking effort.
- Keynes on Newton:

  "I believe that the clue to his mind is to be found in his unusual powers of continuous concentrated introspection … His peculiar gift was the power of holding continuously in his mind a purely mental problem until he had seen straight through it."
- Maxwell on Ampére

  "We can scarcely believe that Ampere really discovered the law of action by means of the experiments which he describes. We are led to suspect, what, indeed, he tells us himself, that he discovered the law by some process which he has not shown us, and that when he had afterwards built up a perfect demonstration, he removed all traces of the scaffolding by which he had built it."

Models in Finance

- The point is not usually divination, which rarely works.
- A typical valuation model: apartment pricing & calibration.
- Models transform intuitive linear quantities into nonlinear dollar values.
  - Price per square foot to apartment price.
  - Future yield to bond price.
  - Future volatility to option price.
- Models in finance calibrate the future, predict the present.
- Models are used to ranks securities by value on a 1-D scale.
- Models interpolate from liquid prices to illiquid ones.

Scientific Foundations of Financial Valuation

- Science (mechanics, electrodynamics) is reductive, seeks principles.
- Engineering is constructive.
- From Newton to Mechanical Engineering.
- From Maxwell to Electrical Engineering.
- From ??? to Financial Engineering?
- Jim Simons vs. Robert Engle.

Finance vs. Science

- Scientists carry out repeatable experiments on isolated systems. History and environment have weak effects.
- Economics deals with coupled systems where history is important and experiments can rarely be repeated.
- Stocks are risky, models are uncertain. Uncertainty is worse than risk.
- Science plays against God.
- Finance plays against people.

The One Law of Financial Modeling

- If you want to know the value of a financial security, use the known price of another security that's as similar to it as possible. All the rest is modeling.
- Any two securities with identical future payoffs, no matter how the future turns out, should have identical current prices.
- To build a model:
  - Specify all future payoffs (science).
  - Prove similarity of payoffs (engineering).

The Efficient Market Model

- Jiu-Jitsu Finance: cf. The Anthropic Principle
  - Pretend a stock is atomic
  - Price = Value
  - Uncertainty = Quantifiable Risk
  - Risk = standard deviation of returns
  - Returns are normally distributed; there is only one kind of risk, diffusion risk
  - **The Question**: What mean return should you expect?
  - The existence of a riskless bond + The law of one price lets you answer the question
  - Engineering: A riskless bond + a risky bond = a less risky one
  - Therefore: for all stocks $\frac{\text{excess return}}{\text{risk}} \equiv \frac{\mu - r}{\sigma} = \lambda$
  - Black-Scholes: Return per unit of risk is same for option and stock;.
  - Brownian motion is a theory for pollen, a model for stocks.
  - Diffusion risk isn't wild enough
  - People know this is naïve, but giving up simplicity isn't always worth it if you don't know what is better.
  - Limit losses rather than estimate their probability

The Right Way to Use Valuation Models:

- Assume the bare minimum: static replication, dynamic replication, risk replication, utility.
- Avoid axiomatization.
- In physics it pays to drop down deep, formulate a principle, then come back up again. In finance, shallow is better.
- Use vulgar variables in a sophisticated way (The Vanna-Volga Method).
- Sweep dirt under the rug, but tell people about it.
- Think of models as *Gedanken* experiments.

Example: Valuing Illiquid Products

- A model is a simplification. There is no correct model. Use a variety of similar but not identical models to estimate value and hedge ratios.
- Does the model contain an accurate description of the terms of the derivative's payoff?
- Does the model provide a plausible description of the factors that affect value?
- Has the model been calibrated to the observed behavior and prices of liquid constituents?
- P&L simulation: Illiquid derivatives may have to be hedged to expiration.
  - The P&L is uncertain, subject to transactions costs, market moves and hedging error.
  - Simulate market behavior and hedging strategy to create histograms of P&L at expiration.
  - There is a tension between the increased accuracy and the increased cost.
  - The P&L distributions indicate how much can be withheld until the trade is unwound. This varies as an option moves in or out of the money, or as volatility changes.

Conclusion

- There is no TOE; you're lucky if there's a TOA.
  To confuse the model with the world is to embrace a future disaster driven by the belief that humans obey mathematical rules.

- *If a fool would persist in his folly he would become wise.*

- A little hubris is good.

- Catastrophes strike when hubris evolves into idolatry. Somewhere between these two extremes, a little north of common sense but still south of idolatry, lies the wise use of conceptual models.

The Financial Modelers' Manifesto

- I will remember that I didn't make the world, and it doesn't satisfy my equations.
- Though I will use the models I or others create to boldly estimate value, I will always look over my shoulder and never forget that the model is not the world.
- I will not be overly impressed by mathematics. I will never sacrifice reality for elegance without explaining to its end users why I have done so.
- I will not give the people who use my models false comfort about their accuracy. I will make the assumptions and oversights explicit to all who use them.
- I understand that my work may have enormous effects on society and the economy, many beyond my apprehension.
- MODELERS OF ALL MARKETS, UNITE!
  You have nothing to lose but your illusions.