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The Triumph of Mediocrity: A Case Study of "Naïve Beta"

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PanAgora Asset Management

Definition

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What do they mean?

- “Naïve”
  - showing unaffected simplicity; a lack of judgment, or information
- “Smart”
  - showing intelligence or good judgment
- “Mediocrity”
  - of ordinary or moderate quality; neither good nor bad; barely adequate
- “Why all the quotation marks”?
  - Few things are what they seem in investment industry.

Definition

What do they really mean?

- “Securities”
  - Risky investments
- “High yield bonds”
  - Junk bonds
- “Private equity”
  - Leveraged buyout, accounting arbitrage
- “60/40 balanced funds”
  - Portfolios with non-diversified equity risk
- “Hedge funds”
  - Unhedged investments for regressive wealth distribution
- “Smart beta”
  - ???

Definition

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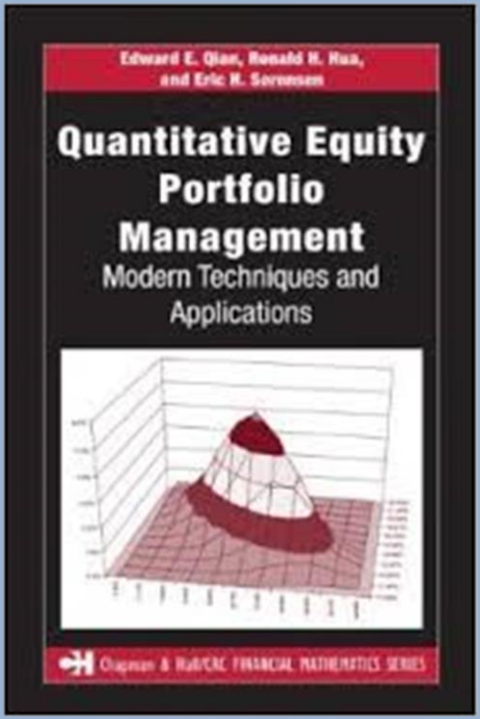
"Smart beta"

- » Quantitative Equity Portfolio Management
  - » Co-authors Ron Hua, Eric Sorensen
  - » 1st Edition – May 11 2007
- » Second edition?
  - » New chapters on "smart beta"
- » "Smart beta"
  - » Factor-based
  - » Diversification-based



CHAPTER 13

FACTOR-BASED "SMART BETA"

13.1 Please refer to chapter 5 on quantitative equity factors

13.2 Discard risk model

13.3 Use equal-weighting or capitalization-weighting method

13.4 Call it smart beta, scientific beta, advanced beta, exotic beta, or indexing

13.5 Make no reference to active quantitative equity

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Naïve beta

- What are naïve betas?
- Why do they outperformed the S&P 500 index?
- Not all naïve betas are created equal

Naïve Beta

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Everybody is mediocre in someway

» Dimension of equality and corresponding naïve beta
    » Equal weight: Equal weight (EQ)
    » Equal expected return: Minimum variance (MV)
    » Equal risk-adjusted return: Maximum diversification (MD)
    » Equal risk contribution: Risk parity (RP)

Naïve Beta

Min variance

- Same expected return, then mean-variance optimal portfolio is min variance portfolio

$$\mathbf{i} = (1, \cdots, 1)'$$

$$\min \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}, \text{subject to } w_1 + w_2 + \cdots + w_N = 1.$$

$$\mathbf{w}' \cdot \mathbf{i} = 1$$

$$\mathbf{w}_{\text{MV}} = \frac{1}{\lambda_{\text{MV}}} \boldsymbol{\Sigma}^{-1}\mathbf{i} = \frac{1}{\left(\mathbf{i}'\boldsymbol{\Sigma}^{-1}\mathbf{i}\right)} \boldsymbol{\Sigma}^{-1}\mathbf{i}.$$

Max diversification

» Same risk-adjusted return, then mean-variance optimal portfolio is maximum diversification portfolio

$$\mu_i = k\sigma_i, i = 1, \cdots, N \qquad \boldsymbol{\mu} = k\boldsymbol{\sigma}$$

$$\mathbf{w}_{\mathrm{MD}} = \frac{1}{\lambda_{\mathrm{MD}}} \boldsymbol{\Sigma}^{-1}\boldsymbol{\sigma} = \frac{1}{\left(\mathbf{i}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\sigma}\right)} \boldsymbol{\Sigma}^{-1}\boldsymbol{\sigma}$$

Naïve Beta

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Risk parity

- Same risk contribution leads to risk parity portfolio
  - Risk contribution = weight x marginal contribution

$$\mathbf{RC} = \mathbf{w} \otimes (\boldsymbol{\Sigma}\mathbf{w})$$

$$\mathbf{w}_{\mathrm{RP}} \otimes (\boldsymbol{\Sigma}\mathbf{w}_{\mathrm{RP}}) = \lambda_{\mathrm{RP}}\mathbf{i}.$$

A Case Study

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Naïve beta versus the S&P 500 index

- Data
  - From Jan 1990 to Nov 2014
  - Monthly return for the S&P 500 index
  - Monthly return for the 10 S&P 500 index sectors
  - Monthly sector weights

- Backtest for EQ/MV/MD/RP
  - Long-only, fully invested
  - In sample
  - Out-of-sample from Jan 1992 to Nov 2014
    - Update covariance matrix (half-life 5 years)

A Case Study

Naïve beta versus the S&P 500 index

- Return statistics
  - High risk sectors: FIN/TEC
  - Low risk sectors: CSS/HLT/UTL

	Return	Volatility	Sharpe Ratio
Consumer Staples (CSS)	**11.40%**	**13.34%**	**0.58**
Consumer Discretionary (CSD)	10.32%	17.86%	0.37
Energy (ENE)	10.67%	18.29%	0.38
Financials (FIN)	**8.62%**	**21.98%**	**0.23**
Health Care (HLT)	**12.24%**	**15.68%**	**0.54**
Industrials (IND)	10.10%	17.41%	0.37
Information Technology (TEC)	**11.06%**	**25.39%**	**0.29**
Materials (MAT)	8.20%	19.92%	0.23
Telecommunication Services (TEL)	5.89%	19.26%	0.12
Utilities (UTL)	8.12%	15.05%	0.30

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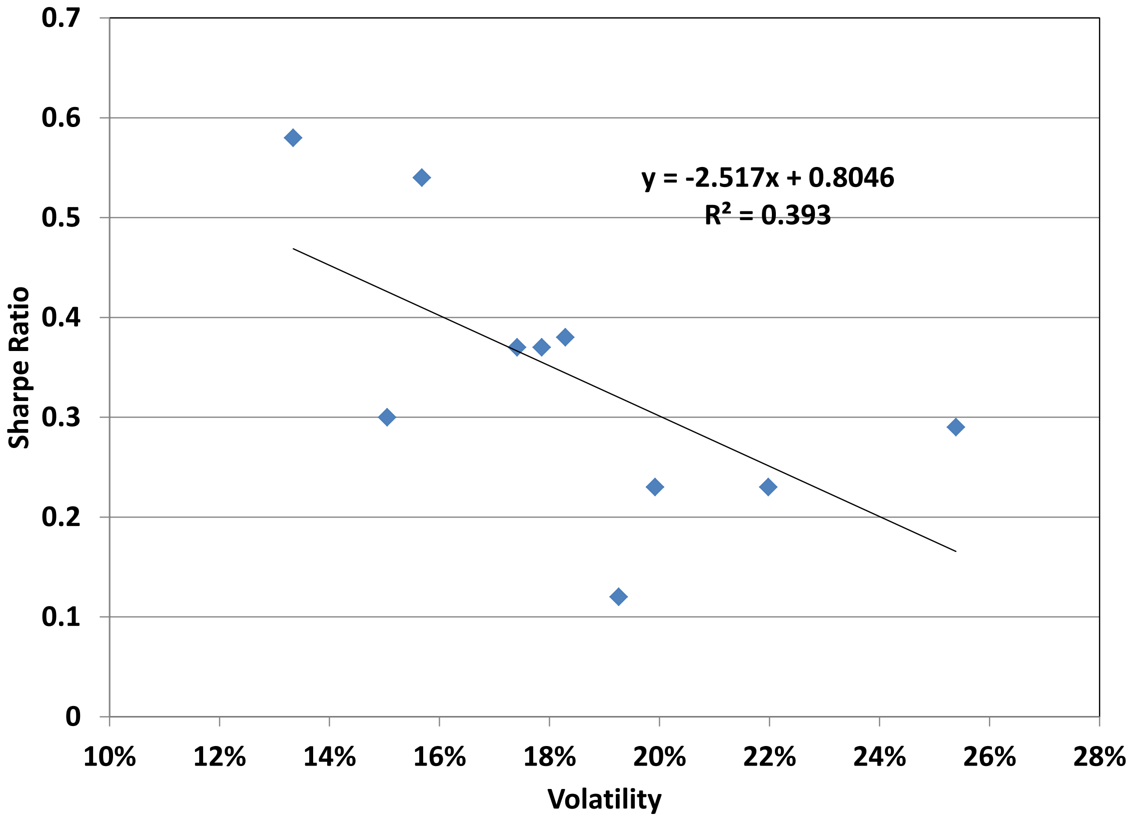
Naïve beta versus the S&P 500 index

- Return statistics
  - CAPM was wrong



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Naïve beta versus the S&P 500 index

- Correlation matrix
  - Cyclical sectors tend to have high correlations with each other

	CSS	CSD	ENE	FIN	HLT	IND	TEC	MAT	TEL	UTL
CSS	1.00	0.56	0.37	0.61	0.71	0.59	0.29	0.49	0.40	0.44
CSD	0.56	1.00	0.44	0.78	0.51	0.85	0.71	0.74	0.53	0.28
ENE	0.37	0.44	1.00	0.48	0.36	0.58	0.37	0.64	0.32	0.50
FIN	0.61	0.78	0.48	1.00	0.59	0.81	0.52	0.69	0.45	0.39
HLT	0.71	0.51	0.36	0.59	1.00	0.55	0.38	0.44	0.41	0.40
IND	0.59	0.85	0.58	0.81	0.55	1.00	0.66	0.83	0.50	0.40
TEC	0.29	0.71	0.37	0.52	0.38	0.66	1.00	0.54	0.49	0.16
MAT	0.49	0.74	0.64	0.69	0.44	0.83	0.54	1.00	0.39	0.33
TEL	0.40	0.53	0.32	0.45	0.41	0.50	0.49	0.39	1.00	0.35
UTL	0.44	0.28	0.50	0.39	0.40	0.40	0.16	0.33	0.35	1.00
Avg	**0.55**	**0.64**	**0.50**	**0.63**	**0.53**	**0.68**	**0.51**	**0.61**	**0.49**	**0.42**

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Naïve beta versus the S&P 500 index

» In sample results sector weights

	MV	MD	RP
Consumer Staples (CSS)	**42.6%**	**16.9%**	13.4%
Consumer Discretionary (CSD)	1.8%	0.0%	8.7%
Energy (ENE)	9.2%	**13.3%**	10.4%
Financials (FIN)	0.0%	0.0%	7.1%
Health Care (HLT)	6.2%	**10.4%**	11.6%
Industrials (IND)	0.0%	0.0%	8.4%
Information Technology (TEC)	3.8%	**16.6%**	7.5%
Materials (MAT)	0.0%	3.2%	8.2%
Telecommunication Services (TEL)	7.7%	**12.9%**	10.2%
Utilities (UTL)	**28.9%**	**26.6%**	14.5%

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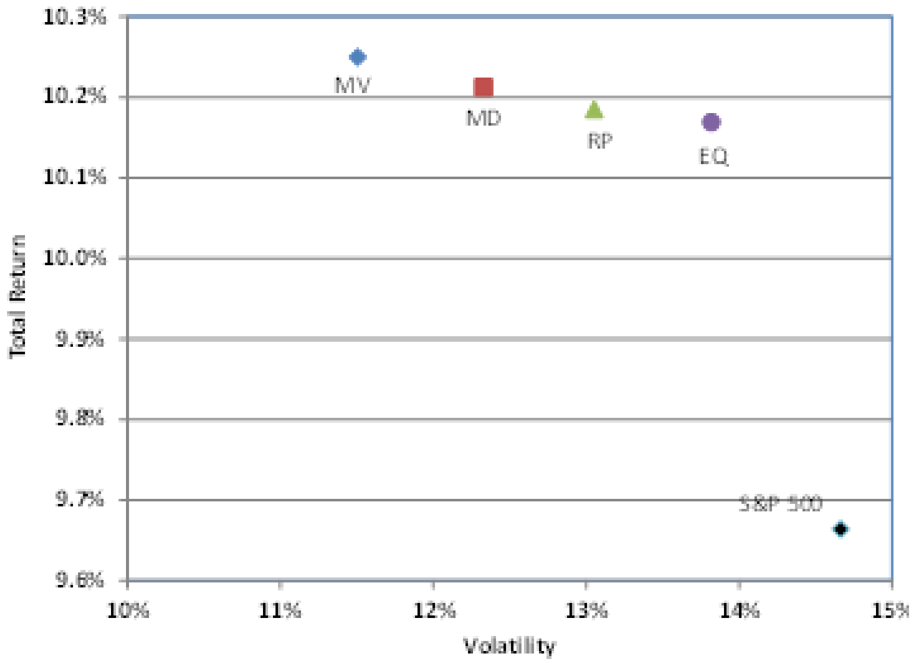
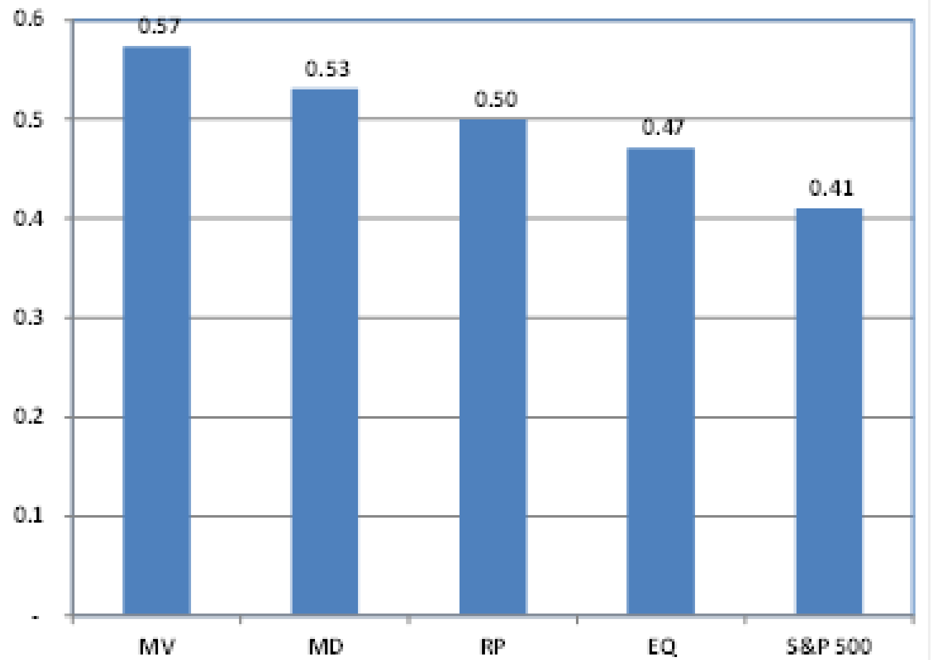
Naïve beta versus the S&P 500 index

- In sample results – performance
    - Turnover 30-35% two-way





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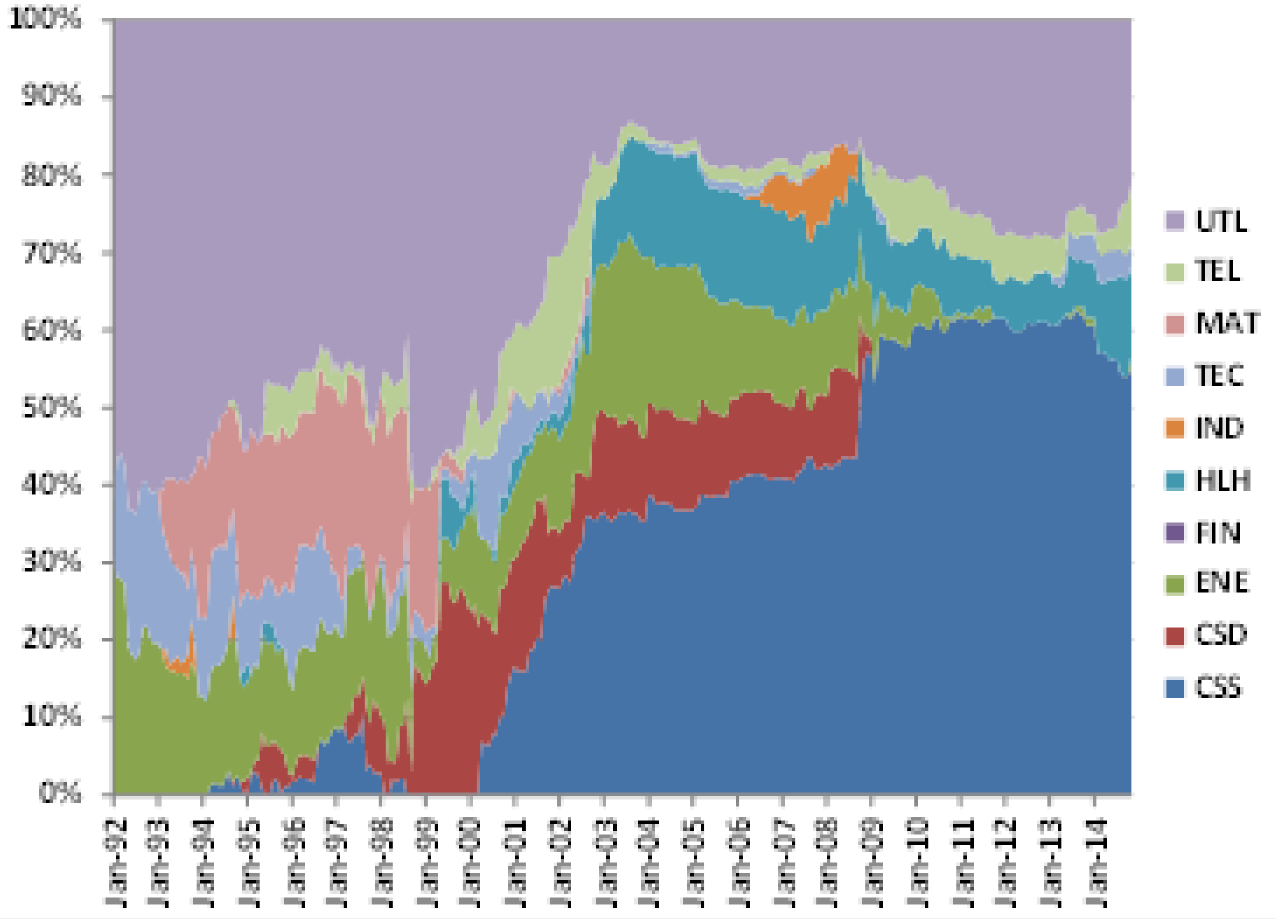
Naïve beta versus the S&P 500 index

» Out-of-sample results – MV sector weights

» Dominated bv UTL and CSS



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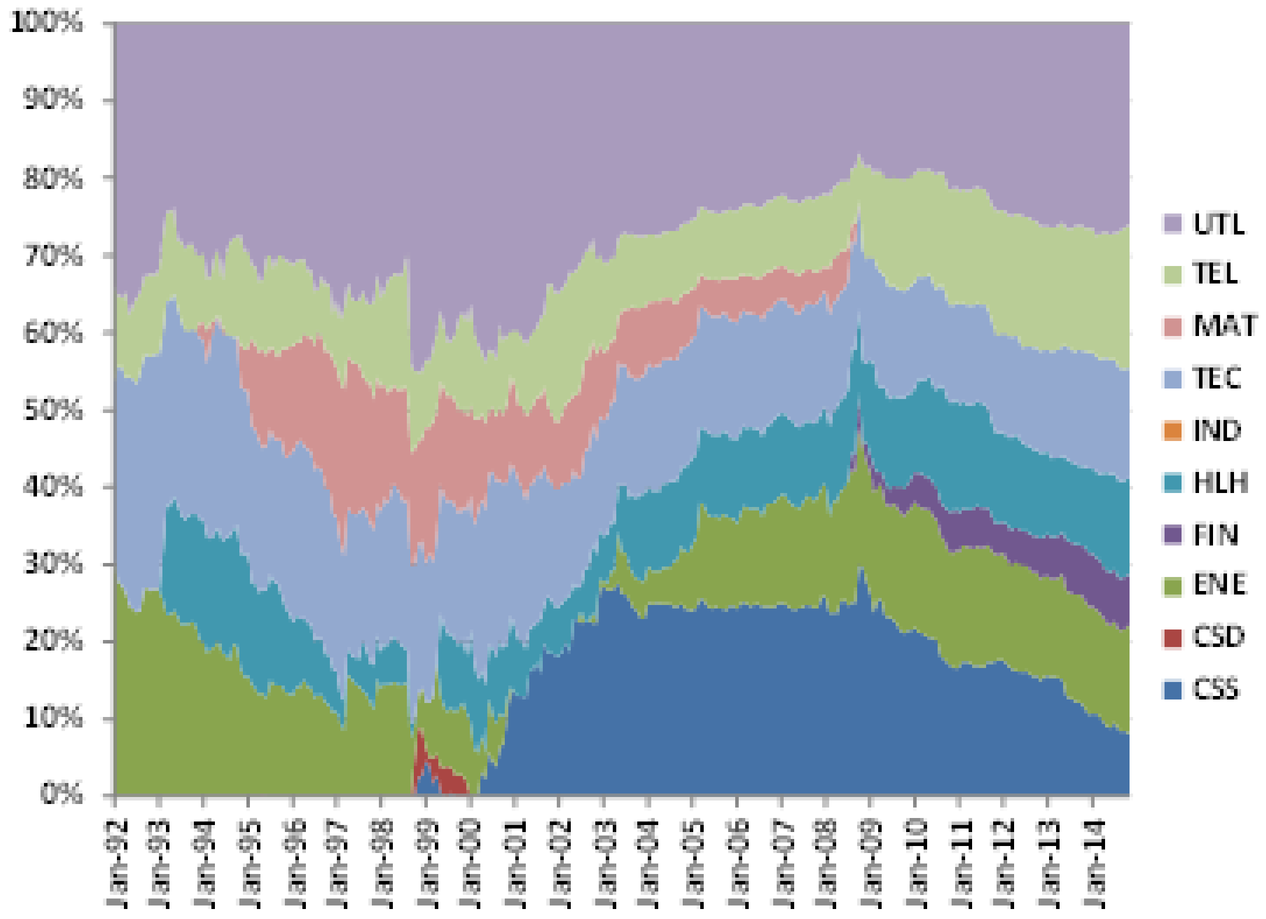
Naïve beta versus the S&P 500 index

» Out-of-sample results – MD sector weights

» UTL/TEL/TEC/HLH/ENE/CSS



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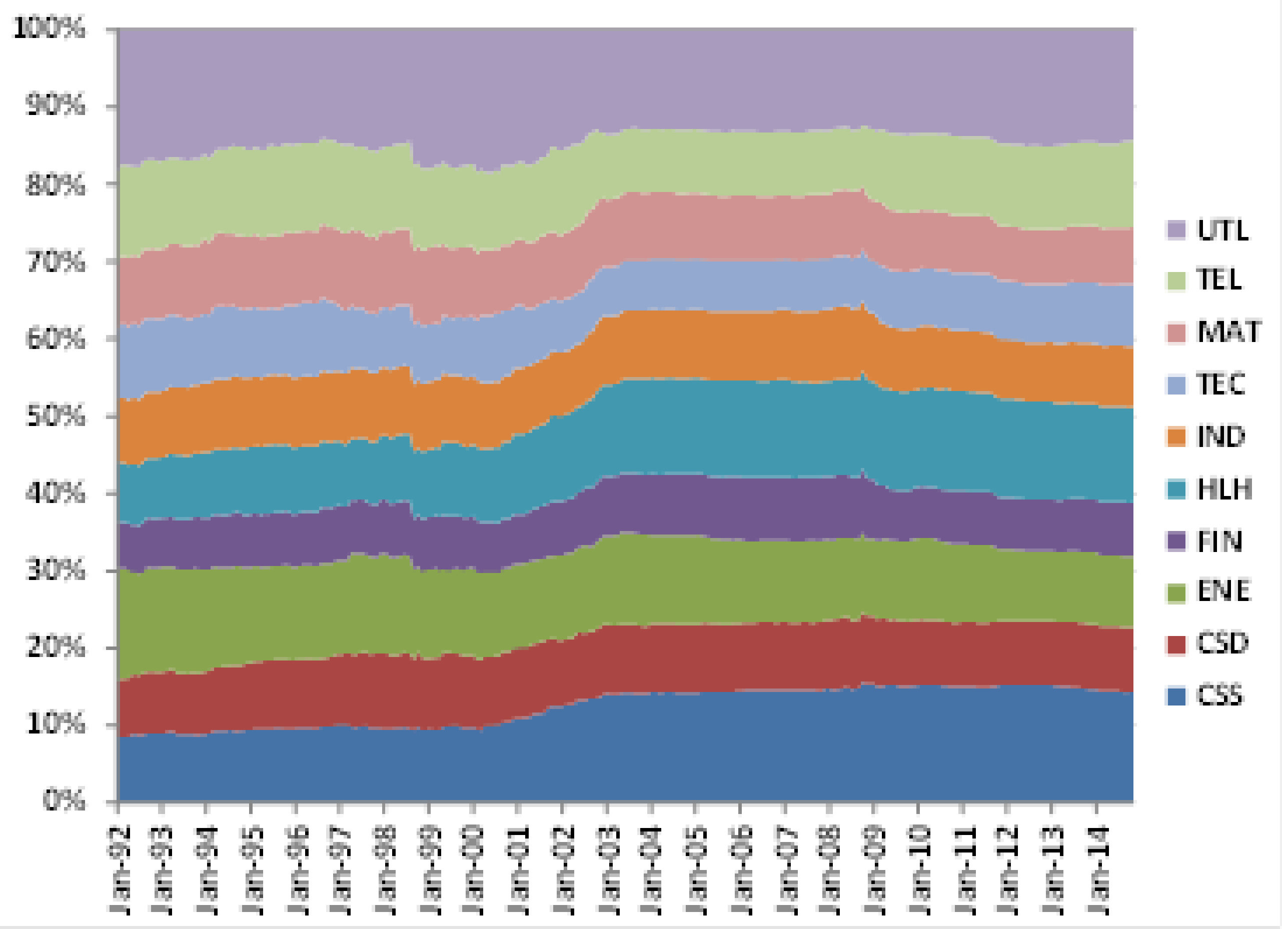
Naïve beta versus the S&P 500 index

» Out-of-sample results – RP sector weights



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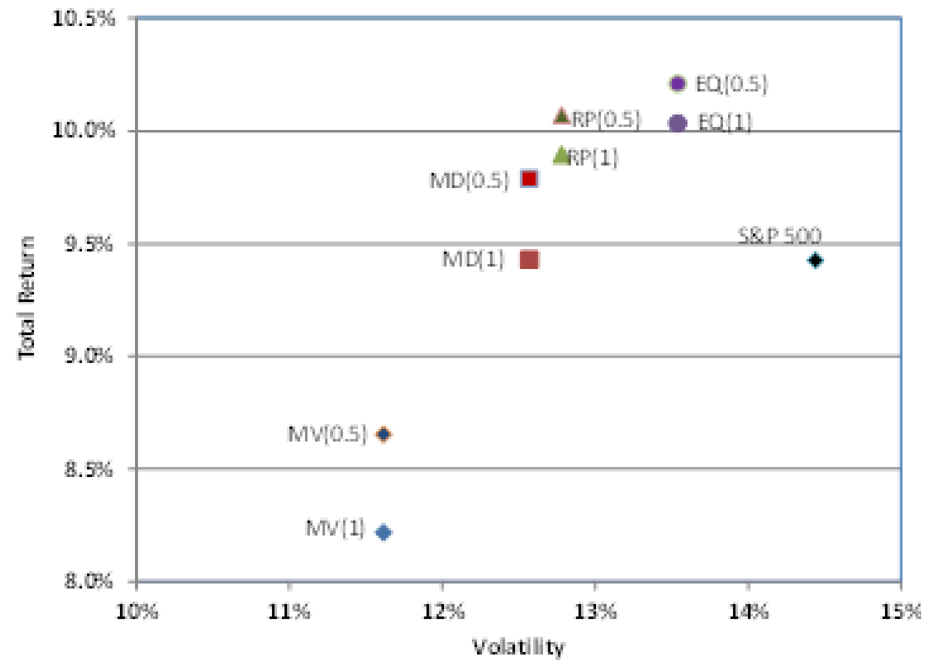
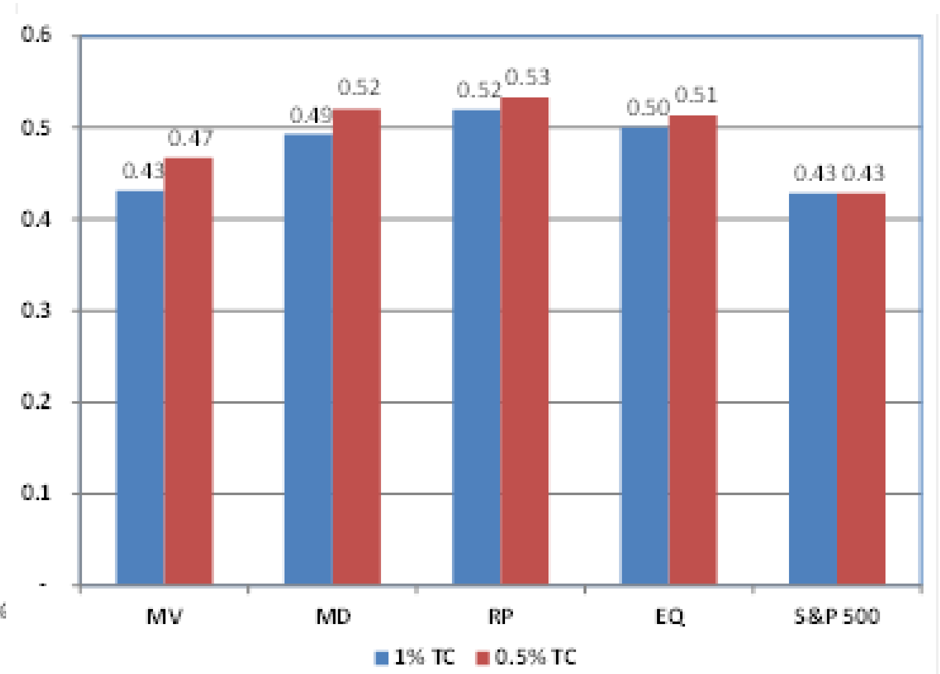
Naïve beta versus the S&P 500 index

- » Out-of-sample results – performance
  - » Turnover – MV 77% MD 64% RP 32%





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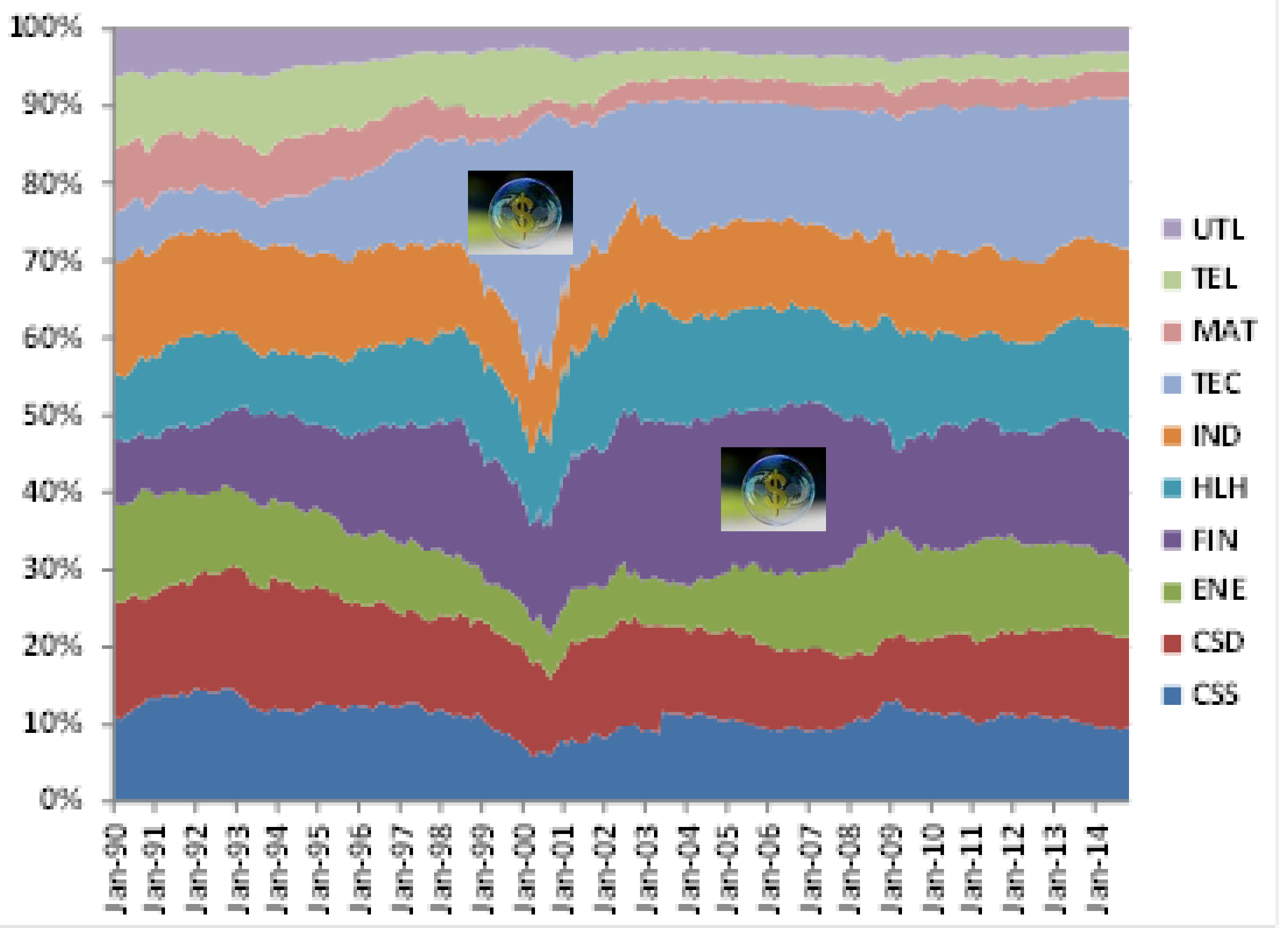
Naïve beta versus the S&P 500 index

» Out-of-sample results – the S&P index sector weights



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Naïve beta versus the S&P 500 index

- Four naïve betas outperformed the index in sample
  - In order of Sharpe MV/MD/RP/EQ/Index
- Four naïve betas still outperformed the index out-of-sample
  - In order of Sharpe RP/EQ/MD/MV/Index
- Sector perspective
  - The index is dominated by cyclical sectors
  - MV is concentrated in low-vol sectors: CSS/UTL
  - MD is concentrated in defensive sectors plus TEC/ENE
  - RP is balanced with tilts to low-vol sectors

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What's wrong with the S&P 500 index?

- Nothing is wrong
  - Rooted in Nobel-prize winning theory – efficient market hypothesis, pretty "smart"
  - Most of active managers don't beat the index

- Something is wrong
  - Why naïve betas beat the index?
  - Why the index is loaded with cyclical sectors?
  - High volatility, tail risks
  - Not truly diversified
  - Is it really passive?

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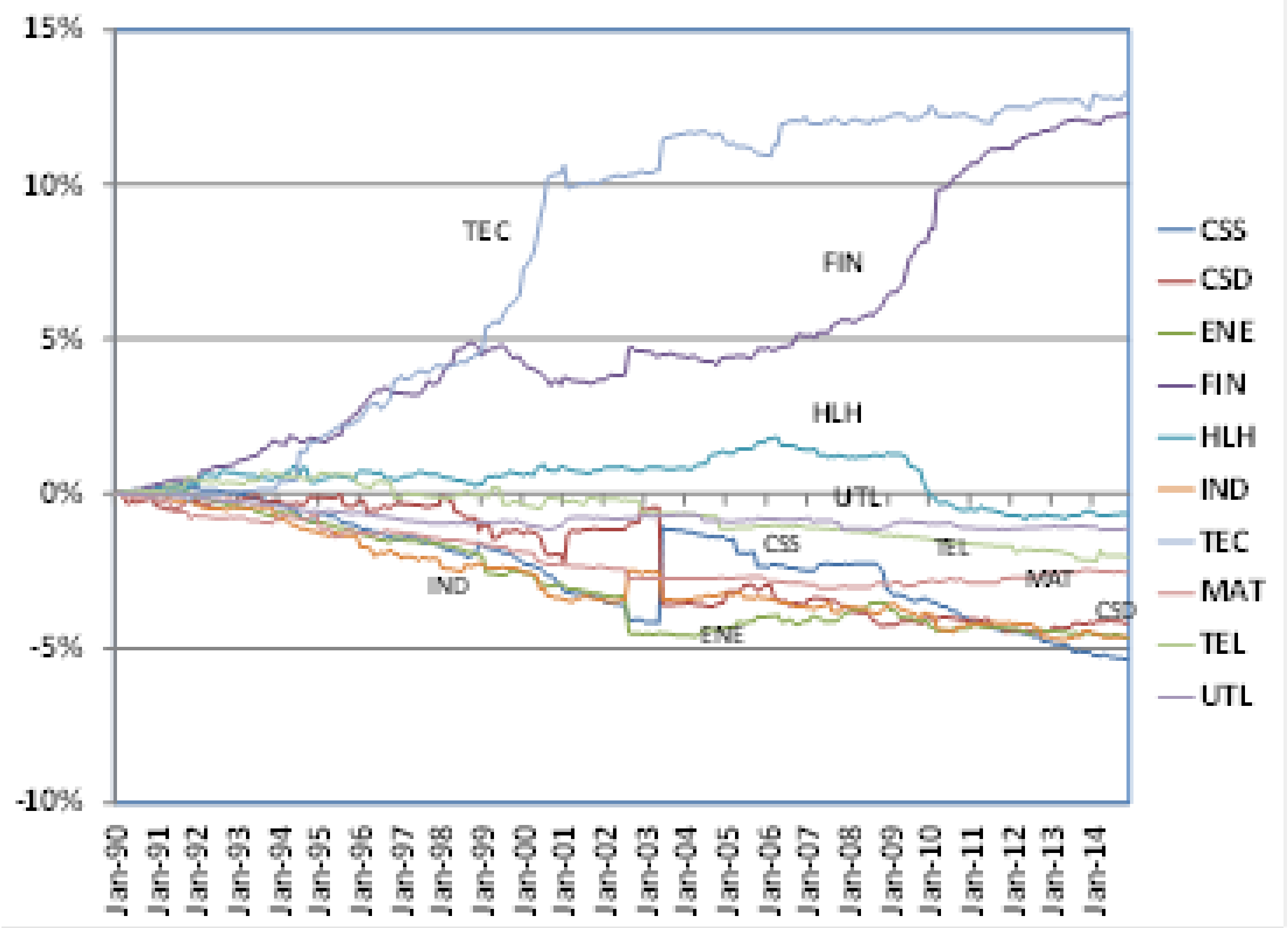
What's wrong with the S&P 500 index?

» Cumulative sector weight change net of drift



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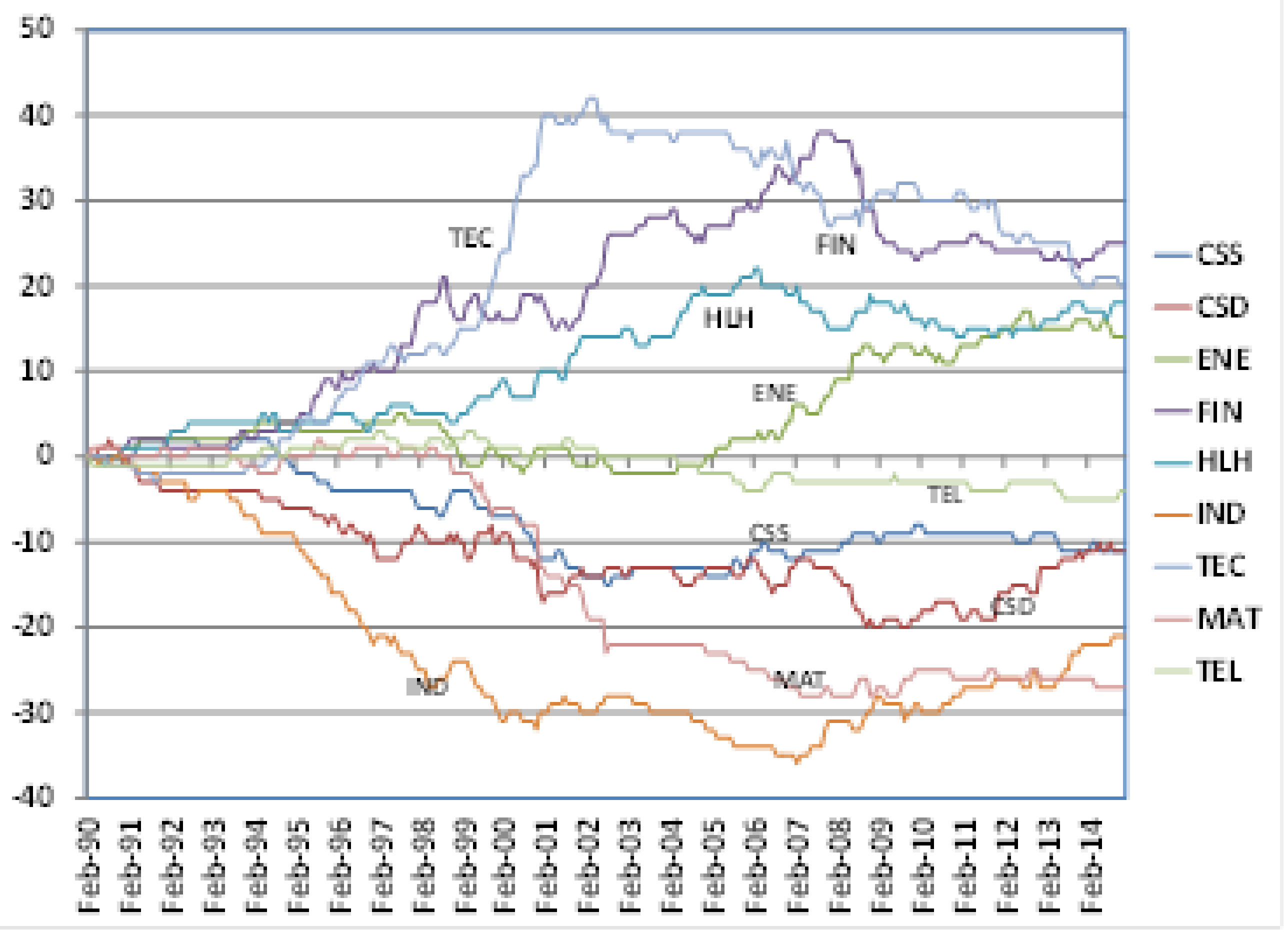
What's wrong with the S&P 500 index?

» Cumulative # of name changes in the sectors



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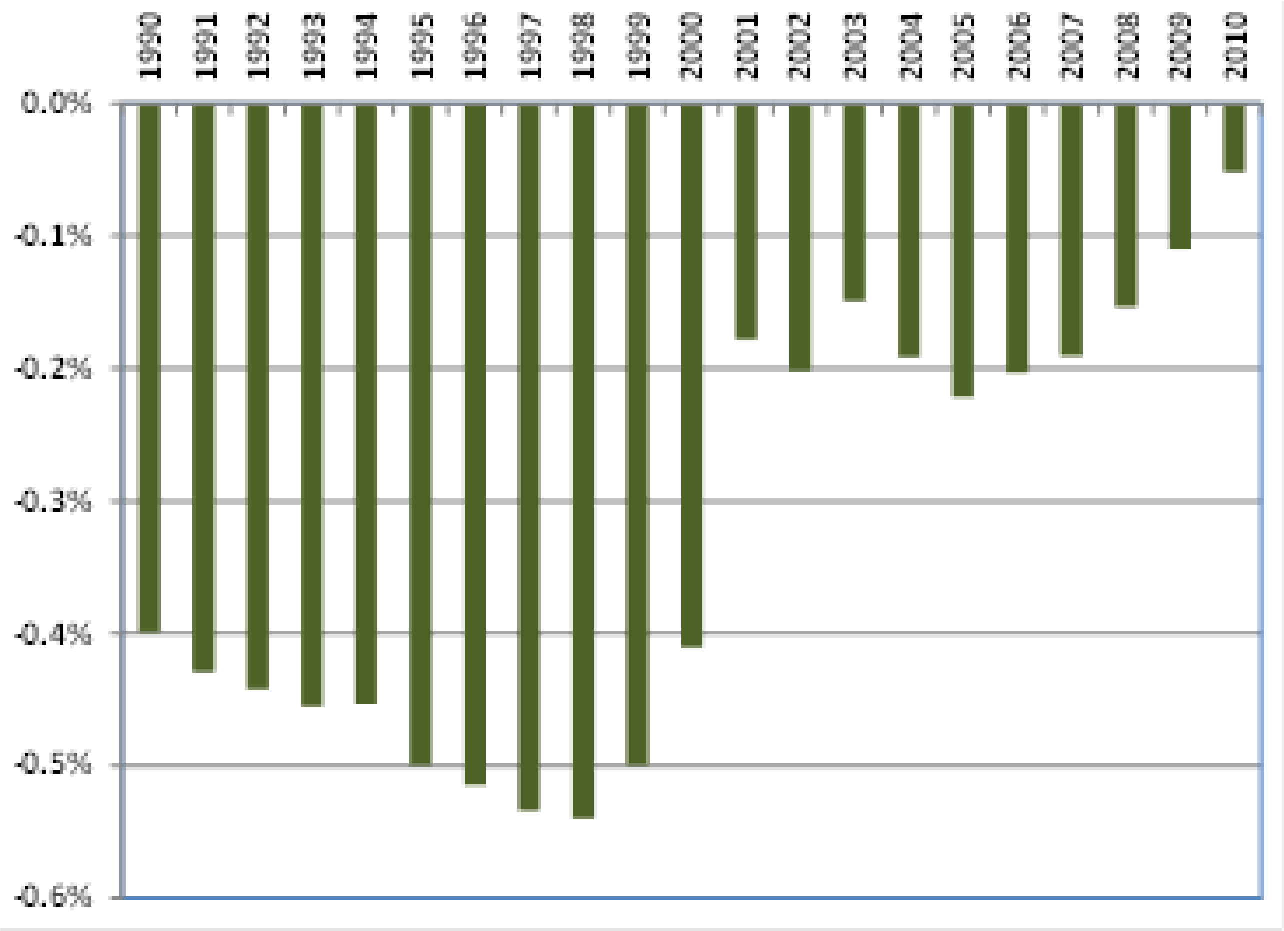
What's wrong with the S&P 500 index?

» "Value added" of sector shifts by the S&P 500 index



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Comparison of three risk-based naïve betas

- MV/MD/RP all use risk models
  - MV/MD use optimization
  - RP uses risk budgeting – no optimization

- MV is concentrated in low vol sectors
- MD is concentrated in defensive sectors with a couple of cyclical sectors
- RP is balanced in sectors with a tilt to low vol sectors

- None of the theoretical solutions is easy to solve

Comparison of three risk-based naïve betas

» Solutions recap

$$\mathbf{w}_{\text{MV}} \propto \boldsymbol{\Sigma}^{-1}\mathbf{i}.$$

$$\mathbf{w}_{\text{MD}} \propto \boldsymbol{\Sigma}^{-1}\boldsymbol{\sigma}$$

$$\mathbf{w}_{\text{RP}} \otimes \left(\boldsymbol{\Sigma}\mathbf{w}_{\text{RP}}\right) \propto \mathbf{i}.$$

$$\boldsymbol{\Sigma}^{-1} = \mathrm{diag}\left(\boldsymbol{\sigma}^{-1}\right)\cdot\mathbf{C}^{-1}\cdot\mathrm{diag}\left(\boldsymbol{\sigma}^{-1}\right).$$

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Comparison of three risk-based naïve betas

» Decomposition of covariance matrix into correlation matrix and volatilities

$$\boldsymbol{\Sigma} = \mathrm{diag}\left(\boldsymbol{\sigma}\right)\cdot\mathbf{C}\cdot\mathrm{diag}\left(\boldsymbol{\sigma}\right).$$

$$\boldsymbol{\Sigma}^{-1} = \mathrm{diag}\left(\boldsymbol{\sigma}^{-1}\right)\cdot\mathbf{C}^{-1}\cdot\mathrm{diag}\left(\boldsymbol{\sigma}^{-1}\right).$$

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Comparison of three risk-based naïve betas

- Risk-modified weights = weight x volatility
  - MV weights inversely proportional to variance
  - MD and RP weights inversely proportional to volatility

$$W_i = \sigma_i w_i, i = 1, \cdots, N, \mathbf{W} = \boldsymbol{\sigma} \otimes \mathbf{w}.$$

$$\mathbf{W}_{\mathrm{MV}} \propto \mathbf{C}^{-1} \boldsymbol{\sigma}^{-1}.$$

$$\mathbf{W}_{\mathrm{MD}} \propto \mathbf{C}^{-1} \mathbf{i}.$$

$$\mathbf{W}_{\mathrm{RP}} \otimes \left(\mathbf{C} \mathbf{W}_{\mathrm{RP}} \right) \propto \mathbf{i}.$$

A Case Study

Comparison of three risk-based naïve betas

- Modeling two groups of securities
  - Homogeneous within each group; heterogeneous across the groups

$$\mathbf{C} = \begin{pmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{pmatrix}.$$

$$\mathbf{C}_{11} = \begin{pmatrix} 1 & \rho_1 & \cdots & \rho_1 \\ \rho_1 & 1 & \cdots & \rho_1 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_1 & \rho_1 & \cdots & 1 \end{pmatrix}_{N_1 \times N_1}, \mathbf{C}_{22} = \begin{pmatrix} 1 & \rho_2 & \cdots & \rho_2 \\ \rho_2 & 1 & \cdots & \rho_2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_2 & \rho_2 & \cdots & 1 \end{pmatrix}_{N_2 \times N_2},$$

$$\mathbf{C}_{12} = \mathbf{C}'_{21} = \rho_{12} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}_{N_1 \times N_2}.$$

A Case Study

Comparison of three risk-based naïve betas

- Modeling two groups of securities
  - Risk-modified weights are the same within each group

$$\mathbf{W} = \begin{pmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{pmatrix}, \mathbf{W}_1 = \begin{pmatrix} W_1 \\ \vdots \\ W_1 \end{pmatrix}_{N_1 \times 1}, \mathbf{W}_2 = \begin{pmatrix} W_2 \\ \vdots \\ W_2 \end{pmatrix}_{N_2 \times 1}$$

  - We are interested in the ratio of the weights for MV/MD/RP portfolios

$$\left(\frac{W_1}{W_2} \right) = ???$$

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Comparison of three risk-based naïve betas

» Ratio of risk-modified weights for the two groups

$$\left(\frac{W_1}{W_2}\right)_{\mathrm{MV}} = \frac{1+(N_2-1)\rho_2-(\sigma_1/\sigma_2)N_2\rho_{12}}{(\sigma_1/\sigma_2)\left[1+(N_1-1)\rho_1\right]-N_1\rho_{12}}.$$

$$\left(\frac{W_1}{W_2}\right)_{\mathrm{MD}} = \frac{1+(N_2-1)\rho_2-N_2\rho_{12}}{1+(N_1-1)\rho_1-N_1\rho_{12}}.$$

$$\left(\frac{W_1}{W_2}\right)_{\mathrm{RP}} = \frac{(N_1-N_2)\rho_{12}+\sqrt{\left[(N_1-N_2)\rho_{12}\right]^2+4\left[1+(N_1-1)\rho_1\right]\left[1+(N_2-1)\rho_2\right]}}{2\left[1+(N_1-1)\rho_1\right]}.$$

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Comparison of three risk-based naïve betas

- Application to the S&P 500 sectors
  - Group 1: consumer discretionary, financials, industrials, and materials
  - Group 2: consumer staples, energy, health care, technology, telecom, and utilities
  - Cross correlation

	Group 1	Group 2
N	4	6
ρ	0.78	0.40
σ	19.3%	17.8%

$$\rho_{12} = 0.51$$

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Comparison of three risk-based naïve betas

- Application to the S&P 500 sectors
  - Group 1: consumer discretionary, financials, industrials, and materials
  - Group 2: consumer staples, energy, health care, technology, telecom, and utilities
  - According to the theoretical solution, MV/MD should have zero weight in group 1 under long-only constraint
  - **The ratio should be 0.81 for RP weights, the actual ratio is 0.805**

$$\left(\frac{W_1}{W_2}\right)_{\mathrm{MV}} < 0, \left(\frac{W_1}{W_2}\right)_{\mathrm{MD}} < 0, \left(\frac{W_1}{W_2}\right)_{\mathrm{RP}} = 0.81.$$

Triumph of "Naïve Beta"

Conclusion

- EQ/MV/MD/RP can be thoughts of as "naïve beta"
  - They are naively diversified in some dimension
  - In contrast, the S&P 500 index is "smart"
  - There is a blurred line between being "smart" and "naïve"
  - "Naïve" beta beating the index is "the triumph of mediocrity"

- MV/MD portfolios are highly concentrated and sensitive to risk inputs and risk models. RP portfolio is the most diversified portfolio

- We provide a two-group correlation matrix, as a model the sector portfolios of MV/MD/RP with reasonable accuracy

Triumph of "Naïve Beta"

Some quotes

» "Prediction is very difficult, especially about the future."

» Niels Bohr

» "The fundamental cause of trouble in the world today is that the stupid are cocksure while the intelligent are full of doubt."

»

» Bertrand Russell